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Semiclassical Treatment of Induced Schwinger Processes at Finite Temperature

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We consider induced pair production in an external field at finite temperature. One-loop correction to the Green function of a meson is calculated semiclassically within the framework of saddle-point analysis of Schwinger proper time integrals. This correction appears to be exponentially small in terms of inverse temperature dependence. Low-temperature limit is shown to be in full agreement with previously obtained zero-temperature results. The corrections in the low-temperature limits are estimated up to the leading exponential and pre-exponential terms. Comparison is made to earlier calculations of vacuum decay.

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1. Motivation. Spontaneous processes of particle production in field theory [1] (also known as Schwinger processes) or string/brane production [2] in string theory in external fields have long been studied. Production of e^+e^- pairs by a constant electric field is the archetypal example for the wide class of these non-perturbative phenomena. They can generally be characterized by the essentially non-analytic behaviour of observables in the external field in the weak field limit, that is, by the presence of $e^{-1/E}$-like terms. A closely related class of phenomena is known as vacuum decay processes [3, 4]. One of the similarities between Schwinger pair production and vacuum decay is that they both can be described in terms of the semiclassical approximation to the tunneling problem in quantum mechanics. That is, the leading probability or another observable is usually organized as e^{-S_E}, where S_E is some action on some classical configuration.

The process being spontaneous means the initial state must be a vacuum state. A generalization of the Schwinger phenomena to the processes in which a non-zero excitation is contained in the initial state (i.e. some particle is present) is referred to as induced Schwinger process. Induced processes of vacuum decay have also been known for quite a long time [5, 6].

Recent progress in understanding induced brane production [2] at zero temperature has lead us to the following question: how would finite temperature influence the dynamics of brane production/decay? This would be of great importance for cosmology. Brane induced decay is apt to be viewed upon as false vacuum-decay in a higher-dimensional theory. This task, when simplified down to field theory level, may be presented as particle decay in an external field. Recently decay of a magnetic monopole was calculated by the authors of the present paper within this string-motivated paradigm, the same was done for a Thirring model meson decay in 2D theory at zero temperature [7]. Here the results of [7] are generalized towards the case of finite temperature.

Studies of spontaneous Schwinger pair production at finite temperatures have a long history. A number of papers have been produced during the last three decades on the spontaneous process of pair production in one-loop approximation. Not claiming to have made a full review in the least part, we cite just a few of them [8 – 10]. A modern picture of one-loop thermal results is reflected in [11]. Two-loop results are available as well [12]. On the side of vacuum decay, the finite-temperature effects have been thoroughly studied by [13] for the spontaneous case, yielding both the exponential and the pre-exponential contributions to the decay rate. On the side of the induced decays, the pre-exponential factor was first calculated in [14].

However, no results dealing with temperature corrections to *induced* decays are available so far. This was one of the main motivations for writing this paper. We do not attempt to describe the full physical picture of a thermal ensemble in an external field, which would require taking into account the non-stationary processes of back-reaction of the produced pairs upon the ensem-

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ble, screening of the field and equilibration of the system. There will also be a competing contribution of spontaneous pair production as well, so the situation becomes quite complicated. A physically reasonable observable for such a complicated object would be equilibrium concentration rather than decay of a single particle. This would be a next step to do, and is beyond the scope of this paper. Our aim is more modest. We want to understand how one-loop corrections to Green function of a scalar particle in an external field at finite temperature. This yields, roughly, non-perturbative decay width, and is a first step towards the physically meaningful problem of non-stationary pair production with back-reaction mentioned above.

While figuring out the simplest one-loop correction to the propagator of a scalar particle due to temperature and external field, semiclassical treatment is applied which is very close to the world-line techniques by Dunne et al. [15, 16]. The semiclassical approach to Schwinger processes has been suggested since Popov's papers, see e.g. [17].

This article is organized as follows. In Section 2.1 a brief reminder of Green function techniques is given in the finite-temperature field theory, and the correction to the meson Green function is calculated, giving a universally valid expression (in terms of any regime in $\beta \equiv 1/T$). In Section 2.2 its asymptotics are studied for $\beta \to \infty$, and comments are made on the opposite limit in Section 2.3. In Section 2.4 the problem of leading and sub-leading asymptotics in semiclassical calculations, as well as on a relationship between loop resummation and multi-instanton resummation is briefly discussed. We conclude in Section 3.

