Pis'ma v ZhETF, vol. 88, iss. 3, pp. 234 – 237

Paramagnetic intrinsic Meissner effect in a bulk

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Submitted 25 June 2008

We calculate a free energy of a quasi-two-dimensional (Q2D) superconductor with $\xi_\perp < d$ in a parallel magnetic field, where $\xi_\perp$ is a perpendicular to the conducting layers coherence length and d is an interlayers distance. It is shown to be different from that in the famous Lawrence-Doniach model. In particular, at high enough magnetic fields, the Meissner currents are found to create an unexpected paramagnetic moment due to shrinking of the Cooper pairs "sizes" in a direction perpendicular to the conducting layers. We suggest to measure this paramagnetic intrinsic Meissner effect in Q2D superconductors and superconducting super-lattices.

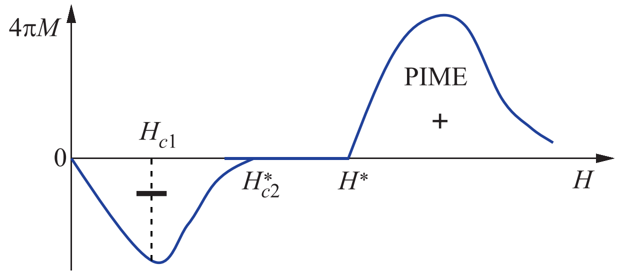
PACS: 74.20.Rp, 74.25.Op, 74.70.Kn

The Meissner diamagnetic effect is known to be responsible for destruction of superconductivity in a magnetic field both in type-I and type-II superconductors. Meanwhile, as shown in Refs. [1 – 4], quantum effects of an electron motion in a magnetic field result in the appearance of superconductivity surviving in high magnetic fields in layered [1 – 3] and isotropic three-dimensional [4] type-II superconductors. In particular, in a layered conductor in a parallel magnetic field, where the Landau levels quantization is not possible, the Bragg reflections are shown [1 – 3, 5, 6] to play an important role. The above mentioned quantum effects result in 3D $\rightarrow$ 2D dimensional crossover of an open electron spectrum in an arbitrary weak magnetic field. This causes the field-induced spin-density-wave (FISDW) and field-induced charge-density-wave (FICDW) instabilities in layered quasi-one-dimensional (Q1D) conductors [5, 6]. More complicated 3D $\rightarrow$ 1D $\rightarrow$ 2D dimensional crossovers are shown [7, 8] to be responsible for the experimentally observed non-trivial angular magnetic oscillations in a metallic phase of different quasi-low dimensional conductors, including Lebed Magic Angle [7] and Lee–Naughton–Lebed [8] oscillations.

The similar 3D $\rightarrow$ 2D dimensional crossovers have to be responsible for a stabilization of superconductivity phase in layered Q1D [1, 3] and quasi-two-dimensional (Q2D) [2] conductors since 2D superconductivity is not destroyed in a parallel magnetic field. As shown in Refs. [1 – 3]: (i) the upper critical field is divergent, $H^{\parallel}_{c2}(T) \rightarrow \infty$ as $T \rightarrow 0$, and (ii) there is some critical filed, H^*, above which superconducting temperature grows in an increasing magnetic field. Such superconducting phase with $dT_c/dH > 0$ is called the Reentrant Superconductivity (RS) [1 – 4]. The original results [1 – 3] have been theoretically confirmed by a number of studies [9 – 15], including Refs. [14, 15] where paramagnetic Meissner effect was suggested for a triplet Q1D superconductor. Despite of a great success [6] of the dimensional crossovers concepts in the explanations of magnetic properties in a metallic, the FISDW, and the FICDW phases of organic conductors, so far there has been no evidence that superconducting temperature can grow in a magnetic field. A possibility of the RS phase to exist was experimentally studied in Q1D layered organic superconductors $(TMTSF)_2X$ ($X = PF_6$ and $X = ClO_4$) [16 – 18]. These experiments gave hints on a possibility for superconductivity to exceed the quasi-classical upper critical field, $H^{\parallel}_{c2}(0)$ – the effect predicted in Refs.[1 – 4, 9 – 15]. On the other hand, they were not able to confirm the appearance of the RS phase with $dT_c/dH > 0$.

The main obvious difficulty in the above mentioned efforts to discover the RS phase is the Pauli spin-splitting destructive mechanism against superconductivity and the related Clogston paramagnetic limiting field, H_p [19]. It is absent only for some triplet superconducting phases which are believed to exist in $(TMTSF)_2X$ conductor [3, 9]. On the other hand, recently there have appeared the NMR measurements [20] in favor of a singlet nature of superconductivity in $(TMTSF)_2ClO_4$ material.

