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Curvature-corrected dilatonic black holes and black hole – string transition

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We investigate extremal charged black hole solutions in the four-dimensional string frame Gauss-Bonnet gravity with the Maxwell field and the dilaton. Without curvature corrections, the extremal electrically charged dilatonic black holes have singular horizon and zero Bekenstein entropy. When the Gauss-Bonnet term is switched on, the horizon radius expands to a finite value provided curvature corrections are strong enough. Below a certain threshold value of the Gauss-Bonnet coupling the extremal black hole solutions cease to exist. Since decreasing Gauss-Bonnet coupling corresponds to decreasing string coupling g_s, the situation can tentatively be interpreted as classical indication on the black hole – string transition. Previously the extremal dilaton black holes were studied in the Einstein-frame version of the Gauss-Bonnet gravity. Here we work in the string frame version of the theory with the S-duality symmetric dilaton function as required by the heterotic string theory.

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I. Introduction. The idea of correspondence between black holes and massive string states suggested by Susskind [1] was elaborated in a number of papers [2 – 8]. Though naively the degeneracy of the string massive levels seems to contradict the Bekenstein-Hawking area formula, it turns out that there is an agreement between the geometric and the statistical entropies for the critical mass at which the size of the string configuration approaches the gravitational radius of the black hole [2]. Some arguments in favor of the black hole – string transition were given in [4] considering classical and string pictures of the wave scattering. Here we would like to push further another argument based on the analysis of the moduli space of the curvature corrected extremal charged dilatonic black holes [9].

Extremal dilatonic black holes are especially suitable to probe the conjectured string – black hole transition for the following reason. To minimize uncertainties related to higher order quantum corrections it is common to consider the BPS states usually associated with black holes exhibiting a degenerate event horizon. At the level of the Einstein-Maxwell-dilaton theory such black holes are problematic since their horizon radius tends to zero implying vanishing Bekenstein-Hawking entropy. Meanwhile, from the string theory side, the entropy is expected to be non-zero. Recently understanding of this situation was improved by realizing that curvature corrections [10] to the Einstein action lead to stretching the horizon to a finite radius [11], so that the geometric entropy calculations fit to microscopic predictions [13, 12] (both results being different from the naive Bekenstein-Hawking value).

However, another problem was noticed in [9]: although the entropies had been claimed to agree, the classical calculations were based on the purely local considerations, like Sen entropy function approach [11], which require solving the Einstein equations only in the vicinity of the event horizon. Meanwhile, to be sure that one deals with the black hole indeed, global integration of the Einstein equations is necessary which should provide an extension of the local solutions to infinity. An attempt of such a continuation in the Einstein-Maxwell-Gauss-Bonnet theory with an arbitrary dilaton coupling [9] revealed existence of the threshold behavior: global solutions cease to exist for the dilaton coupling above some critical value. This was interpreted as an indication for the black hole – string transition anticipated in [4].

Here we would like to push this reasoning further and to mention its possible relevance to the problem of the final stage of Hawking evaporation. Note that the Gauss-Bonnet term is the most popular candidate to imitate string corrections to the Einstein action [14]. The four-dimensional Newton constant G_N, the string inverse tension α', and the string coupling g_s are related in four dimensions as $G_N \sim g_s^2\alpha'$. Thus, decreasing string coupling g_s implies decreasing G_N. Since the coefficient in front of the Gauss-Bonnet term is dimensionless, the decreasing G_N, entering the denominator of the Einstein term, makes this term to dominate the Gauss-Bonnet

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term. Technically, however, it is more convenient to introduce a variable Gauss-Bonnet coupling parameter α and to consider decreasing α for fixed G_N. From the point of view of the black hole – string correspondence principle [2] the black hole case corresponds to large g_s^2, hence, in our treatment, to large α (more precisely, to large dilaton-renormalized α which we will call later $\tilde{\alpha}$). The existence of the lower bound for $\tilde{\alpha}$ in the black hole solution space then can be interpreted as a signal of the black hole – string transition.

Black hole solutions to the Einstein-Maxwell-dilaton theory with the Gauss Bonnet term introduced in the Einstein frame were constructed in [9]. They qualitatively confirm the above expectations. However, from the string theory point of view, the Gauss-Bonnet term has to be introduced in the string frame. Transformation of the Einstein-frame version of the Gauss-Bonnet gravity to the string frame leads to a lagrangian which differs from the string-frame version of the Gauss-Bonnet gravity by the forth-order dilaton derivative term. Therefore these two theories are not equivalent. Here we investigate the extremal black holes in the second version, adding the Gauss-Bonnet term in the string frame.