2. Schwinger Processes at Finite Temperature. 2.1. General Techniques. A simple cubic interaction of a charged scalar ϕ and a neutral χ scalar in a two-dimensional theory is considered,

$$\mathcal{L}=\frac{1}{2}|D_\mu\phi|^2 - \frac{1}{2}\mu^2|\phi|^2 + \frac{1}{2}(\partial_\mu\chi)^2 - \frac{1}{2}m^2\chi^2 + \lambda\,\phi\phi^*\chi, \tag{1}$$

where the covariant derivative given as $D_\mu = \partial_\mu + ieA_\mu$. The masses of the fields being μ and m are first kept arbitrary, but after proceding to the semiclassical approximation it will be assumed that $m/\mu \ll 1$. This situation is known in vacuum decay terms as "an almost spherical bubble" and is used, e.g. in [14]. The charged field interacts with an external Abelian field A_μ.

One can show [18] that in the coordinate representation a free Green function at zero temperature for a particle of field χ with zero charge is

$$G_\chi(x,y) = \frac{1}{(4\pi)^2}\int_0^\infty \frac{d\alpha}{\alpha}\exp\left\{im^2\alpha - \frac{i(y-x)^2}{4\alpha}\right\}, \tag{2}$$

where we have omitted the pole prescription $i\varepsilon$. For the charged particle ϕ in the constant external field $A_\mu = (0, Ex_0)$ it becomes

$$G_\phi(x,y) = \frac{1}{(4\pi)^2}\int_0^\infty \frac{\epsilon d\alpha}{\sinh(\epsilon\alpha)} \times \\ \times \exp\left\{im^2\alpha - \frac{i\epsilon}{4}(y-x)^2\coth(\epsilon\alpha)\right\} \times \\ \times \exp\left\{-\frac{i\epsilon}{4}(y_1-x_1)(y_0+x_0)\right\}, \tag{3}$$

where $\epsilon = eE$; the field is considered to be far below the Schwinger limit $m^2/\epsilon \gg 1$. We shall refer to representation 3 as Schwinger parametrization, and the variable α – Schwinger parameter. When the temperature is not equal to zero, the Green function is periodic in the Euclidean time with a period $\beta \equiv 1/T$, hence [11] its generalization is organized as

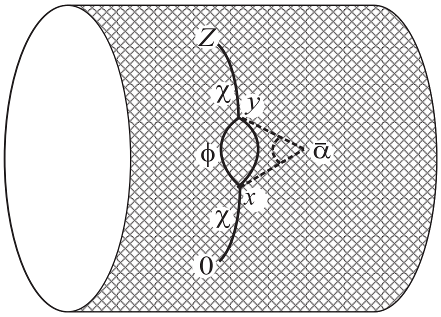
$$G_\phi(x,y) = \sum_n \int_0^\infty \frac{\epsilon\, d\alpha}{(4\pi)^2\sinh(\epsilon\alpha)} \times \\ \times \exp\left\{im^2\alpha\right\} \times \\ \times \exp\left\{-\frac{i\epsilon}{4}\left[(y_0-x_0+n\beta)^2+(y_1-x_1)^2\right]\coth(\epsilon\alpha)\right\} \times \\ \times \exp\left\{-\frac{i\epsilon}{4}(y_1-x_1)(y_0+x_0+n\beta)\right\}. \tag{4}$$

The sum over n naturally appears since one should take into account all the equivalent positions separated by β in Euclidean time, as explained in the cited paper.

Let us consider the one-loop perturbative correction to the Green function of the uncharged particle due to the cubic interaction mentioned above

$$\delta G_\chi(0,z) = \\ \lambda^2\int d^2x d^2y G_\chi(0,x)G_\phi(x,y)G_\phi(y,x)G_\chi(y,z). \tag{5}$$

We think of a one-loop diagram, represented in configuration space in Figure. We stress here that our adherence to configuration space representation of vacuum polarization is not an incidental or technical detail of calculation, but is rather of conceptual meaning. Namely, as we have shown in our previous paper [7], the saddle-point values of α_i correspond directly to the geometric parameters of the classical sub-barrier (Euclidean) trajectory in configuration space. The other reason to keep the position space is the direct relationship of the spatial configuration shown in Figure to a vacuum bubble with external lines attached to it [14] describing induced vacuum decay on a compactified manifold.



