Liquid with gas bubbles as an example of a Korteweg–de Vries–Burgers medium

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It is shown experimentally that an initial momentum in a liquid with gas bubbles evolves in accordance with the Korteweg–de Vries–Burgers equation. By choosing dimensionless complexes that characterize the process, it was possible to observe the propagation of the signal in the form of monotonic and oscillatory shock waves, solitons, and wave packets.

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The equation for the perturbation of the velocity of the liquid with bubbles, obtained in[1,2] in the WKB approximation, is

$$\frac{\partial u}{\partial \tau}+u\frac{\partial u}{\partial \xi}-\frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial \xi^2}+\frac{1}{\sigma^2}\frac{\partial^3 u}{\partial \xi^3}=0 \tag{1}$$

where $u=V/a$ is the dimensionless velocity of the perturbation. In the case of a volume gas content $\phi>10^{-3}$, the connection between V and the true perturbation of the velocity v in the mixture takes the form

$$V=\left[1+\frac{\gamma+1}{2\phi_0}\right]v,$$

where γ is the adiabatic exponent of the gas; a is the amplitude of the initial perturbation; $\tau=at/l_0$ is the dimensionless time; l_0 is the width of the initial perturbation; $\xi=(x-c_0t)/l_0$ is the coordinate; c_0 is the speed of sound; $\mathrm{Re}=al_0/\nu$; ν is the coefficient of the effective viscosity of the mixture, due to viscous, acoustic, and thermal losses produced by pulsations of single inclusions; $\sigma=(\alpha/\beta)^{0.5}l_0$ is a dimensionless complex characterizing the relation between the width of the perturbation, the magnitude of the dispersion, and the level of the perturbation (nonlinearity); $\beta=R_0^2c_0/6\phi_0(1-\phi_0)$, where R_0 is the radius of the equilibrium bubble. The equation for the dimensionless perturbation of the pressure is similar in form.

It is known[3] that at $\mathrm{Re}^{-1}=0$ there exists a value $\sigma=\sigma_c$ such that the character of the solution of Eq. (1) changes. The solution of (1) is represented by solitary waves at $\sigma>\sigma_c$, and at $\sigma\ll\sigma_c$ by wave packets. The presence of dissipation in a gas liquid medium changes somewhat the situation, namely, at $\mathrm{Re}^{-1}\neq 0$ Eq. (1) has as its solution shock waves with oscillating or monotonic structures, depending on the ratio σ/Re. In the case $\sigma/\mathrm{Re}\ll 1$ at $\sigma\ll\sigma_c$ we can expect the initial perturbation to become transformed into a wave packet.

To ascertain the character of the propagation of the perturbations in a gas-liquid mixture in the region of small $\sigma\ll\sigma_c$ and intermediate $\sigma\sim\sigma_c$, experiments were performed with the setup described in[4], constituting a typical shock tube for a gas-liquid mixture. Much smaller σ than in the previous

TABLE I.

Figure	L,M	σ	σ/Re	Characteristic of signal
1a	0	12.1	---	Initial pulse
1b	0.6	---	0.05	Shock wave
1c	1.4	52.8	0.015	Multisoliton perturbation
1d	1.4	30	0.021	Two-soliton perturbation
1e	0.6	12.1	0.034	Single soliton
1f	0.6	3.4	0.071	Wave packet

investigations were attained by decreasing the length of the initial perturbation via shortening the volume of the high-pressure cell. Piezoelectric pickups were placed along the working section at distances L from the entrance of the signal into the medium. The experiments were performed with CO_2 and He bubbles, so that the number Re could be varied by one order of magnitude by changing ν. The working liquid was a water-glycerine solution with density 1.18×10^3 kg/m^3.

We note that the parameter σ could be varied also by varying the initial amplitude of the perturbation. The contraints on the parameters of the experiments were the conditions of the WKB approximation, namely, small amplitude and large lengths of the perturbations, including a sufficient number of bubbles.

The value of σ_c can be determined by the method proposed by Berezin and Karpman.[3] For an initial perturbation in the form of a triangle (Fig. 1a), we have $\sigma_c = 14$.

Figure 1 shows characteristic oscillograms of the perturbations of the pressure for CO_2, while Table I lists the main characteristics of the perturbation and of the medium.

The ratio of the similarity parameter to the Reynolds number is much less than unity, and the evolution of the process agrees well with the main conclusions of the theory of the Korteweg—de Vries equation. At $\sigma < \sigma_c$, the perturbation propagates in the form of a wave packet (Fig. 1f), and at $\sigma > \sigma_c$ the initial pulse breaks up into solitons. The number of solitons is in full agreement with the value of the dispersion parameter. The shape of the soliton is described by the relation

$$\Delta P(x)/\Delta P = \operatorname{sech}^2(x/\delta) \tag{2}$$

and the half-width δ of the soliton is given by

$$\delta = \left(\frac{4\gamma}{\gamma+1}\right)^{0.5} \frac{R_0}{\phi_0^{0.5}} \left(\frac{P_0}{\Delta P}\right)^{0.5}, \tag{3}$$

