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Two Scenarios of the Quantum Critical Point

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Two different scenarios of the quantum critical point (QCP), a zero-temperature instability of the Landau state, related to the divergence of the effective mass, are investigated. Flaws of the standard scenario of the QCP, where this divergence is attributed to the occurrence of some second–order phase transition, are demonstrated. Salient features of a different *topological* scenario of the QCP, associated with the emergence of bifurcation points in equation $\epsilon(p) = \mu$ that ordinarily determines the Fermi momentum, are analyzed. The topological scenario of the QCP is applied to three-dimensional (3D) Fermi liquids with an attractive current-current interaction.

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A statement that the Landau quasiparticle picture breaks down at points of second–order phase transitions has become a truism. The violation of this picture is attributed to vanishing of the quasiparticle weight z in the single-particle state since the analysis of a long wave-length instability in the $S = 1$ particle-hole channel, performed more than forty years ago by Doniach and Engelsberg [1] and refined later by Dyugaev. [2] In non-superfluid Fermi systems, the z-factor is determined by the formula $z = [1 - (\partial\Sigma(p,\varepsilon)/\partial\varepsilon)_0]^{-1}$ where the subscript 0 indicates that the respective derivative of the mass operator Σ is evaluated at the Fermi surface. This factor enters a textbook formula

$$\frac{M}{M^*} = z\left[1 + \left(\frac{\partial\Sigma(p,\varepsilon)}{\partial\epsilon_p^0}\right)_0\right] \tag{1}$$

for the ratio M^*/M of the effective mass M^* to the mass M of a free particle. As seen from this formula, where $\epsilon_p^0 = p^2/2M$, the effective mass diverges at a critical density ρ_c, where z vanishes provided the sum $1 + \left(\partial\Sigma(p,\varepsilon = 0)/\partial\epsilon_p^0\right)_0$ has a *positive and finite value* at this point. Nowadays, when studying critical fluctuations of arbitrary wave-lengths $k < 2p_F$ has become popular, this restriction is often assumed to be met without stipulations. E.g. a standard scenario of the quantum critical point (QCP) where M^* diverges is formulated as follows: in the vicinity of an impending second–order phase transition, "quasiparticles get heavy and die" [3, 4].

However, as seen from Eq.(1), M^* may diverge not only at the points of the second-order phase transitions, but also at a critical density ρ_∞, where the sum $1 + \left(\partial\Sigma(p,\varepsilon)/\partial\epsilon_p^0\right)_0$ changes its sign. Furthermore, we will demonstrate that except for the case of the ferromagnetic instability, [1] M^* cannot diverge at ρ_c without violation of stability conditions.

In what follows we restrict ourselves to one-component three-dimensional (3D) homogeneous Fermi liquids where the particle momentum is conserved, and the Landau equation, connecting the quasiparticles group velocity $\partial\epsilon/\partial\mathbf{p}$ to their momentum distribution $n(p)$ in terms of the interaction function f, has the form [5 – 7]

$$\frac{\partial\epsilon(p)}{\partial\mathbf{p}} = \frac{\mathbf{p}}{M} + \int f(\mathbf{p},\mathbf{p_1})\,\frac{\partial n(p_1)}{\partial\mathbf{p_1}}\,\frac{d^3p_1}{(2\pi)^3}. \tag{2}$$

Setting here $p = p_F$ and introducing the notation $v_F = (d\epsilon(p)/dp)_0 = p_F/M^*$, one obtains

$$v_F = \frac{p_F}{M}\left(1 - \frac{p_F M}{3\pi^2}f_1\right), \tag{3}$$

implying that

$$\frac{M}{M^*} = 1 - \frac{1}{3}\frac{p_F M}{\pi^2}f_1. \tag{4}$$

Hereafter we employ notations of Fermi liquid (FL) theory where f_1 is the first harmonic of the interaction function $f(\theta) = z^2\Gamma^\omega(p_F,p_F;\theta)$, with Γ^ω being the ω-limit of the scattering amplitude Γ of two particles, whose energies and incoming momenta $\mathbf{p}_1,\mathbf{p}_2$ lie on the Fermi surface, with $\cos\theta = \mathbf{p}_1\cdot\mathbf{p}_2/p_F^2$, while the 4-momentum transfer $(\mathbf{q},\omega)$ approaches zero, such that $q/\omega \to 0$.