One-loop vacuum polarization correction to the propagator of field χ in an external field at finite temperature. Geometrical meaning of the saddle-point value $\overline{\alpha}$ for the Schwinger parameter α is illustrated; $0, x, y, z$ correspond to the similar variables in 5; ϕ, χ denote propagator type

The correction to Green function of the neutral χ field becomes then

$$\delta G_{\chi}(0,z) = \frac{\lambda^2\epsilon^2}{(4\pi)^8} \times$$
$$\times \sum_{p,q,k,n} \int_0^{\infty} \frac{d\alpha_1\, d\alpha_2\, d\alpha_3\, d\alpha_4\, d^2x\, d^2y}{\sinh(\epsilon\alpha_1)\sinh(\epsilon\alpha_2)\,\alpha_3\,\alpha_4} \times$$
$$\times \exp\left\{ i\mu^2(\alpha_1+\alpha_2) + im^2(\alpha_3+\alpha_4) - \right.$$
$$\left. - \frac{i(x_0+k\beta)^2}{4\alpha_3} - \frac{i(z_0-y_0+n\beta)^2}{4\alpha_4} \right\} \times$$
$$\exp\left\{ -\frac{ix_1^2}{4\alpha_3} - \frac{i(z_1-y_1)^2}{4\alpha_4} + \frac{i\epsilon\beta}{2}(y_1-x_1)(p-q) \right\} \times$$
$$\exp\left\{ -\frac{i\epsilon}{4}[(y_0-x_0+p\beta)^2+(y_1-x_1)^2]\coth(\epsilon\alpha_1) \right\} \times$$
$$\exp\left\{ -\frac{i\epsilon}{4}[(y_0-x_0+q\beta)^2+(y_1-x_1)^2]\coth(\epsilon\alpha_2) \right\}. \quad (6)$$

Note that due to temperature, Lorentz-invariance is explicitly broken in this expression. We may relate coordinate representation of the Green function correction to

$$\delta G(n,k_1) = \int \delta G(0;z_0,z_1) \times$$
$$\exp\{-i\omega_n z_0 - ik_1 z_1\}\, dz_0\, dz_1, \quad (7)$$

where $\omega_n = 2\pi n/\beta$. Calculation of this correction corresponds, as usually, to the shift of the Green's function pole

$$G(n,\mathbf{k}) = \frac{i}{\omega_n^2 + k_1^2 + m^2 + M^2(n,k_1)}, \quad (8)$$

however, now the shift is not Lorentz-invariant but rather depends on both n and $\mathbf{k}$ separately. Whatever complicated expression for the variation of the Green function one would obtain, physically relevant information is contained in the pole shift and rescaling of the wave function. Further on, only the pole shift will be considered. The mass shift is the value of the M^2 calculated at the point corresponding to the pole. For the zero temperature theory that means that the shift is determined only by the value of bare mass (and a scale). Since in this case $M^2(k^2)$ depends only on $k^2 = k_0^2 - k_1^2$, one should take $k^2 = m^2$. In a theory in a compactified Euclidean space-time it is quite obvious that the shift should depend on n as $M^2(n,k_1)$, where n and k_1 such that $\omega_n^2 + k_1^2 + m^2 = 0$. For the n-th Matsubara mode the pole shift is approximately related to a Green function variation as

$$\delta m_n^2 = -\left[\left(m^2 + k_1^2 + \frac{4\pi^2 n^2}{\beta^2}\right)^2 \delta G(k_1,n)\right]_{\omega_n^2+\mathbf{k}^2+m^2=0}. \quad (9)$$

Moreover the on-shell condition does not have a solution for arbitrary k_1^2 since ω_n is determined by the discrete variable n. Note again that generically every single mode is renormalized in its own way. One can easily understand that due to Lorentz symmetry violation by compactifying the Euclidean time direction there is no invariant mass anymore. Rather, if some physical quantities related to thermal theory are of interest, the partition function $Z(\beta,\mu,\dots)$ has to be calculated, at some values of chemical potential μ and other external potentials. Then some statistically averaged reasonable quantities have to be found[2)], say, $n_i = \partial Z/\partial\mu_i$, which is the equilibrium concentration of particles of the i-th type.

Therefore, 9 is understood merely as a convenient way of writing down the propagator variation. However, there is a range of parameters within which it is still possible to preserve the meaning of this quantity as the mass shift of the particle. This range is the limit of small temperatures or large β. In this case on-shell condition can be solved even for $k_1 = 0$, since for sufficiently large β the value of $m\beta/2\pi$ differs from an integer slightly. So one can treat the imaginary part of the mass shift for such n and $k_1 = 0$ as the decay rate of the particle in the external field with a temperature not equal to zero.

