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Jet quenching with running coupling including radiative and collisional energy losses

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We calculate the nuclear modification factor for RHIC and LHC conditions accounting for the radiative and collisional parton energy loss with the running coupling constant. We find that the RHIC data can be explained both in the scenario with the chemically equilibrium quark-gluon plasma and purely gluonic plasma with slightly different thermal suppression of the coupling constant. The role of the parton energy gain due to gluon absorption is also investigated. Our results show that the energy gain gives negligible effect.

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1. It is widely believed that suppression of the high-p_T hadrons in AA-collisions (jet quenching (JQ)) observed at RHIC (for a review, see [1]) is dominated by the induced gluon emission [2 – 7] in the hot quark-gluon plasma (QGP) produced at the initial stage of AA-collisions. There are currently considerable theoretical efforts in the development of the quantitative methods for computation of JQ [8 – 13] which can be used for the tomographic analysis of the QGP. In the present paper we study JQ using the light-cone path integral (LCPI) approach to the radiative energy loss [3, 4]. In this formalism the probability of gluon emission is expressed through the Green's function of a two-dimensional Schrödinger equation with an imaginary potential. This approach has not the restrictions on the applicability of both the BDMPS approach [2] (valid only for massless partons in the limit of strong Landau–Pomeranchuk–Migdal effect) and the GLV formalism [6] (applicable only to a thin plasma in the regime of small Landau-Pomeranchuk-Migdal suppression). We perform the calculations with accurate treatment of the Coulomb effects. If one neglects these effects the gluon spectrum can be expressed in terms of the oscillator Green's function and the medium may be characterized by the well-known transport coefficient $\hat{q}$ [2, 4]. However, the oscillator approximation can lead to uncontrolled errors since it gives a physically absurd prediction that for massless partons the dominating $N = 1$ rescattering contribution vanishes [14, 15]. Besides the radiation energy loss we include the collisional energy loss. Both the contributions are calculated with the running coupling constant. Also, we investigate the impact of the parton energy gain due to gluon absorption from the QGP on JQ.

We calculate the nuclear modification factor R_{AA}, which characterizes JQ, accounting for the fluctuations of the parton path lengths in the QGP. In the treatment of multiple gluon emission we use a new method which takes into account time ordering of the DGLAP and the induced radiation stages. We compare the theoretical results with the data obtained at RHIC by the PHENIX Collaboration[16] and give prediction for LHC. Our principle purpose in comparing with the RHIC data is to understand whether the observed JQ is consistent with the entropy of the QGP required by the hydrodynamical simulations of the AA-collisions for reproducing the observed particle multiplicities. Our results show that JQ and particle multiplicities can be naturally reconciled. Contrary to the conclusion of Ref. [17] that the observed at RHIC JQ is consistent only with purely gluonic plasma, we find that the scenario with the chemically equilibrium QGP is also possible. A good description of the JQ RHIC data can be obtained in this scenario with the thermal suppression of the coupling constant qualitatively consistent with the lattice results.

2. As usual we define the nuclear modification factor for AA-collisions as

$$R_{AA}(b) = \frac{dN(A+A\to h+X)/d\mathbf{p}_T dy}{T_{AA}(b)d\sigma(N+N\to h+X)/d\mathbf{p}_T dy}, \quad (1)$$

where $\mathbf{p}_T$ is the hadron transverse momentum, y is rapidity (we consider the central region $y = 0$), b is the impact parameter, $T_{AA}(b) = \int d\boldsymbol{\rho} T_A(\boldsymbol{\rho})T_A(\boldsymbol{\rho}-\mathbf{b})$, T_A is the nucleus profile function. The differential yield for high-p_T hadron production in AA-collision can be written in the form

$$\frac{dN(A+A\to h+X)}{d\mathbf{p}_T dy} = = \int d\boldsymbol{\rho} T_A(\boldsymbol{\rho})T_B(\boldsymbol{\rho}-\mathbf{b})\frac{d\sigma_m(N+N\to h+X)}{d\mathbf{p}_T dy}, \quad (2)$$

where $d\sigma_m(N+N\to h+X)/d\mathbf{p}_T dy$ is the medium-modified cross section for the $N+N\to h+X$ process.