It is instructive to rewrite Eq.(4) in terms of the k-limit of the dimensionless scattering amplitude $\nu\Gamma^k =$

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$= A+B\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2$ where $\nu = z^2 p_F M^*/\pi^2$ is the quasiparticle density of states. Simple algebra then yields

$$\frac{M}{M^*} = 1 - \frac{1}{3}A_1. \tag{5}$$

This formula stems from Eq.(4) and relation [5–7] $A_1 = \Phi_1/(1+\Phi_1/3)$, where Φ is the spin-independent part of the product $\nu\Gamma^\omega$. Thus at the density ρ_∞ where the effective mass diverges one has

$$A_1(\rho_\infty) = 3, \quad \Phi_1(\rho_\infty) = \infty. \tag{6}$$

In the following we focus on critical density fluctuations with $k_c \neq 0$, addressed in Ref.[8]. First we notice that there is a strong dependence of the amplitude $\Gamma_{\alpha\beta,\gamma\delta}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}, \omega = 0)$ on the momentum transfer k close to the critical momentum k_c, specifying the spectrum of density fluctuations, that stems from the asymmetry of Γ with respect to the interchange of momenta and spins of colliding particles [2]. In this case, upon neglecting regular components one finds [2]

$$A(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}, \omega{=}0; \rho \to \rho_c) = -D(\mathbf{k}) + \frac{1}{2}D(\mathbf{p}_1 - \mathbf{p}_2 + \mathbf{k}), \tag{7}$$

with

$$D(k \to k_c, \omega = 0) = \frac{g}{\xi^{-2}(\rho) + (k-k_c)^2}, \tag{8}$$

the correlation length $\xi(\rho)$ diverging at $\rho = \rho_c$.

Within the quasiboson approximation [8], the derivative $(\partial\Sigma(p,\varepsilon)/\partial\varepsilon)_0$ diverges at $\rho \to \rho_c$ as $\xi(\rho)$, while the derivative $\left(\partial\Sigma(p,\varepsilon)/\partial\epsilon_p^0\right)_0$ remains finite at any k_c. If these results were correct, then the densities ρ_∞ and ρ_c would coincide, in agreement with the standard scenario of the QCP. However, calculations of harmonics $A_k(\rho)$ of the amplitude $A(p_F, p_F, \cos\theta)$ from Eqs.(7) and (8) yield

$$A_0(\rho{\to}\rho_c) = g\frac{\pi}{2}\frac{k_c\xi(\rho)}{p_F^2}, \quad A_1(\rho{\to}\rho_c) = g\frac{3\pi}{2}\frac{k_c\xi(\rho)}{p_F^2}\cos\theta_0. \tag{9}$$

We see that the sign of $A_1(\rho \to \rho_c)$, coinciding with that of $\cos\theta_0 = 1 - k_c^2/2p_F^2$, turns out to be *negative* at $k_c > p_F\sqrt{2}$. According to Eq.(5) this implies that at the point of the second–order phase transition, the ratio $M^*(\rho_c)/M < 1$. Thus we infer that at $k_c > p_F\sqrt{2}$, the densities ρ_c and ρ_∞ *cannot coincide*. In its turn, this implies that vanishing of the z-factor at ρ_c is compensated by the divergence of the derivative $(\partial\Sigma(p,\varepsilon)/\partial\epsilon_p^0)_0$ at this point, otherwise Eq.(1) fails.