Another new feature of the present work is the use of the S-duality symmetric dilaton function which is also motivated by string theory. It is worth noting that, within the Einstein-Maxwell-dilaton model without curvature corrections, electric and magnetic extremal black holes look very different in the string frame. For magnetic black holes the dilaton conformal factor cancels the divergent metric term on the horizon, so in the string frame the solutions look regular. For electric ones the singularity persists in the string frame too. So the role of curvature corrections is expected to be more significant for the electric solutions. Actual calculations show that purely electric extremal black holes do not exist within the string frame version of the Einstein-Maxwell-Gauss-Bonnet theory: some magnetic charge is necessary. However one can distinguish the electrically and the magnetically dominated dyons. Our interpretation of the boundary in the parameter space as a signal of the black hole – string transition refers to the case of electric dominance.

II. String frame Gauss-Bonnet gravity. We start with the following four-dimensional action:

$$\mathcal{S} = \frac{1}{16\pi G}\int S(R + S^{-2}(\partial S)^2 - F^2)\, d^4x\sqrt{-g} + \\ + \int \Delta\mathcal{L}\, d^4x\sqrt{-g}, \tag{1}$$

where

$$\Delta\mathcal{L} = \frac{\alpha}{16\pi}\psi(S)\, L_{GB}, \\ L_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}. \tag{2}$$

The dilaton factor $\psi(S)$ in front of the Gauss-Bonnet term inheriting the S-duality symmetry of the heterotic string [15–17] can be presented as

$$\psi(S) = -\frac{3}{\pi}\ln(2S|\eta(iS)|^4), \tag{3}$$

where the Dedekind η-function is defined on the complex plane of τ as follows:

$$\eta(\tau) \equiv e^{2\pi i\tau/24}\prod_{n=1}^{\infty}(1 - e^{2\pi i n\tau}). \tag{4}$$

It is a modular form of the weight 1/2 satisfying the functional equations

$$\eta(-\tau^{-1}) = \sqrt{-i\tau}\,\eta(\tau), \quad \eta(\tau+1) = e^{\frac{2\pi i}{24}}\eta(\tau), \tag{5}$$

which represent the action of generators of the modular group. In the general case $\tau = \mathrm{a} + iS$ where a is the axion field. Here we consider only static spherically symmetric configurations for which it is consistent to set the axion $a = 0$. In this truncated theory the function $\psi(S)$ is invariant under the discrete S-duality $S \to S^{-1}$. Note that asymptotically for large S it is a linear function of the dilaton: $\psi(S) \to S$.

Consider the following parametrization of the static spherically symmetric space-time:

$$ds^2 = -w(r)\sigma(r)^2dt^2 + \frac{dr^2}{w(r)} + \rho(r)^2 d\Omega_2^2, \tag{6}$$

and the ansatz for the gauge field:

$$A = -f(r)dt - m\cos\theta d\varphi, \tag{7}$$

with m denoting magnetic charge. Substituting this ansatz into the action we obtain the following one-dimensional lagrangian

$$\mathcal{L} = S\left[\sigma' w\rho\rho' + \frac{\sigma}{2}(\rho'(w\rho)' + 1)\right] + \\ + S'\left(\sigma w\rho\rho' + \frac{\sigma w'\rho^2}{4} + \frac{\sigma' w\rho^2}{2}\right) - \\ - \frac{\alpha\psi'}{\sigma}(w\sigma^2)'(w\rho'^2 - 1) + Sw\sigma\rho^2\frac{S'^2}{4S^2} + \\ + S\left(\frac{\rho^2 f'^2}{2\sigma} - \frac{\sigma m^2}{2\rho^2}\right), \tag{8}$$

where primes denote derivatives with respect to r. The corresponding Euler-Lagrange equations are rather complicated except that for the vector field which can be integrated once to give

$$f' = g/\rho^2 S, \tag{9}$$

where g is an electric charge parameter defined on the horizon (an asymptotical definition of the electric charge differs from this by the dilaton exponent). Therefore, after fixing the gauge $\sigma = 1$, the problem is reduced to three second-order ordinary differential equations for three unknown functions w, ρ, S and a constraint equation.

