

# Non-Fermi liquid criticality and super universality in the quantum Hall regime

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We report the results of a microscopic theory, based on the topological concept of a  $\theta$  vacuum, which show that the Coulomb potential, unlike any finite ranged interaction potential, renders the longstanding problem of the plateau transitions in the quantum Hall regime non-Fermi liquid like. Our present results, which are important for quantum phase transitions in general and composite fermion ideas in particular, provide a novel understanding of the critical exponent values that have recently been (re)taken from a series of state-of-the-art quantum Hall samples.

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W. Li et al. [1] recently revisited the problem of plateau transitions in the quantum Hall regime (QHR) by investigating a series of specially grown samples with varying disorder. Remarkably, both the experimental methodology and the reported value of the universal exponent ( $\kappa = 0.42$ ) are exactly the same as those of the original papers by H.P. Wei et al. [2] more than 15 years ago. Like the original experiments, W. Li et al. put the all the emphasis on samples with *short ranged potential fluctuations* relative to the magnetic length. This fundamental aspect of the theory [3] has traditionally been ignored [4] in the many experiments by many others. [1] Unfortunately, however, W. Li et al. [1] present their results based on Fermi-liquid type of arguments that originally came along with the pioneering work of H.P. Wei et al. [2] For example, the observable exponent  $\kappa = 0.42$  is generally recognized as the ratio of two independent quantities,  $\kappa = p/2\nu$ , with  $\nu$  denoting the correlation or localization length exponent and  $p$  a dynamical exponent that is determined by inelastic processes [5]. W. Li et al. [1] identify  $\nu = 2.3$  which is the free electron value known from numerical work [6, 7]. The exponent  $p$ , on the other hand, which defines the relation between the phase breaking length  $L_\phi$  and temperature  $T$ , popularly written as  $L_\phi \propto T^{-p/D}$  with  $D = 2$  the spatial dimension [5], has been taken as a purely phenomenological quantity with a numerical value  $p \approx 2$  [1].

In view of the dramatic progress made over the years, in the theory of localization and interaction effects [8, 9, 3], there actually exists no reason to believe in

Fermi liquid principles. Completely different and novel insights have emerged which tell us that all the basic aspects of the QHR reveal themselves as *super universal* features of a single principle in quantum field theory, i.e. the topological concept of an *instanton vacuum* [10]. The statement of super universality makes it is easier and more natural to comprehend why such basic phenomena as scaling and quantum criticality are retained by the electron gas also in the presence of the infinitely ranged Coulomb potential. Unlike Fermi liquid theory, however, quantum criticality at  $\theta = \pi$  does not necessarily stay the same. Different applications of the  $\theta$  vacuum concept generally have different exponent values and, hence, fall into different *universality classes* [10]. *Super universality* is important since it resolves many controversies [11] that originally were encountered in QCD where the algebra is in many ways the same. It has not been recognized until recently, for example, that the  $\theta$  vacuum concept generically displays *massless chiral edge excitations* [3] as well as *robust topological quantum numbers* [11] that explain the *precision* and *observability* of the quantum Hall plateaus. These very basic advances clearly indicate that the QHR is an outstanding laboratory where the strong coupling problems in quantum field theory can be explored and investigated in great detail.

In this Letter we present the results of a detailed analysis on topological excitations (instantons) [12] which unequivocally demonstrate that the experimental exponents  $\nu$  and  $p$ , rather than being disconnected pieces in heuristic scenario's, are in fact universal quantities that emerge simultaneously from a unifying microscopic theory [8]. This theory describes the low energy dynam-

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	$A_1$	$A_2$	$\mathcal{D}$	$B_1$	$B_2$	$\mathcal{D}_\gamma$
$c = 0$	0 [16]	1/2 [16]	$16\pi/e$ [10]	1 [17]	0 [17]	$8\pi/e$ [10]
$0 < c < 1$	$2[1 + \frac{1-c}{c} \ln(1-c)]$ [9, 14]	–	$\mathcal{D}(c)$ [12]	1 [9, 14]	–	$\mathcal{D}_\gamma(c)$ [12]
$c = 1$	2 [13]	$4\mathcal{A}$ [9]	$16\pi e^{1-4\gamma_E}$ [12]	1 [13]	$\pi^2/6 + 3$ [9]	$8\pi e^{1-4\gamma_E}/3$ [12]