In analogy to the ordinary pQCD formula, we write it in the form

$$\frac{d\sigma_m(N+N\to h+X)}{d\mathbf{p}_T dy} = \\ = \sum_i \int_0^1 \frac{dz}{z^2} D^m_{h/i}(z,Q) \frac{d\sigma(N+N\to i+X)}{d\mathbf{p}^i_T dy}. \quad (3)$$

Here $\mathbf{p}^i_T = \mathbf{p}_T/z$ is the parton transverse momentum, $D^m_{h/i}$ is the medium-modified fragmentation function (FF) for transition of the parton i to the observed hadron h, and $d\sigma(N+N\to i+X)/d\mathbf{p}^i_T dy$ is the ordinary hard cross section. For the parton virtuality scale Q we take the parton transverse momentum p^i_T. We assume that hadronization of the fast partons occurs after escaping from the QGP. This hadronization process should be described by the FFs at relatively small fragmentation scale, μ_h. Indeed, from the uncertainty relation $\Delta E \Delta t \gtrsim 1$ one can obtain for the L dependence of the parton virtuality $Q^2(L) \sim \max(Q/L, Q_0^2)$, where we have introduced some minimal nonperturbative scale $Q_0 \sim 1-2$ GeV. For RHIC and LHC conditions the size of the QGP is quite large ($\gtrsim R_A$, where R_A is the nucleus radius), and from the above formula one sees that for partons with energy $E \lesssim 100$ GeV the hadronization of the final partons may be described by the FFs at the scale $\mu_h \sim Q_0$. Then we can write

$$D^m_{h/i}(z,Q) \approx \int_z^1 \frac{dz'}{z'} D_{h/j}(z/z',Q_0) D^m_{j/i}(z',Q_0,Q), \quad (4)$$

where $D_{h/j}(z,Q_0)$ is the FF in vacuum, and $D^m_{j/i}(z',Q_0,Q)$ is the medium-modified FF for transition of the initial parton i with virtuality Q to the parton j with the virtuality Q_0. Presently there is no a systematic method for calculation of the medium-modified FFs which treats on an even footing the DGLAP and induced radiation processes. In the present paper we use the picture based on the time ordering of the DGLAP and the induced radiation stages which should be a reasonable approximation for not very high parton energies, say, $E \lesssim 100$ GeV. It uses the fact that at such energies the typical length/time scale of the DGLAP stage is smaller than the longitudinal scale of the induced radiation stage. The gluon emission scale for the DGLAP stage can be estimated using the gluon formation length $l_F(x,k_T^2) \sim 2Ex(1-x)/(k_T^2+\epsilon^2)$, where x is the gluon fractional longitudinal momentum, and ϵ in terms of the effective parton masses reads $\epsilon^2 = m_q^2 x^1 + m_g^2(1-x)$. Using the vacuum spectrum of the gluon emission from a quark

$$\frac{dN}{dk_T^2 dx} = \frac{C_F \alpha_s(k_T^2)}{\pi x}\left(1-x+x^2/2\right)\frac{k_T^2}{(k_T^2+\epsilon^2)^2} \quad (5)$$

one can obtain for the typical formation length $\bar{l}_F \sim$ $\sim 0.3-1$ fm for $E \lesssim 100$ GeV (if one takes $m_q \sim 0.3$ GeV and $m_g \sim 0.75$ GeV [18]). This estimate is obtained in the one gluon approximation. However, it should be qualitatively correct since in the energy interval of interest the number emitted gluons is small $\bar{N}_g \lesssim 2$, and the first hardest gluon dominates the DGLAP energy loss. Thus we see that the DGLAP time scale is about the formation time for the QGP, $\tau_0 \sim 0.5-1$ fm. Since the induced radiation is dominated by the distances from $L \sim \tau_0$ up to $L \sim R_A$ one can neglect the interference between the DGLAP and the induced radiation stages. In this approximation we can write

$$D^m_{j/i}(z,Q_0,Q) = \\ = \int_z^1 \frac{dz'}{z'} D^{\mathrm{ind}}_{j/l}(z/z',E_l) D^{\mathrm{DGLAP}}_{l/i}(z',Q_0,Q), \quad (6)$$

where $E_l = Qz'$, $D^{\mathrm{ind}}_{j/l}$ is the induced radiation FF (it depends on the parton energy E, but not the virtuality), and $D^{\mathrm{DGLAP}}_{l/i}$ is the DGLAP partonic FF. In numerical calculations the DGLAP FFs have been evaluated with the help of the PYTHIA event generator [19].