To verify this assertion let us write down a fundamental FL relation [7] between the k- and ω-limits of the vertex $\mathcal{T}$ that has the symbolic form $\mathcal{T}^k = \mathcal{T}^\omega + \left(\Gamma^k((G^2)^k - (G^2)^\omega)\mathcal{T}^\omega\right)$ where external brackets mean integration and summation over all intermediate momemtum and spin variables. In dealing with the bare vertex $\mathcal{T}^0 = \mathbf{p}$ the extended form of this relation is

$$-\frac{\partial G^{-1}(p)}{\partial \mathbf{p}} = \frac{\partial G^{-1}(p)}{\partial\varepsilon}\frac{\mathbf{p}}{M} + Sp_\sigma \int \Gamma^k(p,q)\left(\left(G^2(q)\right)^k - \left(G^2(q)\right)^\omega\right) \times \frac{\partial G^{-1}(q)}{\partial q_0}\frac{\mathbf{q}}{M}\frac{d^4q}{(2\pi)^4 i}, \tag{10}$$

In writing this equation the Pitaevskii identities [7]

$$\mathcal{T}^k(\mathbf{p}) = -\frac{\partial G^{-1}(p,\varepsilon)}{\partial\mathbf{p}}, \quad \mathcal{T}^\omega(\mathbf{p}) = \frac{\partial G^{-1}(p,\varepsilon)}{\partial\varepsilon}\mathbf{p} \tag{11}$$

are employed. Upon inserting the FL formula

$$\left(G^2(q)\right)^k = \left(G^2(q)\right)^\omega - 2\pi^3 i\frac{\nu}{p_F^2}\,\delta(\varepsilon)\delta(p-p_F) \tag{12}$$

into Eq. (10) and the standard replacement of the spin-independent part of $\nu\Gamma^k$ by A, after some algebra we are led to equation

$$1 + \left(\frac{\partial\Sigma(p,\varepsilon)}{\partial\epsilon_p^0}\right)_0 = \left(1 - \left(\frac{\partial\Sigma(p,\varepsilon)}{\partial\varepsilon}\right)_0\right)\left(1 - \frac{1}{3}A_1\right). \tag{13}$$

Remembering that $-\partial G^{-1}(p,\varepsilon)/\partial\mathbf{p} = z^{-1}d\epsilon(p)/dp$ one arrives [7] at Eq.(5). On the other hand, as seen from Eq.(10), at $k_c > p_F\sqrt{2}$ where $A_1 < 0$, the derivative $\left(\partial\Sigma(p,\varepsilon)/\partial\epsilon_p^0\right)_0$ does diverge at the same density, as the derivative $\left(\partial\Sigma(p,\epsilon)/\partial\varepsilon\right)_0$, in contrast to the result [8]. To correct the defect of the quasiboson approximation [8], the spin-independent part of the scattering amplitude Γ entering the formulas [8] for the derivatives of the mass operator Σ should be replaced by that of the amplitude Γ^ω (for details, see Ref.[2]).

We will immediately see that at finite $k_c < p_F\sqrt{2}$, vanishing of the z-factor is *incompatible* with the divergence of M^* as well. Indeed, as seen from Eq.(9), the harmonics $A_0(\rho_c)$ and $A_1(\rho_c)$ are related to each other by equation $A_0(\rho_c) = A_1(\rho_c)/(3\cos\theta_0)$. If $M^*(\rho_c)$ were infinite, then according to Eq.(5), $A_1(\rho_c)$ would equal 3, and $A_0(\rho_c) = 1/\cos\theta_0$. However, the quantity $A_0 = \Phi_0/(1+\Phi_0)$ cannot be in excess of 1, otherwise the Pomeranchuck stability condition [6, 7] $\Phi_0 > -1$ is violated, and the compressibility turns out to be negative. Thus the QCP cannot be reached without the violation of the stability condition. If so, approaching the QCP, the system undergoes a first–order phase transition, as in the case of 3D liquid ^{3}He.

The finiteness of M^* at the points of vanishing of the z-factor requires an alternative explanation, (see e.g. Ref.[9]), of the logarithmic enhancement of the specific heat $C(T)$, observed in many heavy fermion metals [10] and attributed to contributions of critical fluctuations.