Coefficients of renormalization group functions, see text.  $A_2$  and  $B_2$  for  $0 < c < 1$  are yet unknown,  $\gamma_E \approx 0.577$  is the Euler constant and the analytic expressions for  $\mathcal{D}(c)$ ,  $\mathcal{D}_\gamma(c)$  and  $\mathcal{A} \approx 1.64$  are given in Refs. [12] and [9] respectively.

ics of spin polarized electrons in the presence of electron-electron interactions and quenched (*short ranged*) disorder [3]. It encapsulates not only the different aspects of symmetry, quantum criticality and super universality of the electron gas in the QHR but also the conventional *mobility edge* problem in  $2 + \epsilon$  dimensions [13, 14] which sets the stage for our understanding of quantum phase transitions in general [9]. For example, *exact* global symmetries based on the electrodynamic  $U(1)$  gauge invariance ( $\mathcal{F}$  invariance [8]) quite generally mark the difference between a *Fermi liquid* and a *non-Fermi liquid* universality class depending on whether the *range* of the interactions is *finite* or *infinite* [9]. The main objective of the present Letter is to elucidate the basic principles of quantum transport theory and explain the experimental results of W. Li et al. [1] based on a novel non-Fermi liquid theory of the conductances.

The instanton vacuum representation of the spin polarized electron gas involves unitary (*Grassmannian*) matrix field variables  $Q_{mn}^{\alpha\beta}(\mathbf{r})$  that obey  $Q^2(\mathbf{r}) = \mathbf{1}$ . The integers  $\alpha, \beta$  denote the *replica* indices and  $m, n$  correspond to the discrete set of Matsubara frequencies  $\omega_n = \pi T(2n + 1)$ . The effective action in  $D = 2$  is given as

$$S_{\text{eff}}[Q] = S_\sigma[Q] + S_F[Q], \quad (1)$$

where  $S_\sigma$  represents the well know *free electron* part [10]

$$S_\sigma[Q] = -\frac{\sigma_{xx}}{8} \int d\mathbf{r} \text{tr} \nabla_\mu Q \nabla_\mu Q + 2\pi i \sigma_{xy} \mathcal{C}[Q], \quad (2)$$

$$\mathcal{C}[Q] = \frac{1}{16\pi i} \int d\mathbf{r} \text{tr} \epsilon_{\mu\nu} Q \nabla_\mu Q \nabla_\nu Q, \quad (3)$$

with  $\mathcal{C}[Q]$  denoting the topological charge and  $\sigma_{xx}, \sigma_{xy}$  the mean field parameters for the longitudinal and Hall conductance respectively. The term  $S_F$  contains the *single interaction* amplitude  $zc$  [8] and has been proposed in different physical settings before [13, 14]

$$S_F = \pi z T \int d\mathbf{r} \left[ c \sum_{\alpha n} \text{tr} I_n^\alpha Q \text{tr} I_{-n}^\alpha Q + 4\text{tr} \eta Q - 6\text{tr} \eta \Lambda \right]. \quad (4)$$

The meaning of the symbols is as follows. At the mean field level,  $c$  is determined by the standard Fermi liquid

interaction parameter and  $z$  is proportional to the effective mass. The theory in the range  $0 < c < 1$  describes the electron gas with *finite* ranged electron-electron interactions. The cases  $c = 0$  and  $c = 1$  describe *free* electrons and the problem with *infinitely* ranged interaction potentials such as the Coulomb potential respectively. The matrices  $I_n, \eta$  and  $\Lambda$  are all directly related to the electron dynamic gauge invariance of the theory [8]. The  $\eta$  and  $\Lambda$  are diagonal

$$\eta_{nm}^{\alpha\beta} = n \mathbf{1}_{nm}^{\alpha\beta}, \quad \Lambda_{nm}^{\alpha\beta} = \text{sign}(\omega_n) \mathbf{1}_{nm}^{\alpha\beta}, \quad (5)$$

with  $\eta$  standing for the *frequency* matrix and  $\Lambda$  representing the *classical* value of  $Q$  (a convenient and frequently used representation is  $Q = \mathcal{T}^{-1} \Lambda \mathcal{T}$  with unitary  $\mathcal{T}$ ). The  $I_k$  are shifted diagonal matrices

$$[I_k^\gamma]_{nm}^{\alpha\beta} = \delta_{\alpha\gamma} \delta_{\beta\gamma} \delta_{n,m+k}, \quad (6)$$