The induced radiation FFs have been calculated making use the probability distribution of the $1\to 2$ partonic processes obtained in the LCPI approach. We have taken into account only the processes with gluon emission, and the process $g\to q\bar{q}$ which gives a small contribution has been neglected. For calculation the one gluon emission distribution we use the method elaborated in [20]. To calculate the $D^{\mathrm{ind}}_{j/l}$ one needs to take into account the multiple gluon emission. Unfortunately, up to now, there is no an accurate method of incorporating the multiple gluon emission. We follow the analysis [8] and employ the Landau method developed originally for the soft photon emission. In this approximation the quark energy loss distribution has the form

$$P(\Delta E) = \sum_{n=0}^{\infty} \frac{1}{n!}\left[\prod_{i=1}^{n}\int d\omega_i \times \\ \times \frac{dP(\omega_i)}{d\omega}\right] \delta\left(\Delta E - \sum_{i=1}^{n}\omega_i\right) \exp\left[-\int d\omega \frac{dP}{d\omega}\right], \quad (7)$$

where $dP/d\omega$ is the probability distribution for one gluon emission. This approximation leads to the leakage of the probability to the unphysical region of $\Delta E > E$

[9]. To avoid the quark charge non-conservation we define the renormalized distribution $\bar{P}(\Delta E) = K_q P(\Delta E)$ with $K_q = \int_0^\infty d\Delta E P(\Delta E) / \int_0^E d\Delta E P(\Delta E)$. We use the renormalized distribution to define the in-medium FF $D^{\rm ind}_{q/q}(z) = \bar{P}(\Delta E = E(1-z))$. To ensure the momentum conservation we take into account the $q \to g$ transition as well. At the one gluon level the corresponding FF can be written as $D^{ind}_{g/q}(z) = dP(\omega = zE)/d\omega$. This automatically leads to the FFs which satisfy the momentum sum rule. We use the same form of the $q \to g$ FF for the case with the multiple gluon emission. To satisfy the momentum sum rule (which are not valid after the renormalization of the $q \to q$ distribution) we multiply it by a renormalization coefficient K_g defined from the total momentum conservation. This procedure seems to be reasonable since the nuclear modification factor is only sensitive to the behavior of the FFs at z close to unity [8] where the form of the $q \to g$ distribution should not be very sensitive to the multiple gluon emission. In the case of the $g \to g$ transition we use the following prescription. In the first step we define $D^{ind}_{g/g}$ at $z > 0.5$ through the Landau distribution $P(\Delta E)$, and in the soft region $z < 0.5$ (where the multiple gluon emission and the Sudakov suppression strongly compensate each other) we use the one gluon distribution. Then we multiply this FF by a renormalization coefficient $\bar{K}_g$ to ensure the momentum conservation (since the number of gluons is not conserved the arguments based on the conservation of the probability cannot be used in this case).

In the above discussion we ignored the collisional energy loss. Presently there is no an accurate method for incorporating of the collisional energy loss in the scheme of the medium-modified FFs. In the present work we view the collisional energy loss as a perturbation and incorporate it into our model by a small renormalization of the QGP density according to the change in the ΔE due to the collisional energy loss. To evaluate the collisional energy loss we use the Bjorken method [21] with an accurate treatment of kinematics of the binary collisions (the details can be found in [22]). We use the same infrared cutoffs and parametrization of the coupling constant for the radiative and collisional energy loss, which is important for minimizing the theoretical uncertainties in the fraction of the collisional contribution.

We calculate the cross sections for the $N + N \to \to q(g) + X$ processes using the LO pQCD formula with the CTEQ6 [23] parton distribution functions. To account for the nuclear modification of the parton densities (which leads to some small deviation of R_{AA} from unity even without parton energy loss) we include the EKS98 correction [24]. To simulate the higher order K-factor we follow the prescription used in the PYTHIA event generator [19] with replacement of the Q argument of α_s by a lower value cQ. We take $c = 0.265$ which allows to describe well the data on π^0 p_T-spectrum in the pp-collisions. For the FFs $D_{h/q(g)}(z, Q_0)$ we use the KKP parametrization [25].