Let us now turn to the analysis of another opportunity for the occurrence of the QCP, addressed first in microscopic calculations of the single-particle spectrum of 3D electron gas [11, 12]. It is associated with the change of the sign of the sum $1 + (\partial\Sigma(p,\varepsilon)/\partial\epsilon_p^0)_0$ at $\rho_\infty \neq \rho_c$. In this case, the z-factor keeps its finite value, and hence, the quasiparticle picture holds on both the sides of the QCP. In standard Landau theory, equation

$$\epsilon(p, T = 0) = \mu, \tag{14}$$

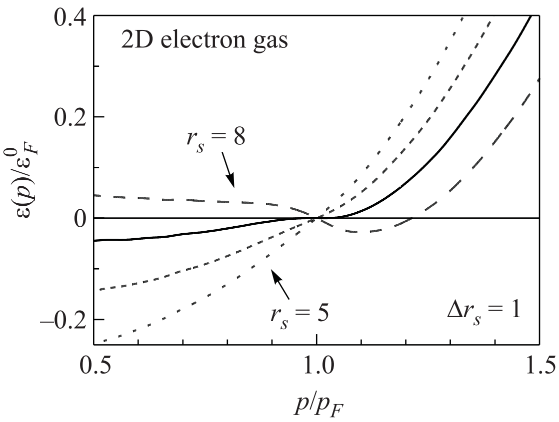
with μ, being the chemical potential, has the single root, determining the Fermi momentum p_F. Suppose, at a critical coupling constant g_T, a bifurcation in Eq.(14) emerges, then beyond the critical point, at $g > g_T$, this equation acquires, at least, two new roots that triggers a *topological* phase transition [13]. In many-body theory, equation, determining critical points of the topological phase transitions, has the form

$$\epsilon_p^0 + \Sigma(p, \varepsilon = 0) = \mu. \tag{15}$$

Significantly, terms, proportional to $\epsilon \ln \epsilon$, existing in the mass operator Σ of marginal Fermi liquids, do not enter this equation.

The bifurcation p_b in Eqs.(14) and (15) can emerge at any point of momentum space. If p_b coincides with the Fermi momentum p_F, then at the critical density the sum $1 + (\partial\Sigma(p,\varepsilon)/\partial\epsilon_p^0)_0$ vanishes, and one arrives at the *topological* quantum critical point. In connection with this scenario, it is instructive to trace the evolution of the group velocity $v_F = (d\epsilon/dp)_0$ versus the first harmonic f_1. As follows from Eq.(3), v_F keeps its positive sign as long as $F_1^0 = p_F M f_1/3\pi^2 < 3$, and the Landau state with the quasiparticle momentum distribution $n_F(p) = \theta(p - p_F)$ remains stable. However, at $F_1^0 > 3$, the sign of v_F changes, and the Landau state is necessarily rearranged. This conclusion is in agreement with results of microscopic calculations of the single-particle spectrum $\epsilon(p, T = 0)$ of 2D electron gas, [14] shown in Figure. As seen, the sign of v_F holds until the dimensionless parameter r_s attains a critical value $r_{sc} \simeq 7.0$. At greater r_s, the derivative $(d\epsilon(p)/dp)_0$, evaluated with the momentum distibution $n_F(p)$, becomes negative, and the Landau state loses its stability, since the curve $\epsilon(p)$ crosses the Fermi level more than one time.

At $T = 0$, two types of the topological transitions are known [13]. One of them, giving rise to the multi-



Single-particle spectrum $\epsilon(p)$ of a homogeneous 2D electron gas in units of $\epsilon_F^0 = p_F^2/2M$, evaluated [14] at $T = 0$ for different values of the dimensionless parameter $r_s = \sqrt{2}Me^2/p_F$

connected Fermi surface, was uncovered [15] and studied later [16–18, 9] in calculations of the single-particle spectrum $\epsilon(p)$ on the base of Eq. (2), with the interaction function $f(k)$, having no singularities at $k \to 0$. In this case, beyond the QCP, Eq.(14) has three roots $p_1 < p_2 < p_3$, i.e. the curve $\epsilon(p)$ crosses the Fermi level three times, and occupation numbers are: $n(p) = 1$ at $p < p_1$, $n(p) = 0$ at $p_1 < p < p_2$, while at $p_2 < p < p_3$, once again $n(p) = 1$, and at $p > p_3$, $n(p) = 0$. As the coupling constant g increases, the number of the roots of Eq.(14) rapidly grows, however, their number remains countable at any $g > g_T$.

