Distribution of vacuum charge near supercharged nuclei

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We obtain the distribution of the charge produced near supercritical nuclei ($Ze^2 \gg 1$) as a result of the restructuring of the electron-positron vacuum. The calculation is carried out in the Thomas-Fermi approximation.

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The Dirac equation for an electron in the Coulomb field of a nucleus with $Z>137$ was considered in[1–3]. This has led to the prediction of the effect of spontaneous production of positrons in a supercritical ($Z > Z_{cr} \approx 170$) Coulomb field. The experimental investigation of this effect, which is possible in the collision of two heavy nuclei with total charge $Z_1+Z_2 > Z_{cr}$, is of interest for a verification of quantum electrodynamics in the region of strong external fields. A discussion of various aspects of this problem and the pertinent literature can be found in[4,5]. A more consistent examination of the phenomena produced when the charge Z goes through the critical value Z_{cr} (with allowance not only for an interaction $\propto Ze^2$, of the electron with the nucleus, but also with an interaction $\propto e^2$ between the electrons and positrons) is contained in[6].

In this article we report the results of an investigation of the case $Z \gg Z_{cr}$. This investigation was stimulated by the studies[7] of the problem of stability of vacuum and production of Bose particles in critical fields. These studies dealt with the possibility of a phase transition with formation of π condensate in ordinary nuclei,[7] as well as the possible existence of superdense, neutron, and supercharged ($Ze^3 \gtrsim 1$) nuclei.[8]

We consider a supercritical nucleus surrounded by a vacuum shell (which remains around the nucleus after the positrons go off to infinity[4]). By virtue of the condition $Ze^2 \gg 1$ it is possible to use the quasi-classical approximation, and the electron momentum is equal to ($\hbar=c=m_e=1$):

$$p(r) = [(\epsilon - V(r))^2 - 1]^{1/2} . \tag{1}$$

The electrons of the vacuum shell constitute a degenerate relativistic Fermi gas and fill all the cells of phase space with momentum $p \le p_{max} = (V^2+2V)^{1/2}$. The value of the maximum momentum p_{max} is obtained from (1) at $\epsilon = \epsilon_{max} = -1$. The density of the electron gas

$$n_e(r) = \frac{p^3_{max}}{3\pi^2} = \frac{1}{3\pi^2}(V^2+2V)^{3/2} \tag{2}$$

differs from zero in that region of space where $V(r) < -2$ (in units of $m_e c^2$). The spatial distribution of the electrons of the vacuum shell is determined by the relativistic Thomas-Fermi equation

$$\Delta V = -4\pi e^2\left[\frac{1}{3\pi^2}(V^2+2V)^{3/2} - n_p(r)\right], \tag{3}$$

where $n_p(r)$ is the proton density. We have used $n_p(r) = n_p\,\theta(R-r)$, neglecting the diffuseness of the edge of the nucleus ($n_p = Zn_0/A \approx 0.25 m_\pi^3$, where $Z/A \sim 0.5$ and n_0 is the ordinary nuclear density). The characteristic parameter in (3) is $Ze^3 \approx Z/1600$. Equation (3) was solved analytically in the limiting cases $Ze^3 \ll 1$ and $Ze^3 \gg 1$, and also numerically in the intermediate region $Ze^3 \sim 1$.

In the region $Ze^3 \ll 1$, the screening by the vacuum shell still distorts the potential of the bare nucleus only slightly and can be taken into account by perturbation theory. The external charge of the system is equal to

$$Z_1 = Z\left\{1 - \frac{4}{3\pi}(Ze^3)^2\left(\ln\frac{Ze^2}{R} + c\right)\right\}, \tag{4}$$

where $Z_1 = Z - N_e$, N_e is the total number of electrons of the vacuum shell, and c is a constant that depends on the distribution of the protons over the volume of the nucleus. In the case constant density ($n_p(r) = \text{const}$ at $r < R$) we have $c = -1.38$. The radius of the vacuum shell is $r_A = Z_1 e^2/2$; in this case a small fraction of the electron cloud is located inside the nucleus.