3. For calculations of the induced gluon spectrum and collisional energy loss with the help of the formulas given in [20, 22] we must specify the form of the coupling constant and the mass parameters (the quasiparticle masses and the Debye mass). In our calculations we use the running coupling constant. We parametrize $\alpha_s(Q^2)$ by the one-loop expression and assume that it is frozen at some value α_s^{fr} for $Q \le Q_{fr}$. This form with $\alpha_s^{fr} \approx 0.7$ ($Q_{fr} \approx 0.82\,\text{GeV}$ for $\Lambda_{QCD} = 0.3\,\text{GeV}$) allows one to describe well the HERA data on the low-x structure functions within the dipole approach [26, 18, 27]. A similar value of α_s^{fr} follows from the relation $\int_0^{2\,\text{GeV}} dQ \frac{\alpha_s(Q^2)}{\pi} \approx 0.36\,\text{GeV}$ obtained in [28] from the analysis of the heavy quark energy loss in vacuum. In vacuum the stopping of the growth of α_s at low Q may be caused by the nonperturbative effects [28]. In the QGP thermal partons can give additional suppression of α_s at low momenta ($Q \sim (2-3)T$). The lattice simulations [29] give $\alpha_s(T)$ smoothly decreasing from ~ 0.5 at $T \approx 175\,\text{MeV}$ to ~ 0.35 at $T \approx 400\,\text{MeV}$. However, the thermal $\alpha_s(T)$ in some sense gives the mean value of α_s. For this reason one can expect that the thermal $\alpha_s(T)$ should be somewhat smaller than the in-medium α_s^{fr}. To clear up whether the RHIC data on jet quenching agree with the thermal suppression of α_s we perform numerical calculations for different values of α_s^{fr}.

As in [22] we use the quasiparticle masses obtained in Ref. [30] from the analysis of the lattice data within the quasiparticle model. For the relevant range of the plasma temperature $T \sim (1-3)T_c$ the analysis [30] gives $m_q \approx 0.3$ and $m_g \approx 0.4\,\text{GeV}$. To fix the Debye mass in the QGP we use the results of the lattice calculations for $N_f = 2$ [31] which give the ratio μ_D/T slowly decreasing with T ($\mu_D/T \approx 3$ at $T \sim 1.5T_c$, $\mu_D/T \approx 2.4$ at $T \sim 4T_c$).

4. We describe the QGP in the Bjorken model [32] with the longitudinal expansion which gives the proper time dependence of the plasma temperature $T^3\tau = T_0^3\tau_0$ (T_0 is the initial plasma temperature). To simplify the numerical calculations for each value of the impact parameter b we neglect the variation of T_0 in the transverse directions. For each b we define its own effective initial temperature evaluated with the help of the entropy distribution adjusted in the hydrodynamic analysis [33] of the RHIC data on the small-p_T hadron spectra. For Au+Au collisions at $\sqrt{s} = 200\,\text{GeV}$ it reads [33]

$$dS(\tau, \boldsymbol{\rho}, \mathbf{b})/d\boldsymbol{\rho} dz =$$

$$= \frac{C}{\tau(1+\alpha)} \left[\alpha dN_{\mathrm{part}}/d\boldsymbol{\rho} + (1-\alpha) dN_{\mathrm{coll}}/d\boldsymbol{\rho}\right],$$

where $C = 24$, and $\alpha = 0.85$, N_{part} and N_{coll} are the number of participants and binary collisions evaluated in the Glauber model. In evaluating the entropy distribution we use the Woods-Saxon nucleus density $\rho_A(r) = C_{\mathrm{norm}}/\{1+\exp[(r-c)/d]\}$ with $c = 1.07A^{1/3}$ fm, and $d = 0.545$ fm. For central Au+Au collisions at $\sqrt{s} = 200$ GeV this gives $T_0 \approx 320$ MeV for $\tau_0 = 0.5$ fm. It was assumed that the QGP occupies the region $r < (c+kd)$, with $k = 2$ (for $k = 1$ R_{AA} changes slightly). To perform the extrapolation to the LHC energy $\sqrt{s} = 5500$ GeV we use the energy dependence of the entropy similar to the energy dependence of the total particle rapidity density $\propto N_{\mathrm{part}} \ln(\sqrt{s}/1.5)$ observed at RHIC [34]. It gives for central Pb+Pb collisions $T_0 \approx 404$ MeV. The fast parton path length in the QGP, L, in the medium has been calculated according to the position of the hard reaction in the impact parameter plane. To take into account the fact at times about $1-2$ units of R_A the transverse expansion should lead to fast cooling of the hot QCD matter [32] we also impose the condition $L < L_{\mathrm{max}}$. We performed the calculations for two values $L_{\mathrm{max}} = 6$ and 8 fm.

