

Magnetoresistance of highly correlated electron liquid

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The behavior in magnetic fields of a highly correlated electron liquid approaching the fermion condensation quantum phase transition from the disordered phase is considered. We show that at sufficiently high temperatures $T \geq T^*(x)$ the effective mass starts to depend on T, $M^* \propto T^{-1/2}$. This $T^{-1/2}$ dependence of the effective mass at elevated temperatures leads to the non-Fermi liquid behavior of the resistivity, $\rho(T) \propto T$ and at higher temperatures $\rho(T) \propto T^{3/2}$. The application of a magnetic field B restores the common T^2 behavior of the resistivity. The effective mass depends on the magnetic field, $M^*(B) \propto B^{-2/3}$, being approximately independent of the temperature at $T \leq T^*(B) \propto B^{4/3}$. At $T \geq T^*(B)$, the $T^{-1/2}$ dependence of the effective mass is re-established. We demonstrate that this $B-T$ phase diagram has a strong impact on the magnetoresistance (MR) of the highly correlated electron liquid. The MR as a function of the temperature exhibits a transition from the negative values of MR at $T \to 0$ to the positive values at $T \propto B^{4/3}$. Thus, at $T \geq T^*(B)$, MR as a function of the temperature possesses a node at $T \propto B^{4/3}$.

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An explanation of the rich and striking behavior of the strongly correlated electron liquid in heavy fermion metals and high-temperature superconductors are, as years before, among the main problems of the condensed matter physics. There is a fundamental question about whether or not these properties can be understood within the framework of the Landau Fermi liquid theory [1]. The basis of the Landau theory is the assumption that the excitation spectrum of the Fermi liquid looks like the spectrum of an ideal Fermi gas. This excitation spectrum is described in terms of quasiparticles with an effective mass M^*, charge e and spin $1/2$. The single-particle excitations, or quasiparticles, define the major part of the low-temperature properties of Fermi liquids. The stability of the ground state of a Landau liquid is determined by the Pomeranchuk stability conditions. The stability is violated when at least one of the Landau effective interaction parameters becomes negative and reaches a critical value. The new phase at which the stability conditions are restored is again described within the framework of the same theory. The Pomeranchuk conditions do not cover all the possible instabilities. The missed instability corresponds to the situation when, at the temperature $T = 0$, the effective mass, the most important characteristic of Landau quasiparticles, can become infinitely large. Such a situation, leading to profound consequences, can take place when the corresponding Landau amplitude being repulsive reaches some critical value. This leads to a completely new class of a strongly correlated Fermi liquids with the fermion condensate (FC) [2, 3], which is separated from that of a normal Fermi liquid by the fermion condensation quantum phase transition (FCQPT) [4, 5].

As any phase transition, the quantum phase transition is driven by a control parameter and is related to the order parameter, which describes a broken symmetry. In our case, the control parameter is the density x of a system and the order parameter is $\kappa(\mathbf{p})$. The existence of the FC state can be revealed experimentally. Since the order parameter $\kappa(\mathbf{p})$ is suppressed by a magnetic field B, a weak magnetic field B will destroy the state with FC, converting the strongly correlated Fermi liquid into the normal Landau Fermi liquid [6]. In this case the magnetic field plays a role of the control parameter. The transition from the strongly correlated liquid into the normal Landau liquid was observed in several experiments [7 – 10]. As soon as FCQPT occurs at the critical point $x = x_{FC}$, the system becomes divided into two quasiparticle subsystems: the first subsystem is characterized by the quasiparticles with the effective mass M^*_{FC}, while the second one is occupied by quasiparticles with mass M^*_L. The quasiparticle dispersion law in systems with FC can be represented by two straight lines, characterized by effective masses M^*_{FC} and M^*_L, and intersecting near the binding energy E_0.

Properties of these new quasiparticles with M^*_{FC} are closely related to the state of the system which is characterized by the temperature T, pressure or by the pres-

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ence of the superconductivity. We may say that the quasiparticle system in the range occupied by FC becomes very "soft" and is to be considered as a strongly correlated liquid. Nonetheless, the basis of the Landau Fermi liquid theory survives FCQPT: the low energy excitations of a strongly correlated liquid with FC are quasiparticles. The only difference between the Landau Fermi-liquid and Fermi-liquid after FCQPT is that we have to expand the number of relevant low energy degrees of freedom by introducing a new type of quasiparticles with the effective mass M^*_{FC} and the energy scale E_0 [4, 11].

