# Viscosity of vortices in pure superconductors

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The vortex viscosity at low temperatures was obtained for pure superconductors with electron mean free path $l$ satisfying the conditions $vT^{-1} << l$, $v\epsilon_F T_c^{-2}$. Accurate to a factor $\ln(\Delta/T)$, the results correspond to the assumption that there exist normal regions with dimensions on the order of $\xi_0 = v/T_c$.

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A rough estimate of the viscosity of the vortices in the mixed state can be obtained by assuming that the internal part of the vortex is in the normal state.[1] In relatively pure superconductors, the vortex dimension at low temperature decreases in proportion to temperature.[2] However, as will be shown below, this decrease of the vortex dimension does not lead to a decrease in viscosity. The reason is that in kinetic phenomena the role of the vortex dimension is played by the distances to which the excitations penetrate. The dimension of the region in which the order parameter is small turns out to be less significant.

The vortex viscosity $\eta$ is connected with the conductivity $\sigma$ by the relation

$$\eta = \pi B\sigma/e.$$

To calculate the conductivity it is necessary to derive a system of equations for the slowly varying corrections to the order parameter and to the vector potential. This system can be written in the form

$$\left(\hat{L} + \hat{K}\frac{\partial}{\partial t}\right)(\Delta_1^{(1)}, \Delta_2^{(1)}, \mathbf{A}^{(1)}) = 0, \tag{1}$$

where the matrix $\hat{L}$ denotes second variational derivatives of the energy release with respect to $\Delta$ and $\mathbf{A}$. The matrix $\hat{K}$ can be obtained from the microscopic theory of superconductivity. The average current density $j$ is expressed in terms of $\hat{K}$ by the formula[3,4]

$$B^2\mathbf{j} = <([\mathbf{B}\times\partial_+\Delta^*], [\mathbf{B}\times\partial_-\Delta], (\mathbf{BH}))\hat{K}(\mathbf{u}\partial_-\Delta, \mathbf{u}\partial_+\Delta^*, [\mathbf{H}\times\mathbf{u}])>, \tag{2}$$

where $\mathbf{B}$ is the mean value of the magnetic field, and $\mathbf{u}$ is the vortex velocity connected with the mean value of the electric field by the relation $\mathbf{E} = \mathbf{B}\times\mathbf{u}$, and $\partial_\pm = \partial/\partial\mathbf{R} \pm 2ieA$.

The deviations, linear in the electric field, of the Green's functions $G^1$ satisfy the relations[4]

$$\mathbf{v}\frac{\partial G^1}{\partial\mathbf{R}} + \hat{\omega}_+ G^1 - G^1\hat{\omega} = -\hat{V}G + G_+\hat{V},$$

$$\hat{V} = -ie\mathbf{v}\mathbf{A}_1\tau_z - i\hat{\Delta}^{(1)} + ie\phi + \frac{1}{2\tau}\int\frac{d\Omega_{\mathbf{v}_1}}{4\pi}G^1(\mathbf{v}_1); \tag{3}$$

  

where

$$\hat{\omega} = \omega \hat{\tau}_z - iev\mathbf{A}\tau_z - i\hat{\Delta} + \frac{1}{2\tau}\int \frac{d\Omega_{v_1}}{4\pi} G(\omega, v_1),$$

$\hat{\omega}_+$ is obtained from $\hat{\omega}$ by the substitution $\omega \to \omega + \omega_0$, $\omega_0$ is the frequency of the external field, $\tau$ is the electron free path time. The Green's functions $G$ are obtained by solving the static problem.[2]

The expressions for the matrix elements of $\hat{L}$ and $\hat{K}$ can be obtained from the Green's function $G^1$ by summing over the frequencies $\omega$ and by analytic continuation in $\omega_0$. The main contribution to the conductivity is made by the frequency region in which $-\operatorname{sign}\omega = \operatorname{sign}(\omega + \omega_0)$. From (2) and (3), carrying out an analytic continuation in $\omega_0$, we obtain

$$B^2 \mathbf{j} = \frac{imp}{8\pi^2} \int \frac{d\Omega_v}{4\pi} \int_{-\infty}^{\infty} d\epsilon \frac{\partial \operatorname{th}\frac{\epsilon}{2T}}{\partial \epsilon} \operatorname{Sp} < [\mathbf{B}\left(\frac{\partial}{\partial \mathbf{R}} - 2ie\mathbf{A}\tau_z\right)\hat{\Delta}] G^1 >, \tag{4}$$