5. We present the numerical results for $\alpha_s^{fr} = 0.7$, 0.5 and 0.4. The higher value seems to be reasonable in the absence of the thermal effects since it comes from the analyses of the HERA data on the low-x structure functions [18] and heavy quark energy loss in vacuum [28]. In Fig.1 we plot R_{AA} for π^0 production in the central Au+Au collisions at $\sqrt{s} = 200$ GeV. The theoretical curves corresponds to $L_{\mathrm{max}} = 8$ fm. The choice $L_{\mathrm{max}} = 6$ fm gives R_{AA} higher just by about 3–8%. The experimental points in Fig.1 are from [16]. The upper and lower panels show the results for the chemically equilibrium and purely gluonic plasmas, respectively. The results are presented for the purely radiative energy loss and with inclusion of the collisional energy loss and the radiative energy gain. We have found that the effect of the radiative energy gain on R_{AA} is practically negligible and can be safely neglected. Besides the total (quarks plus gluons) contribution in Fig.1 we also show separately the contributions from quarks and gluons. The grows of R_{AA} for gluons is due to the $q \to g$ transition which is usually neglected. However, it does not affect strongly the total R_{AA} since for $\sqrt{s} = 200$ GeV the gluon contribution to the hard cross section is small at $p_T \gtrsim 15$ GeV. As can be seen from Fig.1 the collisional energy loss suppresses R_{AA} only by about 15–25%. This is in contradiction with strong collisional JQ

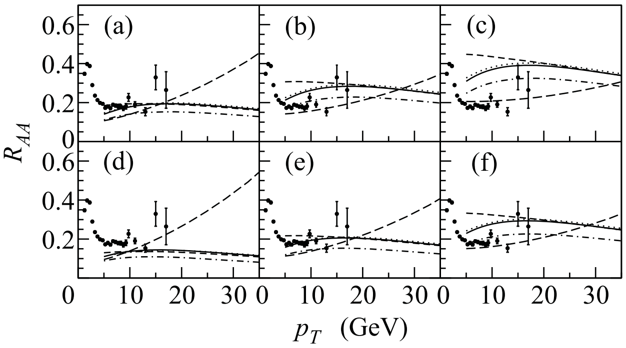

Fig.1. The nuclear modification factor R_{AA} for π^0 production in the central Au+Au collisions at $\sqrt{s} = 200$ GeV for $\alpha_s^{fr} = 0.7$ ((a),(d)), $\alpha_s^{fr} = 0.5$ ((b),(e)), $\alpha_s^{fr} = 0.4$ ((c),(f)). The upper panels are for the chemically equilibrium plasma, and the lower ones for purely gluonic plasma with the same entropy. For all the theoretical curves $L_{\mathrm{max}} = 8$ fm. Solid line: the total (quarks plus gluons) radiative R_{AA}. Dashed line: the radiative R_{AA} for π^0 from quarks. Long-dash line: the radiative R_{AA} for π^0 from gluons. Dash-dotted line: the total (quarks plus gluons) R_{AA} including the radiative plus collisional energy loss and the energy gain due to gluon absorption. Dotted curves show the total (quarks plus gluons) radiative R_{AA} for the inverse time order of the DGLAP and induced gluon emission stages. The experimental points are the data obtained by the PHENIX Collaboration [16] for the most central (0–5%) collisions

found in [35]. To illustrate the effect of the time ordering of the DGLAP and induced radiation stages in Fig.1 we also plot the results for the inverse order of these stages. One can see that the results for these two prescription are very close. This fact may be explained by the dominance of the soft gluon emission at RHIC energies.

From Fig.1 one can see that the theoretical R_{AA} for the chemically equilibrium plasma obtained with $\alpha_s^{fr} = 0.5$ is in qualitative agreement with the experimental one. The scenario with purely gluonic plasma can be consistent with the RHIC data if $\alpha_s^{fr} \approx 0.4$. In the light of the lattice results [29] the value $\alpha_s^{fr} \sim 0.5$ seems to be reasonable for the RHIC (and LHC) conditions. Thus one sees that, contrary to the conclusion of Ref. [17], the scenario with the chemically equilibrated plasma can not be excluded. Note that the distribution of the entropy [33] used in our calculations gives the entropy rapidity density $dS/dy \approx 6500$. If we take a smaller value $dS/dy \approx 5100$ obtained in [36] approximately the same R_{AA} as for $\alpha_s^{fr} = 0.4$ and 0.5 can be obtained with the values $\alpha_s^{fr} \approx 0.45$ and 0.55.