which span a  $U(1)$  algebra  $I_m^\alpha I_n^\beta = \delta_{\alpha\beta} I_{m+n}^\alpha$ . These are recognized as the generators of electrodynamic gauge transformations in Matsubara frequency space which can formally be represented by unitary rotations  $Q \rightarrow \mathcal{W}^{-1} Q \mathcal{W}$  [8]. One of the longstanding and notorious complications of Eqs. (2)–(4), however, is that the electrodynamic gauge invariance is preserved only if the Grassmannian field variables  $Q$  are taken as *infinite size* matrices in frequency space. To ensure that the different limits of the theory (*replica limit* along with *infinite* frequency space) can generally be taken in a unique, cut-off independent manner it is necessary to spell out the different physical constraints that should be rigorously retained by Eqs. (2)–(4) for all values of the bare parameters  $\sigma_{xx}, \sigma_{xy}, c$  and  $z$ . The matter has been investigated in considerable detail in a series of more recent papers on abelian quantum Hall states based on an extended version Eqs. (2)–(4) that includes the Chern Simons statistical gauge fields [8, 3]. The results can be summarized as follows.

(i) The Coulomb interaction problem displays an *exact* global symmetry, termed  $\mathcal{F}$  invariance, which means that the action with  $c = 1$  is invariant under global  $U(1)$  gauge transformations.  $\mathcal{F}$  invariance is broken by finite ranged interactions  $0 < c < 1$  as well as the free electron approximation  $c = 0$  [8].

(ii) In order for the theory to be consistent with the macroscopic conservation laws (continuity equation) it is imperative that the combination  $z(1-c)$  in Eq. (4) does not acquire radiative corrections, neither perturbatively nor in the theory at a non-perturbative level [15, 9].

•  $D = 2 + \epsilon$ . To see these general statements at work in explicit computations we discard the topological charge in Eq. (2) for the moment and present the results of the theory in  $D = 2 + \epsilon$  dimensions. These have recently been extended to one order in  $\epsilon$  higher than what was known previously [9]. The renormalizations of the  $\sigma_{xx}$  and  $zc$  fields can be expressed as follows (see Table)

$$\beta_{\sigma}^0(\sigma_{xx}, c) = \frac{d\sigma_{xx}}{d \ln b} = \epsilon \sigma_{xx} - \frac{A_1(c)}{\pi} - \frac{A_2(c)}{\pi^2 \sigma_{xx}}, \quad (7)$$

$$\gamma_{zc}^0(\sigma_{xx}, c) = \frac{d \ln zc}{d \ln b} = -\frac{B_1(c)}{\pi \sigma_{xx}} - \frac{B_2(c)}{\pi^2 \sigma_{xx}^2}. \quad (8)$$

The dependence on  $c$  is a peculiar feature of interaction terms like  $S_F$  which is quite unlike the conventional role played by ordinary mass terms [16, 17]. This dependence is, in fact, the way in which electrodynamic gauge invariance manifests itself in the theory of Eqs. (2)–(4). For example, on the basis of statement (ii) above one can express the equations for  $c$  and  $z$  in terms of  $\gamma_{zc}^0$  as follows

$$\beta_c^0(\sigma_{xx}, c) = \frac{dc}{d \ln b} = c(1-c)\gamma_{zc}^0(\sigma_{xx}, c), \quad (9)$$

$$\gamma_z^0(\sigma_{xx}, c) = \frac{d \ln z}{d \ln b} = c\gamma_{zc}^0(\sigma_{xx}, c), \quad (10)$$

which are in accordance with the results obtained in explicit computations [9]. In Fig.1 we plot the renormalization group flow lines in the  $\sigma_{xx}$  versus  $c$  plane. The results are reminiscent of those in the quantum Hall regime, to be discussed below, in that the  $\mathcal{F}$  non-invariant theory  $0 < c < 1$  generally lies in the domain of attraction of the *Fermi liquid* line  $c = 0$  whereas the  $\mathcal{F}$  invariant theory  $c = 1$  constitutes a *non-Fermi liquid* universality class all by itself. There are two different critical fixed points, one with  $c^* = 0$ ,  $\sigma_{xx}^* = \mathcal{O}(\epsilon^{-1/2})$  and one with  $c^* = 1$ ,  $\sigma_{xx}^* = \mathcal{O}(\epsilon^{-1})$ , which separate a weak coupling metallic state from a strong coupling insulating phase. The two independent critical exponents  $\nu$  and  $p$  obtained as

$$\nu^{-1} = \partial \beta_{\sigma}^* / \partial \sigma_{xx}, \quad p = D / (D + \gamma_{zc}^*) \quad (11)$$

have exactly the same meaning as those describing the quantum Hall plateau transitions. The non-Fermi liquid values obtained from the  $c^* = 1$  fixed point are given by  $\nu^{-1} = \epsilon + \mathcal{A}\epsilon^2$  and  $p = 1 + (\epsilon/8) + [(\pi^2 + 15)/12 - \mathcal{A}]\epsilon^2/16$ .