When a Fermi system approaches FCQPT from the disordered phase it remains the Landau Fermi liquid with the effective mass M^* strongly depending on the density $x_{FC} - x$, temperature and a magnetic field B provided that $|x_{FC} - x|/x_{FC} \ll 1$ and $T \geq T^*(x)$ [12]. This state of the system, with M^* strongly depending on T, x and B, resembles the strongly correlated liquid. In contrast with a strongly correlated liquid, there is no the energy scale E_0 and the system under consideration is the Landau Fermi liquid at sufficiently low temperatures with the effective mass $M^* \simeq$ const. Therefore this liquid can be called a highly correlated liquid. Obviously, a highly correlated liquid has to have uncommon properties.

In this Letter, we study the behavior of a highly correlated electron liquid in a magnetic field. We show that at $T \geq T^*(x)$ the effective mass starts to depend on the temperature, $M^* \propto T^{-1/2}$. This $T^{-1/2}$ dependence of the effective mass at elevated temperatures leads to the non-Fermi liquid behavior of the resistivity, $\rho(T) \sim \rho_0 + aT + bT^{3/2}$. The application of magnetic field B restores the common T^2 behavior of the resistivity, $\rho \simeq \rho_0 + AT^2$ with $A \propto (M^*)^2$. Both the effective mass and coefficient A depend on the magnetic field, $M^*(B) \propto B^{-2/3}$ and $A \propto B^{-4/3}$ being approximately independent of the temperature at $T \leq T^*(B) \propto B^{4/3}$. At $T \geq T^*(B)$, the $T^{-1/2}$ dependence of the effective mass is re-established. We demonstrate that this $B - T$ phase diagram has a strong impact on the magnetoresistance (MR) of the highly correlated electron liquid. The MR as a function of the temperature exhibits a transition from the negative values of MR at $T \to 0$ to the positive values at $T \propto B^{4/3}$. Thus, at $T \geq T^*(B)$, MR as the function of the temperature possesses a node at $T \propto B^{4/3}$. Such a behavior is of a general form and takes place in both three dimensional (3D) highly correlated systems and two dimensional (2D) ones.

At $|x - x_{FC}|/x_{FC} \ll 1$ and $T \to 0$, the effective mass M^* of a highly correlated electron liquid is given by the equation [12]

$$M^* \sim M \frac{x_{FC}}{x - x_{FC}}. \tag{1}$$

It follows from Eq. (1) that effective mass is finite provided that $|x - x_{FC}| \equiv \Delta x > 0$. Therefore, the system represents the Landau Fermi liquid. On the other hand, M^* diverges as the density x tends to the critical point of FCQPT. As a result, the effective mass strongly depends on such a quantities as the temperature, pressure, magnetic field provided that they exceed their critical values. For example, when T exceeds some temperature $T^*(x)$, Eq. (1) is no longer valid, and M^* depends on the temperature as well. To evaluate this dependence, we calculate the deviation $\Delta x(T)$ generated by T. The temperature smoothing out the Fermi function $\theta(p_F - p)$ at p_F induces the variation $p_F \Delta p/M^*(x) \sim T$, and $\Delta x(T)/x_{FC} \sim M^*(x)T/p_F^2$, with p_F is the Fermi momentum and M is the bare electron mass. The deviation Δx can be expressed in terms of $M^*(x)$ using Eq. (1), $\Delta x/x_{FC} \sim M/M^*(x)$. Comparing these deviations, we find that at $T \geq T^*(x)$ the effective mass depends noticeably on the temperature, and the equation for $T^*(x)$ becomes

$$T^*(x) \sim p_F^2 \frac{M}{(M^*(x))^2} \sim \varepsilon_F(x) \left(\frac{M}{M^*(x)}\right)^2. \tag{2}$$

Here $\varepsilon_F(x)$ is the Fermi energy of noninteracting electrons with mass M. It follows from Eq. (2) that M^* is always finite at temperatures $T > 0$. At $T \geq T^*(x)$, the main contribution to Δx comes from the temperature, therefore

$$M^* \sim M \frac{x_{FC}}{\Delta x(T)} \sim M \frac{\varepsilon_F}{M^* T}. \tag{3}$$