We have left out of (4) the terms proportional to $(BH)$, since they are small at low temperatures $T \ll T_c$ at arbitrary values of the Ginzburg-Landau parameter $\kappa$. In the principal approximation in the large electron mean free path, the solution of the system (3) can be obtained by the method of classical linear trajectories, which is used when solving the static problem.[2] As a result we get

$$G^1 = C(\mathbf{v}, x)\exp(-2\Delta|t|)\begin{pmatrix} 1; & -i\exp[i(\varphi + \eta(t))] \\ i\exp[i(-\varphi + \eta(-t))]; & -1 \end{pmatrix}. \tag{5}$$

The coefficient $C$ depends on the parameters that determine the trajectory, the velocity $\mathbf{v}$ on the trajectory, and the distance $x$ from the vortex axis to the trajectory. In formula (5), $vt$ is the path along the trajectory, measured from the point closest to the vortex axis

$$\eta(t) = 2e\int_{-\infty}^{t} (\mathbf{vQ})dt_1; \quad \mathbf{Q}(\mathbf{R}) = \mathbf{A}(\mathbf{R}) - \frac{1}{2e}\frac{\partial \varphi}{\partial R}; \qquad \mathbf{R}(t) = \mathbf{x} + \mathbf{v}t.$$

The coefficient $C$ which is independent of $t$, can be obtained by taking the trace and the interval over all $t$ of Eq. (3). Then the large terms that do not depend on the path length cancel out in the left-hand side of the equation. The result is an integral equation connecting the values of the coefficient $C$ on the different trajectories. With logarithmic accuracy, this equation reduces to an algebraic one and we obtain for the coefficient $C$

$$C(\mathbf{v}, x) = 4i\tau\Delta_0 (\mathbf{vE})\delta(x)\operatorname{sign}\epsilon/Bv. \tag{6}$$

Substituting (5) and (6) in formula (4), we obtain for the conductivity $\sigma$ the expression

$$\sigma = emp_F\tau\Delta_0^2 \ln\left(\frac{\Delta_0}{T}\right)\Big/2\pi^2 B \quad . \tag{7}$$

Expressing $\Delta_0$ in terms of the critical magnetic field $H_{c2}$, we reduce formula (7) to the form

$$\sigma/\sigma_0 = 0.23(H_{c2}/B)\ln\left(\frac{\Delta_0}{T}\right), \tag{8}$$

where $\sigma_0$ is the conductivity of the normal metal.

In the derivation of (7) it was assumed, just as in[2], that $\Delta(\rho)$ reaches its limiting value $\Delta_0$ at distances $\xi_1 \sim vTT_c^{-2} \ll \xi_0$. If this is not the case, then $\Delta_0$ in (7) must be replaced by $\Delta(\rho_1)$, where $\xi_1 \ll \rho_1 \ll \xi_0$. This leads to a change of the numerical coefficient in (8). The numerical coefficient can change when account is taken of the anisotropy of the scattering by the impurities and of the electron velocity on the Fermi surface.

In a recent paper,[5] Bardeen and Sherman obtained from simple phenomenological considerations an expression for the conductivity; this expression differs from (8) by a numerical factor 4/3. The reason for this difference is that the effective time between the collisions of the excitations with the impurities does not coincide with the free path time of the electrons in the normal metal.

As noted above, the obtained expressions are valid for sufficiently pure superconductors, in which $T\tau \gg 1$. The superconductor, however, must not be too pure, so as to make the reciprocal time between collision larger than the distance between the quantum levels in the vortex. As shown in[2], this distance is $\delta\epsilon \sim \Delta^2/\epsilon_F$ and coincides in order of magnitude with the distance between the levels in the normal metal in the magnetic field $H_{c2}$. For purer samples, the conductivity in a magnetic field, whether of a superconductor or of a normal metal, should decrease with increasing electron mean free path. Pure niobium, which is a type-II superconductor, is used for resonator manufacture.[6] It is possible that at low temperatures the $Q$ of these resonators is determined by the viscous motion of the vortices. We do not know, however, of any quantitative measurements of the viscosity of vortices in pure superconductors.

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