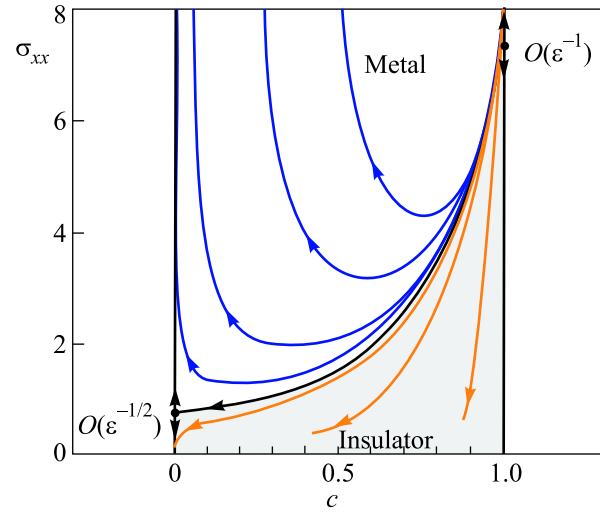


Fig.1. Sketch of scaling in the  $\sigma_{xx}$  versus  $c$  plane in  $2 + \epsilon$  dimensions. For illustration we have taken  $\epsilon = 0.1$ , see text

For finite range potentials  $0 < c < 1$  the problem generally involves three independent scaling fields. For example, let  $\Delta\sigma \approx (\sigma_{xx} - \sigma_{xx}^*)/\sigma_{xx}^* - c$  and  $c$  be the small deviations from the Fermi liquid fixed point then the scaling behavior of conductivity  $\sigma'_{xx}$  is expressed as follows

$$\sigma'_{xx} = b^{\epsilon} \sigma'_{xx} (b^{-1/\nu} \Delta\sigma; b^{D-D/p} c; b^{-D/p} zcT), \quad (12)$$

which generalizes the free particle theory ( $c = 0$ ) where the  $T$  dependence is strictly absent. Besides different exponent values  $\nu^{-1} = 2\epsilon + 3\epsilon^2$  and  $p = 1 + \sqrt{2\epsilon}$  there are also important physical differences in fundamental aspects such as the *multi fractal* singularity spectrum, the specific heat of the electron gas etc. which usually are not being probed in transport measurements [10, 9, 12].

• *QHR*. The metallic phases disappear all together in  $D = 2$ . This dramatic conflict with the QHR demands a complete understanding of the topological piece in Eq. (2). To explore the fundamental significance of the aforementioned *massless chiral edge excitations* we next introduce a change of variables  $Q = t^{-1} Q_0 t$  [10]. Here, the  $Q_0$  represents an arbitrary ‘bulk’ matrix field configuration with the classical value  $Q_0 = \Lambda$  at the edge. The  $t$  are recognized as the ‘edge’ sector of the theory describing the fluctuations about these very special boundary conditions. The distinctly different properties of the ‘bulk’ and ‘edge’ sectors of the theory can generally be expressed in terms of an effective action for the ‘edge’

$$\exp S'_{\text{eff}}[q] = \int_{\partial V} D[Q_0] \exp S_{\text{eff}}[t^{-1} Q_0 t]. \quad (13)$$

Here,  $q = t^{-1}\Lambda t$  and the symbol  $\partial V$  reminds us of the fact that the theory is evaluated with fixed boundary conditions  $Q_0 = \Lambda$ . The theory of Eq. (13), as it now stands, provides a complete description of the low energy dynamics of the electron gas for all values of  $\sigma_{xx}$ ,  $\sigma_{xy}$ ,  $c$  and  $z$  [10, 12]. To show this we elaborate on two extremely important limits of the problem. First, there is the ‘naive’ strong coupling limit of the theory which physically corresponds the situation where the Fermi energy is located in a Landau gap. In this case, all the bare parameters  $\sigma_{xx}$ ,  $z$  and  $c$  in Eqs. (2) and (3) are identically zero except  $\sigma_{xy} = k$  where the integer  $k$  denotes the number of completely filled Landau levels. The action now contains the topological piece only and can be written as [3]