One can investigate the residual symmetries of the lagrangian. Consider first the dilaton shift (rescaling of S): the EMD part is invariant under the following transformation with the global parameter β:

$$\begin{aligned} &S \to \beta S \quad w \to \beta^4 w, \quad \rho \to \frac{\rho}{\beta}, \\ &r \to \beta r, \quad g \to g, \quad m \to \frac{m}{\beta}. \end{aligned} \tag{10}$$

The invariance of the GB term depends on the behavior of the function $\psi(S)$. Another symmetry comes from the rescaling of the electric charge (with the global parameter γ) which causes the following symmetry of the full lagrangian:

$$g \to \gamma g, \quad m \to \gamma m, \quad w \to \frac{w}{\gamma^2}, \quad \rho \to \gamma\rho, \quad \alpha \to \gamma^2\alpha. \tag{11}$$

These symmetries will be used to clarify nature of parameters of the numerical solutions obtained below.

III. Local solutions near the horizon. To construct the extremal black hole solution (by extremality we mean the $AdS_2 \times S^2$ structure of the event horizon) we first construct the local series solution in the vicinity of the event horizon $r = r_0$. We use the gauge $\sigma = 1$. The desired solution must satisfy the conditions:

$$w(r_0) = w'(r_0) = 0, \qquad \rho(r_0) = \rho_0 > 0, \qquad \rho'(r_0) > 0. \tag{12}$$

Thus the expansions must be on the form:

$$\begin{aligned} &w = \sum_{n=2}^{\infty} w_n x^n, \qquad \rho = \sum_{n=0}^{\infty} \rho_n x^n, \\ &S = \sum_{n=0}^{\infty} p_n x^n, \qquad \psi = \sum_{n=0}^{\infty} t_n x^n, \end{aligned} \tag{13}$$

where $x \equiv (r - r_0)$. Recall that ψ is a function of the dilaton field S. Therefore all t_n are the functions of p_i, $i = 0..n$, explicitly one has

$$\begin{aligned} \psi = \Psi_0 + \Psi_1 p_1 x + \left(\frac{\Psi_2}{2}p_1^2 + \Psi_1 p_2\right) x^2 + \\ + \left(\frac{\Psi_3}{6}p_1^3 + \Psi_2 p_1 p_2 + \Psi_1 p_3\right) x^3 + ..., \end{aligned} \tag{14}$$

where $\Psi_i = \Psi_i(p_0) = \psi^{(i)}(S)|_{S=p_0}$ are the i–th derivatives of ψ depending only on p_0. Substituting these expansions into the equations of motion one obtains the sequence of constraints on the coefficients. In the leading order we get the relations between g, m, α. Since we wish to determine the range of α for which the black hole solutions exist, we consider α as a variable parameter on equal footing with the electric and magnetic charges. It is convenient to present the magnetic charge as $m = g\sqrt{u^2 - 1}/p_0$ with u defined on the semi-axis $u^2 > 1$, the remaining parameters will look simpler in terms of u. To avoid using an explicit dependence $\Psi_i(p_0)$ one can also leave p_0 as a parameter and express the expansion coefficients as functions of $(g,\ u,\ p_0,\ \Psi_i(p_0))$, three of which g, u and p_0 are independent.

In the leading order we find the following relations:

$$\alpha = \frac{g^2(2 - u^2)}{4\Psi_1 p_0^2}, \qquad \rho_0 = \frac{gu}{p_0}, \qquad w_2 = \frac{p_0^2}{g^2u^2}. \tag{15}$$

This corresponds to the Bertotti-Robinson metric $AdS_2 \times S^2$ on the horizon with the curvature radius of the anti-de Sitter sector coinciding with the radius of the two-sphere:

$$ds_H^2 = -w_2 x^2 dt^2 + \frac{dx^2}{w_2 x^2} + \rho_0^2 d\Omega_2^2, \tag{16}$$

since $\rho_0^2 = 1/w_2$. With account for the second expansion terms we have:

$$\rho = \frac{gu}{p_0} - \frac{gp_1(2\Psi_1 + 2\Psi_1 u^2 - \Psi_2 p_0 u^2)}{4\Psi_1 u p_0^2} x + O(x^2),$$

$$S = p_0 + p_1 x + O(x^2), \tag{17}$$

$$w = \frac{p_0^2}{g^2u^2}x^2 + \frac{p_0 p_1(2\Psi_1 + 2\Psi_1 u^2 - 3\Psi_2 p_0 u^2)}{6g^2u^4\Psi_1}x^3 + O(x^4).$$

One can notice that the electric charge g is just a scale factor for the radial metric function and the gauge field: the transformation (11) with $\gamma = 1/g$ will remove this parameter from the expansions. We can use this property to achieve a correct asymptotic form of the solution at spatial infinity.