As a result, we obtaian

$$M^*(T) \sim M \left(\frac{\varepsilon_F}{T}\right)^{1/2}. \tag{4}$$

Equation (4) allows us to evaluate the resistivity as a function of T. There are two terms contributing to the resistivity. Taking into account that $A \sim (M^*)^2$ and Eq. (4), we obtain the first term $\rho_1(T) \sim T$. The second term $\rho_2(T)$ is related to the quasiparticle width γ. When $M/M^* \ll 1$, the width $\gamma \propto (M^*)^3 T^2/\varepsilon(M^*) \propto T^{3/2}$, with $\epsilon(M^*) \propto (M^*)^2$ is the dielectric constant [5, 13]. Combining both of the contributions, we find that the resistivity is given by

$$\rho(T) - \rho_0 \sim aT + bT^{3/2}. \tag{5}$$

Thus, it turns out that at low temperatures, $T < T^*(x)$, the resistivity $\rho(T) - \rho_0 \sim AT^2$. At higher temperatures, the effective mass depends on the temperature and the main contribution comes from the first term on the right hand side of Eq. (5). At the same time, $\rho(T)-\rho_0$ follows the $T^{3/2}$ dependence at elevated temperatures.

In the same way as Eq. (4) was derived, we can obtain the equation determining $M^*(B)$ [12]

$$M^*(B) \sim M\left(\frac{\varepsilon_F}{B\mu}\right)^{2/3}, \qquad (6)$$

where μ is the electron magnetic moment. We note that M^* is determined by Eq. (6) as long as $M^*(B) \leq \leq M^*(x)$, otherwise we have to use Eq. (1). It follows from Eq. (6) that the application of a magnetic field reduces the effective mass. Note, that if there exists an itinerant magnetic order in the system which is suppressed by magnetic field $B = B_{c0}$, Eq. (6) has to be replaced by the equation [6],

$$M^*(B) \propto \left(\frac{1}{B-B_{c0}}\right)^{2/3}. \qquad (7)$$

The coefficient of T^2 in the expression for the resistivity $A(B) \propto (M^*(B))^2$ diverges as

$$A(B) \propto \left(\frac{1}{B-B_{c0}}\right)^{4/3}. \qquad (8)$$

At elevated temperature, there is a temperature $T^*(B)$ at which $M^*(B) \simeq M^*(T)$. Comparing Eq. (4) and Eq. (7), we see that $T^*(B)$ is given by

$$T^*(B) \propto (B-B_{c0})^{4/3}. \qquad (9)$$

At $T \geq T^*(x)$, Eq. (9) determines the line in the $B-T$ phase diagram which separates the region of the B dependent effective mass from the region of the T dependent effective mass. At the temperature $T^*(B)$, there occurs a crossover from the T^2 dependence of the resistivity to the T dependence: at $T < T^*(B)$, the effective mass is given by Eq. (7), and at $T > T^*(B)$ M^* is given by Eq. (4).

Using the $B-T$ phase diagram just described above, we consider the behavior of MR

$$\rho_{mr}(B,T) = \frac{\rho(B,T)-\rho(0,T)}{\rho(0,T)}, \qquad (10)$$

as a function of magnetic field B and T. Here $\rho(B,T)$ is the resistivity measured at the magnetic field B and temperature T. We assume that the contribution $\Delta\rho_{mr}(B)$ coming from the magnetic field B can be treated within the low field approximation and given by the well-known Kohler's rule,

$$\Delta\rho_{mr}(B) \sim B^2\rho(0,\Theta_D)/\rho(0,T), \qquad (11)$$

with Θ_D is the Debye temperature. Note, that the low field approximation implies that $\Delta\rho_{mr}(B) \ll \rho(0,T) \equiv \equiv \rho(T)$. Substituting Eq. (11) into Eq. (10), we find that

$$\rho_{mr}(B,T) \sim \sim \frac{c(M^*(B,T))^2T^2 + \Delta\rho_{mr}(B) - c(M^*(0,T))^2T^2}{\rho(0,T)}. \qquad (12)$$

Here $M^*(B,T)$ denotes the effective mass M^* which now depends on both magnetic field and the temperature, and c is a constant.