$$S_{\text{eff}}[t^{-1}Q_0t] \rightarrow \frac{k}{2} \oint dxtrt \nabla_x t^{-1} \Lambda + 2\pi i k C[Q_0], \quad (14)$$

where the integral is over the edge of the system. Since  $C[Q_0]$  is by construction integer valued we conclude that  $S'_{\text{eff}}[q] = S_{\text{eff}}[q]$ , discarding constants and phase factors that are immaterial. Surprisingly, this one dimensional theory is exactly solvable and the spectrum consists of *massless chiral edge excitations* which are completely independent of the details such as the geometry of the edge, the number of field components in the theory etc. [3] It is not difficult to see that  $S'_{\text{eff}}[q]$  must in general be of exactly the same form as Eq. (14) provided the bulk sector  $Q_0$  in Eq. (13) develops a *mass gap*. Eq. (14) is therefore quite generally recognized as the *fixed point action* of the quantum Hall state with the integer  $k$  now standing for the *robustly quantized Hall conductance* rather than the filling fraction. The most important task next, however, is to show whether and how this super universal *strong coupling* feature of the  $\theta$  vacuum can be reconciled with the *weak coupling* results of Eqs. (7)–(10). This takes us to the second limit of the theory where  $S'_{\text{eff}}[q]$  is formally evaluated in terms of a series expansion in powers of both  $T$  and the derivatives acting on  $q$ . Of interest are the lowest order terms which display exactly the same form as the original action  $S_{\text{eff}}[Q]$  except that the bare parameters in Eqs (2) and (3) are now replaced by the *observable* quantities  $\sigma'_{xx}$ ,  $\sigma'_{xy}$ ,  $z'$  and  $c'$  respectively [10, 12, 9]. The expressions for  $\sigma'_{xx}$  and  $\sigma'_{xy}$ , obtained in this way, can quite generally be identified with the linear response formulae for the conductances at  $T = 0$  which now appear as a measure for the response of the bulk of the system to infinitesimal changes in the boundary conditions. The  $z'$  and  $z'c'$ , on the other hand, are analogous to the spontaneous magnetization in the classical Heisenberg ferromagnet [16].

The relation between the *observable* and *bare* theories generally defines  $\beta$  and  $\gamma$  functions according to [12]

$$\sigma'_{xx} = \sigma_{xx}^0 + \int_{b_0}^{b'} \frac{db}{b} \beta_\sigma, \quad \sigma'_{xy} = \sigma_{xy}^0 + \int_{b_0}^{b'} \frac{db}{b} \beta_\theta, \quad (15)$$

$$z'c' = z_0c_0 + \int_{b_0}^{b'} \frac{db}{b} zc\gamma_{zc}, \quad z' = z_0 + \int_{b_0}^{b'} \frac{db}{b} z\gamma_z. \quad (16)$$

A complete quantum theory which includes instanton contributions to the  $\beta$  and  $\gamma$  functions has been developed only recently. The final results satisfy the aforementioned constraints (i) and (ii) and are given by [12]

$$\beta_\sigma = \beta_\sigma^0(\sigma_{xx}, c) - \mathcal{D}(c)\sigma_{xx}^2 e^{-2\pi\sigma_{xx}} \cos 2\pi\sigma_{xy}, \quad (17)$$

$$\beta_\theta = -\mathcal{D}(c)\sigma_{xx}^2 e^{-2\pi\sigma_{xx}} \sin 2\pi\sigma_{xy}, \quad (18)$$

$$\gamma_{zc} = \gamma_{zc}^0(\sigma_{xx}, c) - \mathcal{D}_\gamma(c)\sigma_{xx} e^{-2\pi\sigma_{xx}} \cos 2\pi\sigma_{xy}, \quad (19)$$

$$\beta_c = c(1-c)\gamma_{zc}, \quad \gamma_z = c\gamma_{zc}. \quad (20)$$

Here,  $\beta_\sigma^0$ ,  $\gamma_{zc}^0$  are given by Eqs. (7), (8) with  $\epsilon = 0$  and  $\mathcal{D}(c)$ ,  $\mathcal{D}_\gamma(c)$  for  $c = 0, 1$  are listed in Table. As illustrated in Fig.2, these final results fundamentally recon-