IV. Extending solutions to infinity. We are interested in asymptotically flat metrics such that

$$w(r) \to 1, \quad \rho'(r) \to 1 \qquad \text{as} \quad r \to \infty. \tag{18}$$

We demand the metric to be asymptotically flat in the Einstein frame as well, what implies that the dilaton must be finite at infinity. Consider a solution with the constant asymptotic value w_∞ of the metric component $-g_{00} = w$ at infinity. From the equations of motion one has $w_\infty {\rho'_\infty}^2 = 1$, therefore an asymptotic behavior of ρ'_∞ is determined by w_∞. From the relations (11) it is clear that in order to pass to the solution with another asymptotic value $\hat{w}_\infty$ one should replace g with $\hat{g} = g\sqrt{w_\infty/\hat{w}_\infty}$. The dilatonic derivative coefficient $p_1 = S'(r_0)$ enters the expansions only in some combination with x, thus being the scale factor as well. It can be removed by the transformation $x \to p_1 x$, $w \to p_1^2 w$.

In the case of the linear function $\psi(S) = S$ one has $\Psi_1 = 1$ and all other Ψ_i vanish. Then the parameter p_0 is a scale factor too. Indeed, now the transformation (10) acts on the GB term in the same way as on the EMD term if we rescale $\alpha \to \alpha/\beta$, leading to the symmetry of the full lagrangian under the dilaton shift. Note that the conditions (18) at spatial infinity are not invariant under the dilaton shift, and the Minkowskian asymptotic behavior can be reached only for a unique value of p_0 if the other parameters are fixed. Also, for a purely electric configuration ($u = 1$) all the parameters will act as scale factors. This is the reason why we are motivated to investigate more general dyonic systems.

The solutions with $u = \sqrt{2}$ are possible only for vanishing α. This is exactly the condition on the charges $m = g/p_0$ for which the extremal limit of the EMD black hole exists [18, 19]. It has the $AdS_2 \times S^2$ horizon and the Minkowski asymptotic behavior. In the *string* frame the solution takes the form

$$\rho = \gamma^{-1}\left[\mathcal{W}_0\left(\delta e^{\gamma x+\delta}\right) + 1\right], \quad \gamma = \frac{S_\infty p_0}{\sqrt{2}g(S_\infty - p_0)},$$
$$\delta = \frac{p_0}{S_\infty - p_0}, \quad S = S_\infty \rho', \quad w{\rho'}^2 = \left(1 - \frac{\sqrt{2}g}{p_0\rho}\right)^2. \tag{19}$$

Here S_∞ is the asymptotic value of the dilaton at spatial infinity, and $\mathcal{W}_0$ denotes the real branch of the Lambert $\mathcal{W}$-function which satisfies the following functional and differential equations:

$$z = \mathcal{W}(z)e^{\mathcal{W}(z)}, \qquad z\frac{d\mathcal{W}}{dz} = \frac{\mathcal{W}}{\mathcal{W}+1}. \tag{20}$$

Let us discuss the S-duality symmetry of the back hole solutions we are looking for. The action of the S-duality transformations on the charges and the dilatonic background on the horizon in the general case of our model with an axion present is [20, 11]:

$$\begin{pmatrix} g' \\ m' \end{pmatrix} = \begin{pmatrix} l & n \\ r & s \end{pmatrix}\begin{pmatrix} g \\ m \end{pmatrix},$$
$$a'_0 + ip'_0 = \frac{l(a_0 + ip_0) + n}{r(a_0 + ip_0) + s}; \tag{21}$$
$$(l,\, n,\, r,\, s) \in \mathbb{Z}, \quad ls - nr = 1.$$

If we set zero the horizon value of the axion $a'_0 = a_0 = 0$, we will get the constraints on the parameters of the transformation $(l,\, n,\, r,\, s)$. There are two possibilities:

$$n = r = 0,\ s = 1/l \Rightarrow g' = lg,\ m' = m/l,\ p'_0 = l^2 p_0; \tag{22}$$

$$l{=}s{=}0,\ r{=}-1/n \Rightarrow g'{=}nm,\ m'{=}-g/n,\ p'_0{=}n^2/p_0. \tag{23}$$

The first transformation is a kind of rescaling (10), (11) with $\beta = l^2$, $\gamma = l$. The second transformation is the discrete S-duality: it interchanges the electric and magnetic charges and inverts the dilaton.