The situation changes in Fermi liquids with a singular attractive long-range current-current term

$$\Gamma^0(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}, \omega = 0) = -g\frac{\mathbf{p}_1\mathbf{p}_2 - (\mathbf{p}_1\mathbf{k})(\mathbf{p}_2\mathbf{k})/k^2}{k^2}, \tag{16}$$

since in these systems, e.g. in dense quark-gluon plasma, solutions with the multi-connected Fermi surface are unstable. Indeed, the group velocity $d\epsilon(p)/dp$ evaluated with $n_F(p) = \theta(p - p_F)$ from Eq. (2) has the form $d\epsilon(p)/dp = p_F/M - g\ln(2p_F/|p_F - p|)$, implying that Eq.(14) has three different roots p_1, p_2, p_3, corresponding to the Fermi surface, having three sheets at any $g > 0$ [19]. However, at the next iteration step, the new Fermi surface has already five sheets, the Fermi surface has seven sheets and so on [19]. With increasing the number of iterations, the distance between neighbour sheets rapidly shrinks. In this situation, a minute elevation of temperature renders the momentum distribution $n(p, T)$ a smooth *T-independent* function $n_*(p)$, different from

0 and 1 in a domain $\mathcal{C}$ between the sheets. In this case, the ground–state stability condition,

$$\delta E = \int (\epsilon(p) - \mu)\, \delta n(p) \frac{d^3p}{(2\pi)^3} > 0, \qquad (17)$$

requiring the nonnegativity of the variation δE of the ground state energy E at any admissable variation of $n_*(p)$, is met provided

$$\epsilon(p) = \mu, \quad p \in \mathcal{C}. \qquad (18)$$

As a result, we arrive at another type of the topological transitions, the so-called fermion condensation [20–24, 13, 25], where the roots of Eq.(14) form an *uncountable* set, called the fermion condensate (FC). Since the quasiparticle energy $\epsilon(p)$ is nothing but the derivative of the ground state energy E with respect to the quasiparticle momentum distribution $n(p)$, Eq.(18) can be rewritten as variational condition [20]

$$\delta E/\delta n(p) = \mu, \quad p \in \mathcal{C}. \qquad (19)$$

The FC Green function has the form

$$G(p, \varepsilon) = \frac{1 - n_*(p)}{\varepsilon + i\delta} + \frac{n_*(p)}{\varepsilon - i\delta}, \quad p \in \mathcal{C}. \qquad (20)$$

As seen, only the imaginary part of the FC Green function differs from that of the ordinary FL Green function. This difference exhibits itself in a *topological charge*, given by the integral [21, 13]

$$N = \int_\gamma G(p, \varepsilon)\, \partial_l G(p, \varepsilon) \frac{dl}{2\pi i}, \qquad (21)$$

where integration is performed over a contour in complex energy plane, embracing the Fermi surface. For conventional Fermi liquids and systems with the multi-connected Fermi surface, the topological charge N is integer, while for the states with a FC, its value is *half-integer* [21, 13].

For illustration of the phenomenon of fermion condensation, let us address dense quark-gluon plasma, on the Lifshitz phase diagram of which, as we have seen, there is no room for the conventional FL phase [26]. Upon inserting into Eq.(2) only leading divergent terms in the interaction function f, constructed from Eq.(16), one finds

$$0 = 1 - \lambda \int \ln \frac{1}{|x - x_1|} \frac{\partial n_*(x_1)}{\partial x_1} dx_1, \quad x, x_1 \in \mathcal{C}, \quad (22)$$

where dimensionless variables $x = (p_F - p)/2p_F$ and λ are introduced. A numerical solution of this equation will be found elsewhere. Here we simplify Eq.(22), replacing the kernel $\ln(1/|x - x_1|)$ by $\ln(1/x)$ provided $x > x_1$, and by $\ln(1/x_1)$, otherwise, to obtain

$$0 = 1 + \lambda n_*(x) \ln x + \lambda \int_x^{x_m} \ln x_1 \frac{\partial n_*(x_1)}{\partial x_1} dx_1, \quad x, x_1 \in \mathcal{C}. \qquad (23)$$