Evaluating elementary integrals, one gets a formal expression for the mass shift

[2)]The authors are extremely grateful to Professor H. M. Kleinert for a discussion on this point.

$$\delta m^2 = \frac{\lambda^2}{\beta\,\epsilon^{3/2}} \sum_{r,s\in\mathbb{Z}} \delta_{n+r+s} \times$$

$$\times \int\limits_0^{+\infty+i0} \frac{d\alpha_1 d\alpha_2}{\sqrt{\sinh(\alpha_1+\alpha_2)\cosh(\alpha_1-\alpha_2)}} \times$$

$$\times \exp\left\{\frac{4\pi^2 i}{\epsilon\beta} \times\right.$$

$$\times \frac{(r\tanh(\alpha_1)-s\tanh(\alpha_2))^2 \sinh(2\alpha_1)\sinh(2\alpha_2)}{4\sinh(\alpha_1+\alpha_2)\cosh(\alpha_1-\alpha_2)} +$$

$$\left. + ir^2\tanh(\alpha_1)\right\} \times$$

$$\exp\left\{i\left[s^2\tanh(\alpha_2) + \frac{\mu^2}{\epsilon}(\alpha_1+\alpha_2) + \right.\right.$$

$$\left.\left. + \frac{4\pi^2 n^2}{\epsilon\beta^2}\frac{1}{\coth(\alpha_1)+\coth(\alpha_2)}\right]\right\}. \tag{10}$$

By using the approximate on-shell condition $n \approx \mu\beta/2\pi$ and Poisson resummation formula

$$\sum_{n=-\infty}^{\infty} f(n) = \sum_{n=-\infty}^{\infty} \tilde{f}(2\pi n),$$

where

$$\tilde{f}(k) = \int_{-\infty}^{\infty} f(t) e^{-ikt} dt,$$

this expression can be written down in two equivalent representations. Now and further we retain its imaginary part solely, rather then the full (possibly divergent) pole shift in the propagator. These two representations are

$$\Gamma = \mathrm{Im}\frac{\lambda^2}{m\beta\,\epsilon^{3/2}} \times \tag{11}$$

$$\times \sum_{s=-\infty}^{+\infty} \int \frac{d\alpha_1\, d\alpha_2\, \mathrm{e}^{\frac{i\mu^2}{\epsilon}(\alpha_1+\alpha_2)}}{\sqrt{\sinh(\alpha_1+\alpha_2)\,\cosh(\alpha_1-\alpha_2)}} \times$$

$$\times \exp\left\{-\frac{im^2}{\epsilon}\frac{1}{\coth(\alpha_1)+\coth(\alpha_2)} + \frac{4i\pi^2 A}{\epsilon\beta^2}(s-s_0)^2\right\}$$

and

$$\Gamma = \mathrm{Im}\frac{\lambda^2}{m\epsilon}\sum_{s=-\infty}^{+\infty}\int \frac{d\alpha_1\, d\alpha_2\, \mathrm{e}^{\frac{i\mu^2}{\epsilon}(\alpha_1+\alpha_2)}}{\sinh(\alpha_1+\alpha_2)}$$

$$\times\left\{-\frac{im^2}{\epsilon}\frac{1}{\coth(\alpha_1)+\coth(\alpha_2)} - \frac{i\epsilon\beta^2 s^2}{4A} - 2\pi i s s_0\right\}, \tag{12}$$

where

$$A = \frac{\sinh(\alpha_1+\alpha_2)}{\cosh(\alpha_1-\alpha_2)},$$

$$s_0 = \frac{m\beta}{2\pi}\frac{1}{\tanh(\alpha_2)\coth(\alpha_1)+1}.$$

The sums above can formally be converted to Jacobi theta-functions, however, that would not be of much use, since the integrals over Schwinger parameters would then get out of feasibility. On the contrary, one can do the integrals in Schwinger parameters by saddle-point method for each term in the sum, provided saddle-point works at all. Then in principle one could try to do the sum exactly.

When dealing in such way with 1-dimensional saddle-point integrals

$$\int dz \sum_n \mathrm{e}^{if_n(z)}$$

it will be necessary to restrict the domain of applicability of this approximation by imposing the conditions in saddle-point values $z = \bar{z}_n$ for the nth function f_n [19]:

$$|f_0''(\bar{z}_0)|^{3/2} \gg |f_0'''(\bar{z}_1)|, \tag{13}$$

$$|f_1''(\bar{z}_0)|^{3/2} \gg f_1'''(\bar{z}_1), \tag{14}$$

$$\mathrm{Im}[f_0(\bar{z}_0)] \gg \mathrm{Im}[f_1(\bar{z}_1)]. \tag{15}$$

Condition 13 ensures the possibility of doing saddle-point approximation for the zero mode. It checks whether the next-to-leading order terms in the decomposition of f_0 may be neglected. Condition 14 provides the same check for the first mode. To ensure dominance of the zero mode over the first one, we impose 15.