In Fig.2 we show R_{AA} as a function of N_{part}. One sees that the model reproduces qualitatively the growth of R_{AA} with decrease of N_{part}. But for very peripheral

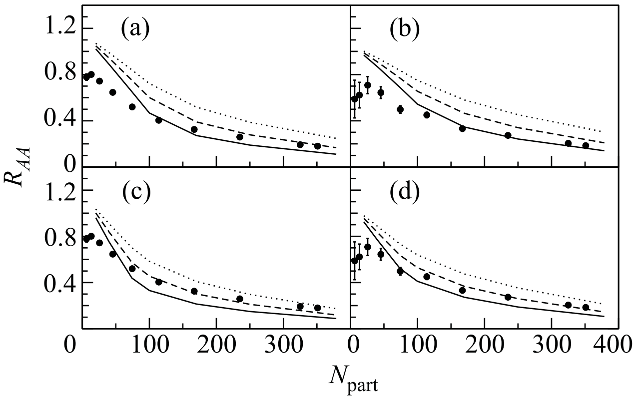

Fig.2. The nuclear modification factor R_{AA} for π^0 production in Au+Au at $\sqrt{s}$ = 200 GeV for p_T > 5 GeV (left panels) and p_T > 10 GeV (right panels) as a function of $N_{\rm part}$. The upper panels are for the chemically equilibrium plasma, and the lower ones for purely gluonic plasma with the same entropy. The theoretical curves show the total (quarks plus gluons) R_{AA} including the radiative plus collisional energy loss and the energy gain due to gluon absorption for α_s^{fr} = 0.7 (solid line), α_s^{fr} = 0.5 (dashed line), α_s^{fr} = 0.7 (dotted line). The experimental points are from [16]

collisions with small $N_{\rm part}$ it overestimates the observed R_{AA}. This may be connected with inadequacy of the neglect of the transverse motion of the matter for thin plasma. Also, in this region the neglect of the variation of T_0 in the impact parameter space may be inadequate as well. Probably for similar reasons the model underestimate the ellipticity parameter v_2 (which is also sensitive to the evolution of the QGP for the peripheral collisions). Our calculations give $v_2 \sim 0.05 - 0.08$ for $p_T \sim 5$ GeV while the experiment gives $v_2 \sim 0.1 - 0.15$ [37].

In Fig.3 we plot the theoretical results similar to that shown in Fig.1 but for Pb+Pb collisions at LHC for $\sqrt{s}$ = 5500 GeV. One can see that the effect of the collisional energy loss becomes smaller for LHC conditions. But the effect of the time ordering of the DGLAP and induced radiation stages is bigger as compared to the RHIC. The difference between the results for the chemically equilibrium and non-equilibrium plasmas is relatively small.

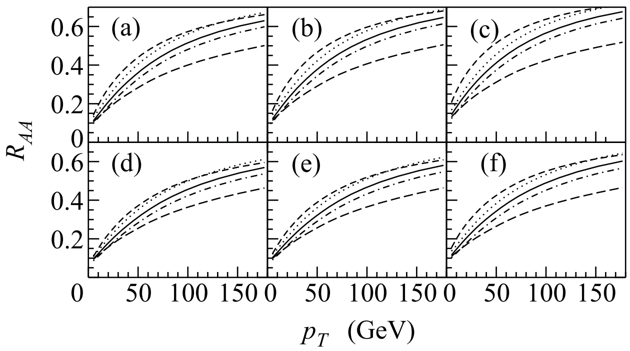

Fig.3. The same as in Fig.1 for Pb+Pb collisions at $\sqrt{s}$ = 5500 GeV

6. In summary, we have calculated the nuclear modification factor for the RHIC and LHC conditions accounting for both the radiative and collisional energy losses with the running α_s. The radiative energy loss has been calculated within the LCPI approach [3]. The collisional energy loss has been evaluated in the Bjorken model of elastic binary collisions with an accurate treatment of kinematics of the binary collisions. In contrast to [35] we find relatively small effect of the collisional energy loss on JQ. We also investigated the effect of the parton energy gain due to the induced gluon absorption. We find that this effect is negligible for the RHIC and LHC conditions.

The calculations are performed using a new algorithm for the multiple gluon emission which takes into account the time ordering of the DGLAP and induced gluon emission stages. We find that the effect of the ordering of these two stages is relatively small for RHIC conditions, but becomes bigger for LHC.

Comparison of our theoretical results with the RHIC data show that R_{AA} can be described in the scenario with the chemically equilibrium QGP with the entropy extracted from the hydrodynamical simulation of the AA collisions at RHIC energies. The scenario with the purely gluonic plasma is also possible, but requires somewhat stronger thermal suppression of α_s. This contradicts to the conclusion of Ref. [17] that the observed JQ and total entropy of the QGP are incompatible with the chemically equilibrium plasma scenario.

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