

O'Raifeartaigh theorem for groups containing spinor generators

B. G. Konopel'chenko

Nuclear Physics Institute, Siberian Division, USSR Academy of Sciences

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The O'Raifeartaigh theorem is generalized to the case of finite groups containing spinor generators.

As is well known, the main obstacle to relativization of internal symmetry groups is the O'Raifeartaigh theorem.^[1] It follows from this theorem that in the irreducible representations of finite Lie groups, the operator of the momentum squares can assume either a single fixed value, or a continuous spectrum of values.

In recent years, symmetry groups different from the Lie group have been considered, the so-called supersymmetry groups.^[2-5] The supersymmetry groups contain transformations whose generators are spinors with respect to the Lorentz group. Consequently, to retain the correct connection between spin and statistics we should consider the anticommutators between such generators. Since the permutation relations for groups containing spinor generators include also anticommuta-

tors, hope was expressed that one can get around the O'Raifeartaigh theorem.^[4-6]

We show in this article that the O'Raifeartaigh theorem can be generalized to include the case of arbitrary finite groups containing spinor generators.

Let us consider a finite group, whose algebra G contains with generators $J_{\mu\nu}$ and P_ρ ($\mu, \nu, \rho = 0, 1, 2, 3$). We denote the operators of the algebra G , which transform like four-component spinors, by $Q_{\alpha i}$, where α are the spinor indices ($\alpha = 1, 2, 3, 4$) and i is a set of indices, say of the internal-symmetry group, and all the remaining operators of algebra G (with the exception of $J_{\mu\nu}$ and P_ρ) are designated A_i .

We shall prove that if the momentum-squared opera-

tor $P^2 = P_\mu P_\mu$ and any power of this operator are self-adjoint operators in Hilbert space H of the representation of the algebra G , and if the spectrum of the operator P^2 in H contains a discrete point m^2 , then the space H_m belonging to the eigenvalue m^2 is closed and is invariant to the operators that represent the algebra G in H (generalized O'Raifeartaigh theorem).

The proof of the closure of H_m coincides with the analogous proof of^[1]. We shall show that the space H_m is invariant with respect to operators representing the algebra G . We consider the operators A_i . Inasmuch as the commutation relations of the operators A_i with one another and with the operator P_μ contain only commutators, it follows that the proof of Lemma I of^[1] holds for the operators A_i . Further, using the same reasoning as in^[1], we verify that the space H_m is invariant with respect to the operators that represent A_i .

Lemma I of^[1] holds true also for the spinor operators $Q_{\alpha i}$. Indeed, by virtue of the Lorentz-invariance, the commutation relations of the operators P_μ and $Q_{\alpha i}$ take the form

$$[P_\mu, Q_{\alpha i}] = \alpha (V_\mu Q)_{\alpha i}, \quad (1)$$

where V_μ ($\mu = 0, 1, 2, 3$) are 4×4 matrices and α is an arbitrary number (in particular, zero). Recognizing that $[P_\mu, P_\nu] = 0$, we obtain from the Jacobi identities for the operators P_μ and $Q_{\alpha i}$

$$[P_\mu, [P_\nu, Q_{\alpha i}]] + [P_\nu, [Q_{\alpha i}, P_\mu]] = 0. \quad (2)$$

It follows from (1) and (2) that

$$V_\mu V_\nu = 0, \quad \mu, \nu = 0, 1, 2, 3, \quad (3)$$

i. e. ,

$$V_\mu \sim (1 \pm \gamma_5) \gamma_\mu,$$

where γ_μ are the Dirac γ matrices and $\gamma_5^2 = 1$. Consequently

$$[P_\mu, [P_\nu, Q_{\alpha i}]] = 0,$$

which is in fact a statement of Lemma I of^[1] for the operators $Q_{\alpha i}$. Next, using Lemma II of^[1], we verify, in analogy with^[1], that the space H_m is invariant with respect to the operators that represent $Q_{\alpha i}$.

Thus, the space H_m is invariant with respect to all the operators of algebra G . This proves the theorem.

It follows from the generalized O'Raifeartaigh theorem that in the irreducible representations of arbitrary finite symmetry groups (whether they do or do not contain spinor generators) the operator P^2 assumes either one fixed value or a continuous spectrum of values. By the same token, inclusion of the spinor operators in the symmetry group does not afford a possibility of explaining the particle-mass spectrum within multiplets.

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