For multi-dimensional expressions the criteria of saddle-point method applicability become more complicated. Namely, instead of 13 or 14 one must require that

$$\frac{\partial^3 f}{\partial z_i \partial z_j \partial z_k} P_{il} P_{jm} P_{kn} \frac{1}{\sqrt{\lambda_l \lambda_m \lambda_n}} \ll 1, \tag{16}$$

where λ_i are eigenvalues of second derivatives matrix $\partial^2 f/\partial z_i \partial z_j$, and P_{ij} is diagonalisation matrix for $\partial^2 f/\partial z_i \partial z_j$, summation implied over i, j, k, l, m, n. Further it will be examined whether these conditions are satisfied for a particular saddle-point function under consideration.

2.2. Limit $\beta \to \infty$. The expression 12 seems to be an appropriate representation for the Γ in the case $\beta \to \infty$, since even the naive condition of the saddle point method applicability fails for 11, namely the factor in the exponent $1/\epsilon\beta^2$ becomes small. So, let us take 12 and make sure that it indeed corresponds to the low-temperature limit. It is supposed that the saddle-point is a symmetric point $\alpha_1 = \alpha_2 = \alpha$. This is possible due to having particles of identical mass in the internal lines. Since the saddle-point values of Schwinger proper times, as shown in [7], correspond to the geometric parameters of the classical Euclidean loop configuration, only a symmetric configuration is expected to be the physically relevant solution of saddle-point equations.

The full "decay width" is a sum over Matsubara contributions

$$\Gamma = \sum_{n=-\infty}^{\infty} \Gamma_n.$$

It is expected that the higher the mode, the more suppressed it is. It will be shown below by means of saddle-point integral that this is indeed true for the zeroth and first modes. One can trivially see that the zero Matsubara mode exponential is

$$f_0 = \frac{\mu^2}{\epsilon} 2\alpha - \frac{m^2}{\epsilon} \frac{\tanh(\alpha)}{2},$$

identical to that of [7]. This function is extremized for $\alpha = \bar{\alpha}_0$ given by

$$\cosh(\bar{\alpha}_0) = m/2\mu.$$

For simplicity the case of a very light external particle ("almost spherical bubble") will be considered, that is $m/\mu \ll 1$. This is indeed the case of interest, as for a particle with $m > 2\mu$ the process will become perturbatively allowed. Checking the conditions of saddle-point applicability 16, one gets

$$\frac{3}{\sqrt{2}\sqrt{m\mu/\epsilon}} \ll 1.$$

This condition is satisfied in our setting, due to having a sufficiently small field $m^2/\epsilon \gg 1$, imposed from the very beginning for the applicability of the saddle-point method (there always must be a significant exponential suppression). Thus the leading order (zero dual Matsubara mode) contribution to the sum 12 is

$$\Gamma_0 \sim \frac{\pi}{\sqrt{\det_{i,j} \partial_i \partial_j f_0}} e^{-f_0(\bar{\alpha})}.$$

In the leading order in both small parameters m/μ and $1/\epsilon\beta^2$ one obtains

$$\det_{i,j} \partial_i \partial_j f_0 = \frac{4\mu^4}{\epsilon^2},$$

hence

$$\Gamma_0 \sim \frac{\lambda^2}{m^2 \mu} e^{-\frac{\pi\mu^2}{\epsilon}}, \tag{17}$$

in agreement with [7]. The next-to-leading term is given by the modes with $s = \pm 1$. In the limit $\epsilon\beta^2 \gg 1, \mu \gg m$ one obtains

$$f_{\pm 1} = \frac{\mu^2}{\epsilon} 2\alpha - \frac{\epsilon\beta^2}{4\sinh 2\alpha}.$$

That amounts to the saddle-point equations

$$\cosh 2\alpha = 4\mu^2/\epsilon^2\beta^2,$$

solved by

$$\bar{\alpha} \approx \pi i/4$$

in the said approximation. Collecting all the terms, one gets the first correction

$$\Gamma_{\pm 1} \sim \frac{\lambda^2}{m\epsilon^2\beta^2} e^{-\epsilon\beta^2/4}. \tag{18}$$

It can be seen that the dependence on a temperature is essentially non-perturbative.