The goal of our Letter is a two-fold one. Firstly, we show that the RS phase reveals itself in Q2D superconductors as another unique phenomenon – paramagnetic intrinsic Meissner effect (PIME) (see Figure). And secondly, we extend the microscopic theory [2] to describe the most important from experimental point of view Q2D superconductors with $\xi_\perp < d$, where $\xi_\perp$ is a coherence length perpendicular to the conducting layers and d is an inter-layer distance. We suggest to mea-



In-plane magnetization, $4\pi\mathbf{M}$, is sketched as a function of a magnetic field in the absence of the Pauli spin-splitting effects, where at high magnetic fields, $H \geq H^*$, the Reentrant Superconducting phase reveals itself as a paramagnetic intrinsic Meissner effect (PIME)

sure in-plane magnetization in high magnetic fields to discover the PIME and RS phenomena in Q2D superconductors. For these purposes, we derive a free energy of a Q2D superconductor with $\xi_\perp < d$ in a parallel magnetic field from the Gor'kov formulation [21–23] of the microscopic superconductivity theory. Our results coincide with that of the Lawrence-Doniach (LD) model [24,25] only at low enough magnetic fields, $H \ll H^*$, where the Meissner effect is diamagnetic. We show that, at high magnetic fields, $H \sim H^* \leq H_p$, the field starts to shrink the Cooper pair "sizes" in perpendicular to conducting layers direction due to 3D $\to$ 2D crossovers in a parallel magnetic field. The above mentioned 3D $\to$ 2D crossovers of the Cooper pairs are not taken into account in the LD model and, as shown below, are responsible for the unique PIME phenomenon.

Let us consider a layered superconductor with a Q2D electron spectrum,

$$\epsilon(\mathbf{p}) = (p_x^2 + p_y^2)/2m + 2t_\perp \cos(p_z d),\qquad t_\perp \ll \epsilon_F = p_F^2/2m, \tag{1}$$

in a parallel magnetic field,

$$\mathbf{H} = (0, H, 0), \quad \mathbf{A} = (0, 0, -Hx), \tag{2}$$

where p_F is the Fermi momentum, $t_\perp$ is an overlapping integral of electron wave functions in a perpendicular to the conducting planes direction, ϵ_F is the Fermi energy. Electron spectrum (1) can be linearized near 2D Fermi surface (FS), $(p_x^2 + p_y^2)/2m = \epsilon_F$, in the following way,

$$\epsilon(\mathbf{p}) - \epsilon_F = v_F \sin\phi[p_x - p_F \sin\phi] + 2t_\perp \cos(p_z d), \tag{3}$$

where $v_F = p_F/m$ is the Fermi velocity, ϕ is angle between in-plane component of electron momentum and magnetic field. In the gauge (2), electron Hamiltonian in a magnetic field can be obtained from Eq.(3) by means of the Peierls substitution method, $p_x \to -i(d/dx)$, $p_z \to p_z + (e/c)Hx$ [5]. Therefore, electron Green functions in a magnetic field satisfy the following differential equation,

$$\begin{gathered}\left\{ i\omega_n - v_F \sin\phi\left[-i\frac{d}{dx} - p_F \sin\phi\right] + \right.\\ \left. + 2t_\perp \cos\left(p_z d + \frac{eHdx}{c}\right) + 2\mu_B H s \right\} \times \\ \times G_{i\omega_n}(x, x_1; \phi, p_z; s) = \delta(x - x_1),\end{gathered} \tag{4}$$

where ω_n is the Matsubara frequency [21], μ_B is the Bohr magneton, $s = \pm(1/2)$ is an electron spin projection along quantization y-axis; $\hbar \equiv 1$.

It is important that Eq.(4) can be solved analytically. As a result, we obtain,

$$\begin{gathered} G_{i\omega_n}(x, x_1; \phi, p_z; s) = -i\frac{\mathrm{sgn}\,\omega_n}{v_F \sin\phi} \times \\ \times \exp\left[-\frac{\omega_n(x - x_1)}{v_F \sin\phi}\right] \exp[ip_F \sin\phi(x - x_1)] \times \\ \times \exp\left[\frac{2i\mu_B s H(x - x_1)}{v_F \sin\phi}\right] \exp\left\{\frac{i\lambda}{2\sin\phi} \times \right. \\ \left. \times \left[\sin\left(p_z d + \frac{eHdx}{c}\right) - \sin\left(p_z d + \frac{eHdx_1}{c}\right)\right]\right\}, \end{gathered} \tag{5}$$

where $\lambda = 4t_\perp c/ev_F H d$.