The picture is different at $Ze^3 \gg 1$. In this case it is convenient to use the substitutions $V = -(3\pi^2 n_p)^{1/3}\chi$ and $x = \kappa(r-R)$, where $\kappa^2 = 4(\pi/3)^{1/3} e^2 n_p^{2/3}$. At $\kappa R \gg 1$, Eq. (3) takes the form

$$\chi'' = \chi^3 - \theta(-x) \tag{5}$$

and admits of an exact solution. Otuside the nucleus, at $x > 0$, we have

$$\chi(x) = \sqrt{2}(x+b)^{-1}, \quad b = \frac{4}{3}\sqrt{2} = 1.886. \tag{6}$$

Inside the nucleus ($x < 0$) the exact solution is somewhat cumbersome, and we confine ourselves to indication of a simple approximate solution:

$$\chi(x) = 1 - Ce^{x\sqrt{3}}; \quad C = 0.2374. \tag{7}$$

the accuracy of which is 1.5% (compare curves 1 and 2). Inside the nucleus, the densities $n_e(r)$ and n_p are practically equal, there is no electric field, and the potential is constant, namely $V(r) = -V_0$ at $R - r \ll \kappa^{-1}$ $V(r) = -V_0$, where $V_0 = (3\pi^2 n_p)^{1/3} \approx 1.94 m_\pi$. The proton charge is completely cancelled here by the electron cloud. Near the edge of the nucleus is located a transition layer of width $\sim x^{-1} = 13$ F (see the figure), in which the uncompensated charge is located. The electric field attenuates exponentially in the interior of the nucleus and reaches a maximum value at the edge of the nucleus: $E_{max} = (9\pi\sqrt{2}/16) \times (3/\pi)^{1/6} n_p^{2/3} = 7.2\cdot 10^{-3}\, m_\pi^2/e$, or 5500 times larger than the characteristic quantum-electrodynamics field intensity $E_0 = m_e^2/e = 1.3\times 10^{16}$ V/cm at which nonlinear effects become significant (the polarization of vacuum and the production of e^+e^- pairs by a homogeneous electric field). The total charge inside the nucleus $Z' = Z - \int_0^R n_e(r)\, 4\pi r^2\, dr$ is determined from the condition $R^2 E_{max} = Z' e$, whence

$$Z'/Z = 0.95(Ze^3)^{-1/3}, \text{ at } Ze^3 \gg 1. \tag{8}$$

The formation of the electrically-neutral plasma inside the nucleus greatly decreases the Coulomb energy $E_Q = 3(Ze)^2/5R \approx 0.6(Ze^3)^{2/3} Zm_\pi$, which is the obstacle to the stability of the supercharged nuclei. There remains, however,

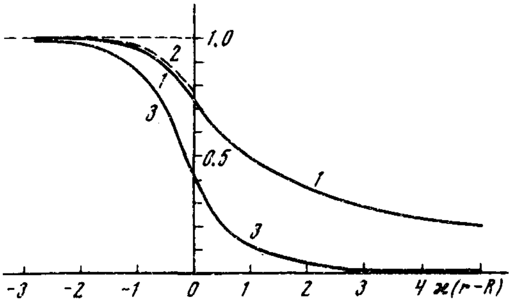

FIG. Plots of the potential and of the density of the electron cloud near the edge of a supercharged nucleus at $Ze^3 \gg 1$. Curves 1 and 2 represent the function $\chi(x) = -V(r)/V_0$; 1—exact solution of (5), 2—the approximate solution (7). Curve 3 shows the ratio of the electron density $n_e(r)$ to the density $n_e(0)$ at the center of the nucleus; at the edge of the nucleus $n_e(R) = 0.42\, n_e(0)$.

also the kinetic energy of the degenerate electron gas: $E_e = p_F^4 R^3/3\pi = \frac{3}{4} Zp_F \approx 1.5 Zm_\pi$. The question of the stability of supercharged nuclei calls for an additional study and will be considered subsequently.

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