Bremsstrahlung of electrons from ions with allowance for screening

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Exact numerical calculations were performed of the bremsstrahlung from a multielectron ion with allowance for screening, using a Thomas-Fermi potential. The results lead to the conclusion that the methods previously developed for calculating bremsstrahlung {V.D. Kirillov *et al.*, Fiz. Plazmy **1**, 218 (1975) [Sov. J. Plasma Phys. **1**, 117 (1975)]; V. I. Gervids and V. I. Kogan, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 308 (1975) [JETP Lett. **22**, 142 (1975)]} are accurate.

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Introduction. Bremsstrahlung of electrons of energy 10^2–10^3 eV from ions of heavy elements was considered in earlier studies.[1–5] Most of these studies pertain to bremsstrahlung from neutral fully ionized atoms.[3–5] Bremsstrahlung from partially-ionized atoms was considered in two recent papers.[1,2] Kirillov *et al.*[1] used a semi-Born approximation and took screening into account with the aid of a Thomas-Fermi-like model potential. [Gervids and Kogan[2] developed a semiclassical approach with allowance for screening with the aid of the Sommerfeld approximation of the Thomas-Fermi potential.]

The most interesting case corresponding to thermodynamic equilibrium, when the electron energy $-V^2/2$ (the atomic system of units is used) is close to the ionization potential $Z_i^2/2$ of an ion with charge Z_i. The main contribution to the bremsstrahlung is made by approach distances $r \sim 1/V$, when the relations between these quantities are $V \sim Z_i$ and $r \sim 1/V$, the potential energy is close to the kinetic energy, and therefore the Born approximation cannot be used; the quasiclassical condition, which is the basis of the semiclassical theory,[2] is violated to an equal degree, since $|d\lambda/dr| \sim 1$. The total bremsstrahlung intensity obtained in[2] was therefore smaller by one order of magnitude than the total bremsstrahlung intensity obtained in[1] in the main energy region. The violation of the criteria of the applicability of the quasiclassical and Born approximations and the large difference between the results obtained in[1,2] make is necessary to carry out exact quantum calculations of the bremsstrahlung.

Formulation of problem. In the nonrelativistic case $V/c \ll 1$ (c is the speed of light), the dipole-radiation intensity of protons with frequency ω, following scattering of an electron, into a solid-angle element $d\theta_{V_1}$, is equal to[6]

$$\frac{dW}{d\omega} = \frac{4\omega^4 V_1}{3(2\pi c)^3 V} \int |< \psi_1^* | \mathbf{r} | \psi >|^2 d\theta_{V_1}, \tag{1}$$

where V and ψ are the velocity and wave function prior to the radiation, V_1 and ψ_1 are the velocity of wave function after the radiation in the case of scattering to a solid-angle element $d\theta_{V_1}$. Using the relation between the dipole matrix element and the acceleration of the matrix element, the spherical symmetry of the ion potential $U(r)$, and the theorem for the addition of Legendre polynomials, we transform (1) into

$$\frac{dW}{d\omega} = \frac{32 V_1}{3c^3 V} \sum_{l=0}^{\infty} (l+1)\left[|M_{l,l+1}|^2 + |M_{l+1,l}|^2\right],$$
$$M_{l,l'} = \int_0^{\infty} \phi_l(V,r)\,\phi_{l'}(V_1,r)\,\frac{dU}{dr}\,dr, \tag{2}$$

where $\phi_l(V,r)$ are solutions of the radial equations, with

$$\phi_l(V,r)_{r\to\infty} \to \frac{1}{V}\sin\left(Vr + \frac{Z_i}{V}\ln 2Vr - \frac{\pi l}{2} + \delta_l\right).$$

The matrix elements M in formulas (2) were obtained by numerically integrating the radial Schrödinger equations, the boundary conditions being specified near the origin by expanding the functions $\phi_l(V,r)$ in powers of r. We took into account angular momenta with $l \lesssim 8$. To take screening into account, we used exact Thomas-Fermi potentials for the ions $U(r) = -Z\Phi(r)/r + V_0$, where Z is the charge of the nucleus, V_0 is the potential at the boundary of the ion, and Φ is the Thomas-Fermi function for an ion with given degree of ionization.

Results and discussion. We have calculated numerically the bremsstrahlung intensity for electron collisions in the energy range 10—3 keV with ions of the Fe atom, with different degree of ionization. The results for singly and five- and 10-fold ionized iron are shown in Fig. 1. The accuracy of the obtained curves is monitored against pure Coulomb numerical calculations by the Sommerfeld formula[7] and amounts to 1—3% at $\omega/E \geq 0.25$. The radiation of the low-frequency photons receives contributions from many angular momenta, the sum (2) converges slowly, and therefore at $\omega/E \leq 0.25$ the accuracy is worse and amounts to 3—10%. Figure 2 shows the total intensity W of the bremsstrahlung for the Fe^+, $W = \int_0^E (dW/d\omega)\,d\omega$. At low velocities, the bremsstrahlung intensity is close to that from a Coulomb center with charge E_i,

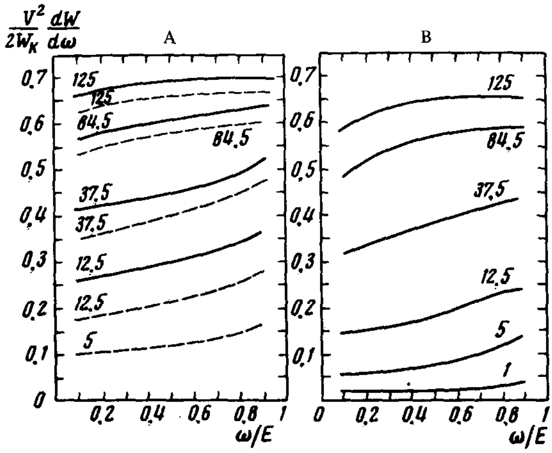

FIG. 1. Bremsstrahlung intensity multiplied by the electron intensity and divided by the total intensity in the Kramers approximation[8] $W_K = 8\pi Z^2/3\sqrt{3}c^3$. The plots are marked with the electron energies in atomic units. A—solid curves for the ion Fe^{10+}, dashed—for the ion Fe^{5+}. B—curves for the ion Fe^+.

and when the velocity is increased the screening decreases and the bremsstrahlung intensity approaches the intensity for a Coulomb center with charge Z.

Our calculations of the total density agree well (10—30%) with the results of the semiclassical approximation[2] (see, e.g., Fig. 2), thus offering proof in favor of the semiclassical approach in comparison with the semi-Born approach.[1] It must be noted, however, that the bremsstrahlung spectral

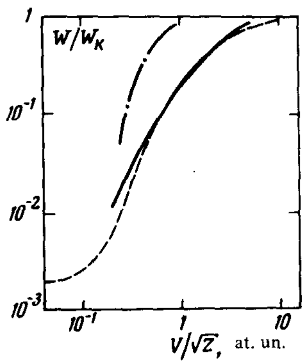

FIG. 2. Ratio of the total intensity of bremsstrahlung from Fe^+ ion to the total intensity from a bare nucleus in the Kramers approximation. Solid curve—our calculation, dashed—calculations of[2], dash dot—averaging of the results of[1] over a Maxwellian distribution.

intensities given in[2] seem to contain computational errors, since they do not agree with the results obtained for the total intensity by the same authors, and are much smaller (by 2—3 times) than our results.

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