Linearized gap equation, determining superconducting transition temperature, $T_c(H)$, can be derived using Gor'kov equations for non-uniform superconductivity [2,22,23]. As a result, we obtain,

$$\begin{gathered} \Delta(x) = \\ = g^* \int_0^{2\pi} \frac{d\phi}{2\pi} \int_{|x - x_1| > |v_F \sin\phi|/\Omega}^{\infty} \frac{2\pi T dx_1}{v_F \sin\phi \sinh\left[\frac{2\pi T |x - x_1|}{v_F \sin\phi}\right]} \times \\ \times \cos\left[\frac{2\mu_B H(x - x_1)}{v_F \sin\phi}\right] \times \\ \times J_0\left\{\frac{2\lambda}{\sin\phi} \sin\left[\frac{eHd(x - x_1)}{2c}\right] \times \right. \\ \left. \times \sin\left[\frac{eHd(x + x_1)}{2c}\right]\right\} \Delta(x_1), \end{gathered} \tag{6}$$

where g^* is an effective coupling constant, Ω is a cut-off energy. (Note that, although Eq.(6) is derived for singlet s-wave superconductors, it is also valid for d-wave superconductors, if we redefine properly anisotropic coherence lengths and the effective coupling constant g^*.)

We point out that Eq.(6) is the most general one among the existing equations to determine the parallel upper critical field in a layered superconductor. In particular, it takes into account the Bragg reflections and related 3D $\rightarrow$ 2D dimensional crossovers of electrons, which move in the extended Brillouin zone in a parallel magnetic field. As shown in Ref. [6], the above mentioned quantum effects result in a momentum quantization law for an electron momentum component along x-axis. This is a reason why the kernel of the integral Eq.(6) is periodic [1–3] with respect to variables x and x_1. In the case, where the destructive Pauli spin-splitting effects against superconductivity are absent (i.e., at $\mu_B = 0$ in Eq.(6)), Eq.(6) possesses a periodic solution for $\Delta(x)$ at any magnetic field. In this case, superconductivity is stable in an arbitrary strong magnetic field and exists at high fields in a form of the RS phase with $dT_c/dH > 0$. In the case of a singlet superconductivity, which is considered in the Letter, the Pauli spin-splitting effects may eliminate the superconductivity with $dT_c/dH > 0$.

As the most general equation, Eq.(6) contains the Ginzburg–Landau and Lawrence–Doniach descriptions as its limiting cases at low enough magnetic fields, $H \ll H^*$. For the so-called Josephson coupled layered superconductors with $\xi_\perp < d$ [24, 25], Eq.(6) may be simplified and rewritten in the following differential form,

$$\left[\frac{T_c - T}{T_c} - C_1\left(\frac{\mu_B H}{\pi T_c}\right)^2 + \xi_\parallel^2 \frac{d^2}{dx^2} - C_2(H) + \right.$$
$$\left. + C_3(H)\cos\left(\frac{2x\omega_c}{v_F}\right)\right]\Delta(x) = 0, \tag{7}$$

with

$$C_1 \simeq 2.1,$$

$$C_2(H) = \frac{8t_\perp^2}{\omega_c^2}\left\langle \frac{1}{\sin^2\phi}\int_0^\infty \frac{dz}{\sinh(z)}\sin^2\left[\frac{\omega_c \sin\phi}{4\pi T_c}z\right]\right\rangle_\phi \tag{8}$$

and

$$C_3(H) = \frac{8t_\perp^2}{\omega_c^2}\left\langle \frac{1}{\sin^2\phi}\int_0^\infty \frac{dz}{\sinh(z)} \times \right.$$
$$\left. \times \sin^2\left[\frac{\omega_c\sin\phi}{4\pi T_c}z\right]\cos\left[\frac{\omega_c\sin\phi}{4\pi T_c}z\right]\right\rangle_\phi. \tag{9}$$

(Here, $\omega_c = eHv_F d/c$ is a characteristic frequency of an electron motion along open FS (1) [3]; $< ... >_\phi$ means an average over the angle ϕ, $\xi_\parallel = \sqrt{7\zeta(3)}v_F/4\sqrt{2}\pi T_c$ is in-plane anisotropic Ginzburg–Landau coherence length, $\omega_c(H) \sim T_c \geq \mu_B H$.)