Consider MR given by Eq. (12) as a function of B at some temperature $T = T_0$. At low temperatures $T_0 \leq \leq T^*(x)$, the system behaves as common Landau Fermi liquid, and MR is an increasing function of B. When the temperature T_0 is sufficiently high, $T^*(B) < T_0$, and the magnetic field is small, $M^*(B,T)$ is given by Eq. (4). Therefore, the difference $\Delta M^* = |M^*(B,T)-M^*(0,T)|$ is small and the main contribution is given by $\Delta\rho_{mr}(B)$. As a result, MR is an increasing function of B. At elevated B, the difference ΔM^* becomes a decreasing function of B, and MR as the function of B reaches its maximum value at $T^*(B) \sim T_0$. In accordance with Eq. (9), $T^*(B)$ determines the crossover from T^2 dependence of the resistivity to the T dependence. Differentiating the function $\rho_{mr}(B,T)$ given by Eq. (12) with respect to B, one can verify that the derivative is negative at sufficiently large values of the magnetic field when $T^*(B) \simeq T_0$. Thus, we are led to the conclusion that the crossover manifests itself as the maximum of MR as the function of B.

We now consider MR as a function of T at some B_0. At low temperatures $T \ll T^*(B)$, it follows from Eqs. (4) and (7) that $M^*(B)/M^*(T) \ll 1$, and MR is determined by the resistivity $\rho(0,T)$. Note, that B has to be comparatively high to ensure the inequality, $T^*(x) \leq T \ll T^*(B)$. As a result, MR tends to -1, $\rho_{mr}(B_0, T \to 0) \simeq -1$. Differentiating the function $\rho_{mr}(B_0,T)$ with respect to B_0 we can check that its slope becomes steeper as B_0 is decreased, being proportional $\propto (B_0 - B_{c0})^{-7/3}$. At $T = T_1 \sim T^*(B_0)$, MR possesses a node because at this point the effective mass $M^*(B) \simeq M^*(T)$, and $\rho(B,T) \simeq \rho(0,T)$. Again, we can conclude that the crossover from the T^2 resistivity to the T resistivity, which occurs at $T \sim T^*(B_0)$, manifests itself in the transition from negative MR to positive MR. At $T > T^*(B)$, the main contribution in MR comes from $\Delta\rho_{mr}(B)$, and MR reaches its maximum value. Upon using Eq. (11) and taking into account that at this point T has to be determined by Eq. (9), $T \propto (B_0 - B_{c0})^{4/3}$, we obtain that the maximum value

$\rho^m_{mr}(B_0)$ of MR is $\rho^m_{mr}(B_0) \propto (B - B_{c0})^{-2/3}$. Thus, the maximum value is a decreasing function of B_0. At $T^*(B) \ll T$, MR is a decreasing function of the temperature, and at elevated temperatures MR eventually vanishes since $\Delta\rho_{mr}(B)/\rho(T) \ll 1$.

The recent paper [14] reports on measurements of the resistivity of $CeCoIn_5$ in a magnetic field. With increasing field, the resistivity evolves from the T temperature dependence to the T^2 dependence, while the field dependence of $A(B) \sim (M^*(B))^2$ displays the critical behavior best fitted by the function, $A(B) \propto (B - B_{c0})^{-\alpha}$, with $\alpha \simeq 1.37$ [14]. All the data are in a good agreement with the $B - T$ phase diagram given by Eq. (9). The critical behavior displaying $\alpha = 4/3$ [12] and described by Eq. (8) is also in a good agreement with the data. Transition from negative MR to positive MR with increasing T was also observed [14]. We believe that an additional analysis of the data [14] can reveal that the crossover from T^2 dependence of the resistivity to the T dependence occurs at $T \propto (B - B_{c0})^{4/3}$. As well, this analysis could reveal supplementary peculiarities of MR.

In conclusion, we have described the behavior of a highly correlated electron liquid in a magnetic field. The highly correlated liquid exhibits the strong dependence of the effective mass M^* on the temperature and the magnetic field. This strong dependence is of a crucial importance when describing the $B - T$ phase diagram and such properties as MR and the critical behavior. We have also identified the behavior of the heavy fermion metal $CeCoIn_5$ in magnetic fields displayed in [14] as the highly correlated behavior of a Landau Fermi liquid approaching FCQPT from the disordered phase.

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