Upon calculating the second and the third derivatives of $f_{\pm 1}$, one gets the following inequality as the condition for saddle-point method applicability by evaluating 16

$$16\mu^2/\beta^3\epsilon^{5/2} \ll 1, \tag{19}$$

which can be rewritten as

$$\beta \gg \frac{1}{\mu} \left(\frac{\mu^2}{\epsilon}\right)^{5/6}. \tag{20}$$

Obviously this condition is fulfilled provided that the temperature is high enough. The value of the exponential $f_{\pm 1}$ on the saddle point in the leading asymptotics is

$$f_{\pm 1}(\bar{\alpha}_0) = \frac{i}{4}\beta^2\epsilon.$$

In the aforementioned limit the s-th mode will be suppressed like $e^{-\epsilon\beta^2 s^2}$. The condition for the effective suppression roughly is

$$\beta \gg \mu/\epsilon,$$

which is satisfied for $\beta \to \infty$ limit. Thus one makes sure that the chosen form of the series 12 indeed suites low-temperature region description.

2.3. Limit $\beta \to 0$? Naively, it seems that one can easily perform similar calculations for the opposite case of extremely high temperatures. Just using the dual representation for the decay rate 11 and applying the saddle point method one could get the answer. However, this is not necessarily so for several reasons. In both representations for Γ the pole has already been chosen by setting $k_1^2 = 0$. It is easy to see that it is impossible to set $k_1^2 = 0$ and find such an integer value n which would satisfy the on-shell condition $4\pi^2 n^2/\beta^2 + m^2 = 0$ for small β. So, in order to fulfill the on-shell condition and, thus, to find the proper expression for the rate in the limit of high temperature, one should keep $\mathbf{k}^2$ non-zero, which reflects violation of Lorentz symmetry by introducing a temperature. It is also worth mentioning that in this case, since one cannot get rid of the dependence on n, it is absolutely necessary to consider the renormalization for each Matsubara mode separately. The result certainly cannot be interpreted then as a high temperature decay rate of the initial particle. It is just some correction to the Green function, the physical meaning of which is not well defined. It may be meaningful in a compactified theory rather than in a thermal one. Intuitively one expects that at high temperatures tunneling term e^{-S_0} (instanton contribution to the semiclassical expression) will be dominated over by the over-barrier term $e^{-\beta E}$("sphaleron" contribution). However, in the saddle-point analysis performed by us, the "sphaleron" term has not appeared in the limit $\beta \to 0$. Thus no final judgement is passed on the applicability of saddle-point method at $\beta \to 0$, but it is doubtful that it can work as directly as it has worked for $\beta \to \infty$.

2.4. Resummation Hierarchy. The values of $\bar{\alpha}$ given above are, of course, not unique. The true solution to the saddle-point equation is a series of roots like, say, $\bar{\alpha} = \pm i \arccos(m/2\mu) + 2\pi i n, \quad n \in \mathbb{Z}$. Therefore, an additional resummation to include all these roots is to be performed in principle. It is not necessary from a practical point of view, the terms in the series being suppressed by the factor $e^{-\mu^2/\epsilon}$, however, this resummation is noted in order to stress the similarity of the saddle-point configuration, on which the 1-loop integral is essentially calculated, and the world-line instanton configuration, proposed in the semiclassical approach by Dunne et al. [15, 16].

3. Conclusion. An example of a calculation of the one-loop thermal corrections to the propagator of a neutral scalar particle interacting with a charged one in an external field has been given. We have found the LO thermal correction to decay width in the semiclassical limit for $\beta \to \infty$, in the case of an "almost spherical bubble", i.e. $m/\mu \ll 1$, far below Schwinger limit $m^2/\epsilon \gg 1$, up to its preexponential factor

$$\delta\Gamma \sim \frac{\lambda^2}{m\epsilon^2\beta^2}\, e^{-\epsilon\beta^2/4}. \tag{21}$$

Thus the result of [7] has been generalized towards thermal/compactified backgrounds. Of peculiar interest would be extension of the presented semiclassical techniques to strings and branes in thermal backgrounds. Our result supports that of Garriga [13]. Namely, the technique of Garriga does not intercept any correction for vacuum decay in two dimensions at low temperatures, whereas we give an estimate of this correction, and point out that due to its rapid decrease it can't have been noticed in the framework of Garriga's method.

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