An approximate solution of this equation is $n_*(x) = x/x_m$ where $x_m = e^{-1/\lambda}$ is determined from condition $n(x_m) = 1$. We see that the range of the interval $[0, x_m]$ of fermion condensation, adjacent to the Fermi surface, is exponentially small.

Nontrivial smooth solutions $n_*(p)$ of Eq.(19) exist even in weakly correlated Fermi systems. However, in these systems, the Pauli restriction $n_*(p) < 1$ is violated, rendering such solutions meaningless. Even at the QCP, where the nonsingular interaction function f is already sufficiently strong, no consistent FC solutions $n_*(p)$ exist, satisfying the requirement $n(p) < 1$ wherever. These solutions emerge at a critical constant g_{FC}, and at $g > g_{FC}$, they win the contest with any other solutions. Thus on the Lifshitz phase diagram of systems with nonsingular repulsive interaction functions f, the standard FL phase occupies the interval $g < g_T$, the phase with the multi-connected Fermi surface, the interval $g_T < g < g_{FC}$, while the phase with the FC exists at $g > g_{FC}$.

In dealing with the full $(T - g)$ phase diagram of such systems we notice that the temperature evolution of the quasiparticle momentum distribution, associated at $T = 0$ with the multi-connected Fermi surface, depends on the departure of the difference $|\epsilon(p) - \mu|$ from 0 in the domain $\mathcal{C}$. Its maximimu value ϵ_m determines a new energy scale $\epsilon_m \simeq d^2/M^*(0)$, where d is the average distance between the sheets of the Fermi surface that rapidly falls with the increase of the sheets number [15, 16]. If temperature T attains values, comparable with ϵ_m, then, as seen from the Landau formula $n(p) = [1 + \exp{(\epsilon(p) - \mu)/T)}]^{-1}$, the distribution $n(p)$ becomes a smooth function of p. Employing the FC notation $n_*(p)$ for this function, one finds that at $T \geq \epsilon_m$ the spectrum

$$\epsilon(p, T) = T \ln \frac{1 - n_*(p)}{n_*(p)}, \quad p \in \mathcal{C} \qquad (24)$$

does coincide with the FC spectrum [22]. We infer that at $T \simeq \epsilon_m$, a crossover from a state with the multi-connected Fermi surface to a state with the FC occurs. As a result, FL thermodynamics of the systems with the multi-connected Fermi surface completely alters at

$T \simeq \epsilon_m$, since properties of systems with the FC, in particular the magnetic susceptibility $\chi(T)$, resemble those of a gas of localized spins [27]. Such a transition was recently observed in the heavy–fermion metal $YbIr_2Si_2$, transition temperature being merely 1 K [28].

Let us now turn to systems of fermions, interacting with a "foreign" bosonic mode, e.g. phonons or photons. In the Frölich model, [7, 29] aimed for the elucidation of electron and phonon spectra in solids, electrons share momentum with the lattice due to the electron-phonon interaction. The non-conservation of the electron momentum results in the *violation* of the second of the relations (11), and Eq.(4) acquires the form

$$\frac{M}{M^*} = z\frac{(\mathbf{n}\mathcal{T}^\omega(\mathbf{p}))_0}{p_F}\left(1 - \frac{1}{3}A_1\right), \qquad (25)$$

where $\mathbf{n} = \mathbf{p}/p$. The departure of the ratio M/M^* from the Landau value (4), is well pronounced in the limit $c_s \ll v_F$. For illustration, let us consider the weak coupling limit of the Frölich model, where the first harmonic A_1, evaluated from the phonon propagator $D(|\mathbf{p}_1 - \mathbf{p}_2|, \omega = 0)$ equals 0 due to its isotropy. If Eq.(4) were correct, then M^*/M would equal 1. However, this is not the case: $M^*/M = 1 + g^2 p_F M/2\pi^2$, where g is the electron-phonon coupling constant [29]. Evidently, if the ratio v_F/c_s drops, then the departure from Eq. (4) falls due to weakening of the contribution of the pole of the boson propagator. Such a situation occurs just in the vicinity of the QCP, since at this point $v_F = p_F/M^* = 0$.