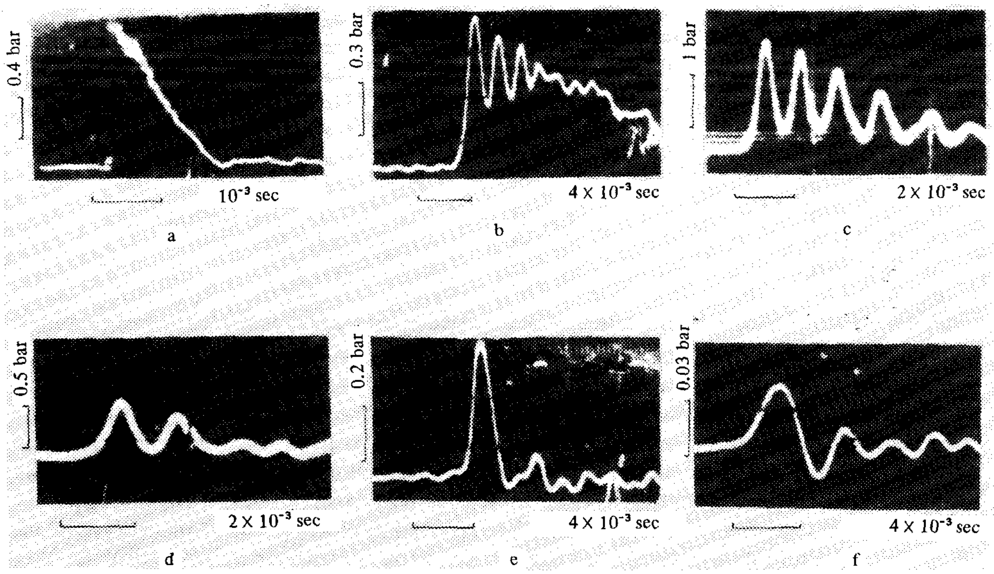

FIG. 1. Evolution of pressure perturbation in a liquid with CO_2 bubbles.

where ΔP is the amplitude of the pressure perturbation and P_0 is the initial pressure in the medium.

A test of relations (2) and (3) is shown in Fig. 2. We present here data on a comparison of the form of the soliton from experiment 1e with calculation in

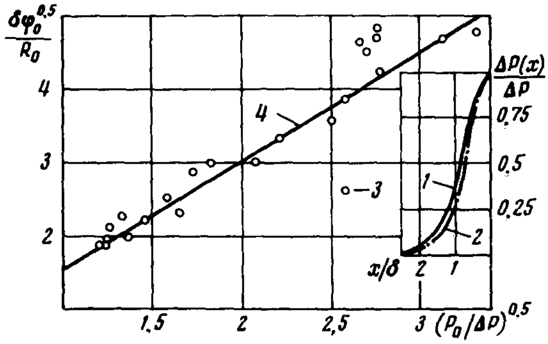

FIG. 2. Dependence of the soliton half-width on its amplitude.

accordance with formula (2), and data on the dependence of the width of the soliton on its amplitude are compared with calculation in accordance with formula (3). Figure 3 shows oscillograms characterizing the evolution of a triangular pulse in the case when the parameter σ/Re is large enough. The data pertaining to the oscillograms are given in Table II. The soliton produced in Figs. 3a and 3b tends to spread out and acquire the form of a "Burgers triangle."

At small values of the parameter (Figs. 3c and 3d) the formation of the wave packet is barely discernible, and the initial pulse spreads out, apparently in accordance with the solution of the linear Burgers equation.

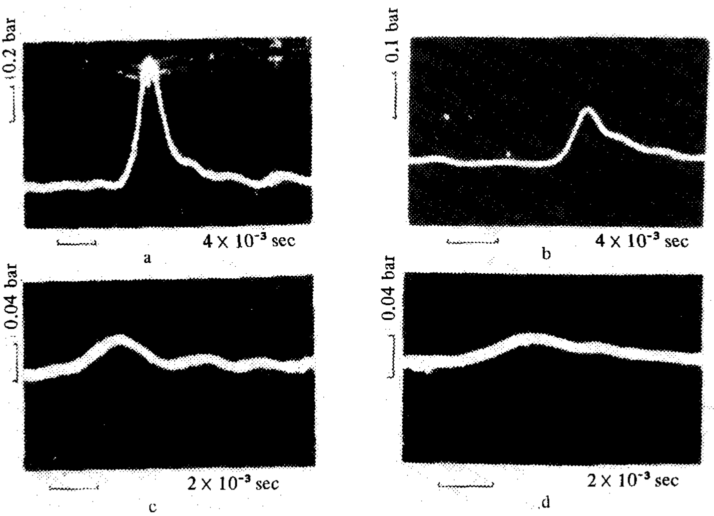

FIG. 3. Evolution of single soliton (a,b) and of a wave packet (c,d) in a liquid with He bubbles.

TABLE II.

Figure	L, M	σ	σ/Re
3a	0.6	13.1	0.21
3b	1.4	13.1	0.21
3c	0.6	2.2	0.46
3d	1.15	2.2	0.46

The results of this paper can be used to analyze the dynamics of the passage of waves through two-phase media and in simulation of processes in a plasma with the aid of processes in gas-liquid media.

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