Transverse acoustic impedance of a normal Fermi liquid

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An expression is obtained for the real and imaginary parts of the transverse acoustic impedance of a normal Fermi liquid. The results are compared with available experimental data.

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The Fermi-liquid parameters for helium-3 are such that transverse zero-sound can propagate in it in accordance with the Landau theory.[1] Experimental observation of this sound mode was recently reported.[2] In the cited study, besides the establishment of the very fact of the propagation of transverse oscillations, they measured the acoustic impedance $Z = \Pi_{ik} u_i n_k / u^2$ for the transverse oscillations of a solid wall in liquid helium-3. Here Π_{ik} is the momentum-flux tensor, u_i is the velocity of the wall, and n_k is the normal to it. The qualitative agreement between the temperature dependences of Z at different frequencies with Bolton's theoretical calculation [3] was regarded by the authors of [2] as an argument that the transverse oscillations observed by them are actually zero sound. In fact, however, Bolton did not take into account the Fermi-liquid interaction, without which zero sound does not exist at all.

In this article we present the results of a calculation of the transverse acoustic impedance of helium-3 in the high-frequency region, i.e., when the oscillation frequency ω and the time τ between the collisions of the quasiparticles satisfy the strong inequality $\omega\tau \gg 1$, with the Fermi-liquid effect taken into account, provided that the function $F(\theta)$, which describes in the Landau theory the interaction of the quasiparticles, can be approximated by two spherical harmonics: $F(\theta) = F_c + F_1 \cos\theta$. We do not present here the calculations themselves, since they duplicate mainly Sec. 2 of the article by Bekarevich and Khalatnikov [4] dealing with the Kapitza jump at the interface between helium-3 and a solid. The formula obtained there for the real part of the

transverse acoustic impedance can unfortunately not be used directly, for at the time of the investigation in [4] it was assumed that $F_1 < 6$ and transverse zero sound cannot propagate in helium-3. The existence of zero sound leads to the appearance of a pole in the Laplace transform of the distribution function, and consequently to an additional term in the acoustic impedance. The character of the necessary changes is clear from that section of the article [4] which pertains to longitudinal zero-sound (see also [5]). The quasiparticle collisions, just as in [5], are taken into account in the τ approximation.

Using the notation of [2], $Z = R + iX$, and retaining only the principal terms in $1/\omega\tau$, we have

$$R = q\rho \frac{3p_0}{mF_1}(\eta - 1 + \Phi), \tag{1}$$

$$X = q\rho \frac{3p_0}{mF_1}\left[\zeta + \frac{1}{\omega\tau}\frac{3 + F_1}{F_1}(1 - \eta + \Psi - \Phi)\right]. \tag{2}$$

Here ρ is the density of the liquid, p_0 is the Fermi momentum, m is the mass of the helium-3 atom, η and ζ are respectively the real and imaginary parts of the velocity of the transverse sound in units of Fermi velocity (for details see [5]),

$$\Phi = \frac{1}{\pi}\int_0^1 \operatorname{arc\,tg}\left[\frac{\pi}{2}\frac{u(1-u^2)}{(1-u^2)(1-u\,\mathrm{ar\,th}\,u) - (F_1 - 6)/3F_1}\right]du, \tag{3}$$

the arctangent is defined in the interval $(0, \pi)$ and

$$\Psi = \frac{1}{F_1}\int_0^1 \frac{u(1-u^2)}{u^2(1-u^2)^2(\pi^2/4) + [(1-u^2)(1-u\,\mathrm{ar\,th}\,u) - (F_1-6)/3F_1]^2}du. \tag{4}$$

The coefficient q in (1) and (2) stands for the fraction of the quasiparticles that experience diffuse scattering by the wall; it is understood that the fraction $1-q$ is specularly scattered. The solution in [4] pertains to pure diffuse reflection, i.e., $q = 1$. However, by virtue of the linearity of the problem, and also since tangential oscillations do not excite oscillations in the liquid in the case of specular reflection, the allowance for the possibility of specular reflection of the quasiparticles reduces to the introduction of a coefficient.

The table lists the numerical values of the quantities contained in (1) and (2) for certain values of F_1 at $q = 1$. The calculations were not made for values of

TABLE I.

P, bar	F_1	$\eta - 1$	$\omega\tau\zeta$	Φ	Ψ	R/ρ, 10^3 cm/sec	$\omega\tau X/\rho$ cm/sec
9	10.15	0.093	0.345	0.346	0.074	2.28	− 664
18	11.79	0.136	0.391	0.357	0.063	2.27	− 683
23	12.95	0.166	0.417	0.362	0.057	2.24	− 692
33	15.31	0.227	0.468	0.408	0.047	2.32	− 861

F_1 close to 6, inasmuch as the approximation of $F(\theta)$ by two terms may turn out to be insufficient in this region. It follows from the paper by Dyugaev, [6], however, that the next term in the expansion of (6) can be taken into account, and in those cases when the first two harmonics in the expansion of $F(\theta)$ do not suffice, it is necessary to take into account a large number of harmonics simultaneously. Allowance for only one of the next higher terms does not lead to a more accurate result.

The third row of the table corresponds (see [7]) to a pressure of 23 bar, for which experimental data are available. [2] The value of R is in good agreement with experiment, and this can be regarded as an indication that the quasiparticle scattering is close to diffuse. A quantitative comparison for X is impossible because of the uncertainty of τ. Let us discuss qualitatively the behavior of X. It is known that $\tau \propto T^{-2}$, where T is the temperature. From (2), (4), and formula (21) of [5] it follows that in the high-temperature region we have $X \propto \omega\tau^2$. In the hydrodynamic region, where the tangential oscillations of the wall excite the usual viscous wave, we have $X \propto \sqrt{\omega\tau}$. Thus, the $X(T)$ corresponding to different frequencies should intersect at $\omega\tau \sim 1$. This intersection does indeed take place for the curves of [2] corresponding to the frequencies 60 and 36 MHz. On the other hand, the behavior of the curve for 108 MHz cannot be explained by the advanced arguments.

In the experiments of [2] they failed also to investigate another interesting frequency and temperature region, $T \lesssim \hbar\omega/2\pi$, in which a quantum analysis is essential. Using Landau's result [1] for the damping of sound in the quantum region, we conclude that the necessary modification of the obtained formulas reduces to the substitution of $1/\tau \to [1+(\hbar\omega/2\pi T)^2]/\tau$, i.e., dependence of the imaginary part of the impedance on frequency and temperature is described by the formula

$$X \sim \frac{T^2}{\omega}\left[1+\left(\frac{\hbar\omega}{2\pi T}\right)^2\right],$$

from which it is seen that the $X(T)$ curves corresponding to different frequencies should again intersect at $T \lesssim \hbar\omega/2\pi$. For the frequencies used in [2], the quantum region corresponds to temperatures lying above the temperature of the transition of helium-3 into the superfluid state, but the quantum region is perfectly attainable already at 300 MHz.

We note in conclusion that besides the sound, the single-particle excitation is scattered by a tangentially-vibrating wall make an appreciable contribution to the acoustic impedance, and the character of the dependences of X and R on T and ω is the same for both contributions, so that measurement of Z can serve as an argument favoring the existence of transverse zero sound only if the experimental values are in quantitative agreement with the theoretical ones.

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