Note that Eqs.(7)–(9) extend the Lawrence–Doniach model [24, 25] to the case of strong magnetic fields and can be called the extended Lawrence–Doniach equations. In contrast to the traditional equations, the coefficients $C_1(H)$ and $C_2(H)$ in Eqs.(7)–(9) depend on a magnetic field, which means that a probability for the Cooper pair to jump from one conducting layer to another depends on the field. This important feature of Eqs.(7)–(9) is a consequence of shrinking of the Cooper pairs "sizes" due to 3D $\rightarrow$ 2D dimensional crossovers in a parallel magnetic field [1–3, 6].

Below, we are interested in descriptions of the RS and PIME phenomena, therefore, we consider Eqs.(7)–(9) at high magnetic fields. It is possible to show that at $H \geq H^*$ the solution of Eq.(7) can be represented as $\Delta(x) = \Delta = \text{const}$, which corresponds to the RS phase [1, 2]. In this case, the corresponding second order term of a free energy with respect to the order parameter Δ can be written in the following simple form,

$$F^2(T,H) = -N(\epsilon_F)\left[\frac{T_c(H) - T}{T_c}\right]\Delta^2, \tag{10}$$

with

$$T_c(H) = T_c - 2.1\frac{t_\perp^2}{\pi^2 T_c} -$$
$$- 2.1\frac{(\mu_B H)^2}{\pi^2 T_c} + 0.25\frac{t_\perp^2}{\pi^2 T_c}\left(\frac{\omega_c}{\pi T_c}\right)^2, \tag{11}$$

where $N(\epsilon_F)$ is a density of states per one electron spin projection at $\epsilon = \epsilon_F$.

Note that the third term in Eq.(11) describes destruction of a singlet superconducting phase by the Pauli spin-splitting effects, whereas the last term in Eq.(11) is responsible for the restoration of superconductivity at high magnetic fields and for the appearance of the RS phase and the PIME phenomenon. If we take into account that the fourth order term of a free energy with respect to the order parameter Δ can be calculated at $H \geq H^*$ in a standard manner [22], $F^4 = 7\zeta(3)N(\epsilon_F)\Delta^4/16\pi^2T_c^2$, then we can minimize the total free energy and find that

$$\delta F(T,H) = -\frac{4\pi^2}{7\zeta(3)}N(\epsilon_F)[T_c(H) - T]^2. \tag{12}$$

Magnetization can be found by a differentiation of the free energy with respect to a magnetic field,

$$M(T,H) = -\frac{\partial \delta F(T,H)}{\partial H} + M_0 =$$
$$= \frac{8\pi^2}{7\zeta(3)}N(\epsilon_F)(T_c - T)\frac{dT_c(H)}{dH} + M_0, \tag{13}$$

$$M(T,H)=\frac{8}{7\zeta(3)}N(\epsilon_F)\left(\frac{T_c-T}{T_c}\right)\times$$
$$\times\left[-4.2\ \mu_B^2+0.5\left(\frac{et_\perp v_F d}{\pi T_c c}\right)^2\right]H+M_0, \qquad (14)$$

where M_0 is a magnetic moment in a metallic phase.

Eqs.(11)–(14), which are valid at $H^* \leq H \leq H_p$, are the main results of the Letter. Note that, in Eq.(14), the first term corresponds to a destruction of superconductivity due to the Pauli spin-splitting effects, whereas the second term represents unusual paramagnetic orbital contribution to a magnetic moment due to the Meissner currents (i.e., the PIME phenomenon) (see Figure).

It is important that the last term, M_0, in Eqs.(13), (14) is normally negligible, therefore, the Meissner magnetic moment becomes paramagnetic one, $M > 0$, in the RS phase, where $dT_c/dlH > 0$. As it follows from Eqs.(11), (13), (14), both the RS phase and the PIME phenomenon occur if $ev_F d/c > 2.9\mu_B(\pi T_c/t_\perp)$, which is equivalent to

$$\omega_c(H)=\frac{ev_F dH}{c} > 2.9\left(\frac{\pi T_c}{t_\perp}\right)\ \mu_B H. \qquad (15)$$

Therefore, the most direct method to discover the RS and PIME phenomena is to find such layered compound or to create such superconducting superlattice, where the condition (15) is fulfilled. There is also indirect method [26], which allows to detect the PIME phenomenon in superconductor with $\xi_\perp < d$ even in the case, where inequality (15) is not fulfilled.

The author is thankful to N.N. Bagmet, P.M. Chaikin, and M.V. Kartsovnik for the useful discussions. This work was supported by the NSF grant DMR-0705986.

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