So far information on the QCP properties of Fermi liquids is extracted from measurements, carried out in 2D electron gas, 2D liquid ^{3}He and heavy–fermion metals. Here we restrict ourselves to several remarks, reserving a more detailed analysis for a separate paper. Accurate measurements of the effective mass M^* in dilute 2D electron gas are made on (100)- and (111)-silicon MOSFET's. [30 – 33]. In principle, the divergence of M^*, observed in these experiments, can be associated with critical spin-density fluctuations. However, experimental data rules out a significant enhancement of the Stoner factor. In many theories, (see e.g. Ref.[34]), the enhancement of M^* is related to disorder effects. However, the effective masses, specifying the electron spectra of (100)- and (111)-silicon MOSFET's, where disorder is different, almost coincide with each other provided dimensionless parameters r_s of the 2D Coulomb problem, are the same [33]. On the other hand, this coincidence that agrees with results of microscopic calculations [14] is straightforwardly elucidated within the topological scenario of the QCP. There are reports on the divergence of the effective mass in 2D liquid ^{3}He, (see e.g. Refs. [35 – 38]). Furthermore, following [36], authors of Ref.[38] reported that at the density $\rho > 9.00nm^{-2}$, the low-temperature limit of the product $T\chi(T)$ quickly increases with increasing ρ. In addition, the ratio of the specific heat $C(T)$ to T does not obey FL theory in this density region, since it increases with lowering T. These facts can be interpreted as evidence for the presence of the FC [24]. Unfortunately, so far the accuracy of extremely difficult measurements of the ratio $C(T)/T$ at $T \le 1K$, is insufficient to properly evaluate a low-temperature part of the entropy S and compare it with the respective FC entropy, extracted from data on $\chi(T)$.

The divergence of the ratio $C(T)/T$, associated with the QCP, is observed in several heavy–fermion compounds [10, 39, 40]. Authors of the experimental article [40] claim that data on the Sommerfeld-Wilson ratio $R_{SW} = \chi(T)/C(T)$ in a doped compound $YbRh_2(Si_{0.95}Ge_{0.05})_2$ point to an enhancement of the Stoner factor that has to be infinite at the point of the ferromagnetic phase transition. However, evaluation of the Stoner factor from experimental data in heavy–fermion metals encounters difficulties, discussed in Ref. [27]. Furthermore, with a correct normalization experimental data [40] are explained without any enhancement of the Stoner factor [27] that rules out the relevance of the ferromagnetic phase transition to the QCP in this metal. A different H–T phase diagram is constructed for the heavy–fermion metal YbAgGe in Refs.[41]. On this diagram, the FL phase is separated from a phase with magnetic ordering by a significant domain of NFL behavior. Such a separation is easily explained within the topological scenario of the QCP, discussed in this article.

In conclusion, in this article, two different scenarios of the quantum critical point (QCP), a low-temperature instability of the Landau state, related to the divergence of the density of states $N(0) \sim M^*$, are analyzed. We discuss shortcomings of the conventional scenario of the QCP, where the divergence of the effective mass M^* is attributed to vanishing of the quasiparticle weight in the single-particle state. In a different, topological scenario, associated with the change of the topology of the Fermi surface at the QCP, the quasiparticle picture holds on both the sides of the QCP. This scenario is in agreement with microscopic calculations of the QCP in 2D electron gas and does not contradict relevant experimental data on 2D liquid ^{3}He and heavy–fermion metals.

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