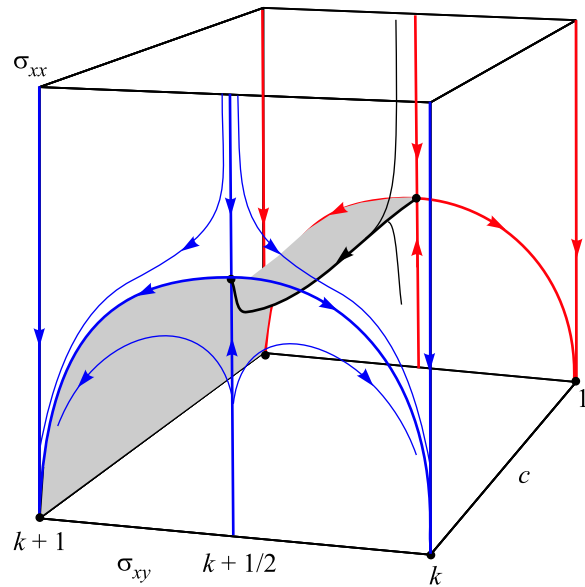


Fig.2. Sketch of the renormalization group flow in the parameter space  $\sigma_{xx}$ ,  $k \leq \sigma_{xy} \leq k+1$  and  $0 \leq c \leq 1$ , see text

cile the quasi metallic behavior (Eqs. (7)–(10)) of the electron gas at short distances and the quantum Hall effect (Eq. (14)) that generally appears at much larger length scales only.

In summary, the spin polarized electron gas in  $D = 2$  displays all the super universal features of a  $\theta$  vacuum in asymptotically free field theory that have not been recognized otherwise [10, 12]. Analogous to the theory

in  $D = 2 + \epsilon$  there are two different critical fixed points  $\sigma_{xy}^* = k + 1/2$  and finite  $\sigma_{xx}^*$  in Fig.2. The plateau transitions in the QHR therefore fall into two different universality classes with distinctly different exponent values [5]

$$\nu^{-1} = \partial\beta_\theta^*/\partial\sigma_{xy}, \quad p = 2/(2 + \gamma_{zc}^*), \quad y_\sigma = \partial\beta_\sigma^*/\partial\sigma_{xx}. \quad (21)$$

Eqs. (17)–(20), when evaluated at the Fermi liquid fixed point  $c^* = 0$  [10], are in remarkable agreement with the exponent values known from numerical work and the best estimates for Eqs. (21) lie in the range  $\nu = 2.30 - 2.38$  [7],  $p = 1.22 - 1.48$  [18] and  $y_\sigma = -(0.34 - 0.42)$  [19]. Similar to Eq. (12) we conclude that  $\sigma'_{\mu\nu} = \sigma'_{\mu\nu}(X, Y, Z)$  for finite range potentials  $0 < c \ll 1$ , where

$$X = (zcT)^{-\kappa} \Delta\theta, \quad \Delta\theta \approx \sigma_{xy} - k - 1/2 \quad (22)$$

denotes the *relevant* scaling variable and  $Y = (zcT)^{\mu_\sigma} \Delta\sigma$ ,  $Z = (zcT)^{\mu_c} c$  with  $\Delta\sigma \approx (\sigma_{xx} - \sigma_{xx}^*)/\sigma_{xx}^* + c$  are the *irrelevant* ones. The Fermi liquid exponents  $\kappa = p/2\nu = 0.29 \pm 0.04$ ,  $\mu_\sigma = -py_\sigma/2 = 0.26 \pm 0.05$  and  $\mu_c = p - 1 = 0.35 \pm 0.15$  are clearly in conflict with the experimental scaling results  $\sigma'_{\mu\nu} = \sigma'_{\mu\nu}((zT)^{-\kappa}(\sigma_{xy} - k - 1/2), (zT)^{\mu_\sigma}(\sigma_{xx} - \sigma_{xx}^*))$  [20] with the reported best exponent values  $\kappa = 0.42 \pm 0.01$  [1] and  $\mu_\sigma = 2.5 \pm 0.5$  [20] respectively. The experiment should in general be associated with a novel  $\mathcal{F}$ -invariant universality class  $c^* = 1$  in Fig.2 which therefore disqualifies any approach based on Fermi liquid ideas [1]. Although the non-Fermi liquid exponent values (Eq. (21)) are beyond the present theory, the scaling results reported in this Letter nevertheless elucidate the fundamental super universal features of the QHR which cannot be obtained in any different manner [12, 9].

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