Now consider the unique value of the parameter $u = \sqrt{2}$, for which the horizon expansions are compatible with switching off the Gauss-Bonnet term. The S-duality transformation acts on u as follows:

$$(22) \Rightarrow u' = \sqrt{\frac{{m'}^2 p_0^2}{{g'}^2} + 1} = \sqrt{\frac{m^2 p_0^2}{g^2} + 1} = u,$$
$$(23) \Rightarrow u' = \sqrt{\frac{{m'}^2 {p'_0}^2}{{g'}^2} + 1} = \sqrt{\frac{g^2}{m^2 p_0^2} + 1}. \tag{24}$$

If $u = \sqrt{2}$, then $m^2p_0^2/g^2 = 1 = g^2/(m^2p_0^2)$ and $u' = u$. Therefore the surface $u(g, m, p_0) = \sqrt{2}$ in the parameter space maps into itself under the S-duality.

Our goal is to find the region of parameters for which asymptotically flat extremal black holes do exist. From numerical calculations it follows that this is only possible in some region of the parameters m, α bounded from below. Note that the transformation to the Einstein frame reads:

$$ds_E^2 = Sds^2 = -wSdt^2 + \frac{d\tilde{r}^2}{wS} + \rho^2 S d\Omega_2^2, \tag{25}$$
$$\text{where} \quad d\tilde{r} = Sdr.$$

So the solution corresponding to the parameter $\hat{g} = g\sqrt{w_\infty S_\infty}$ asymptotically will satisfy the relations $w_\infty = 1/S_\infty$, $\rho'_\infty = \sqrt{S}$ which correspond to the Minkowski space in the Einstein frame. It turns out that the local solutions defined by the series expansions on the horizon can be extended to spatial infinity not for all values of the parameters. As we have discussed, the family of solutions we are interested in is characterized by two free parameters, if one considers the Gauss-Bonnet coupling α as a parameter. Since in our treatment the dilaton S generically is not equal to one at infinity, it is more appropriate to use a dilaton-renormalized GB coupling constant $\tilde{\alpha} = \alpha\Psi_0$ instead of α. As the second parameter we use the quantity u replacing the magnetic charge.

We explored the regions in the parameter plane of $\tilde{\alpha}$, u for which regular solutions exist both for the linear and for the self-dual forms of the function $\psi(S)$. Numerical calculations reveal that for the linear dilaton function $\psi(S) = S$ there are no regular black hole solutions at all. But for the S-duality symmetric function $\psi(S)$ given by (3) such solutions *do exist* in the region $\tilde{\alpha} \geq \tilde{\alpha}_{\min}$ for $u < \sqrt{2}$, and in the region $\tilde{\alpha} \leq \tilde{\alpha}_{\max}$ for $u > \sqrt{2}$, as shown in Fig.1. Passing from u to the electric and

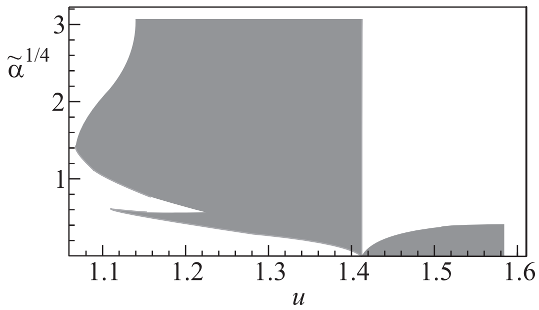

Fig.1. The domain of existence of regular black hole solutions in the plane of the magnetic parameter u and the GB coupling $\tilde{\alpha}$

magnetic charges one finds that the first region corresponds to the electrically dominated configurations for which $mp_0 < g$, while the second – to the magnetically dominated ones. Therefore the electrically dominated black holes with $AdS_2 \times S^2$ horizons exist when the (dilaton renormalized) GB coupling $\tilde{\alpha}$ exceeds some minimal value, while the magnetically dominated black holes exist for $\tilde{\alpha}$ bounded from above. In what follows we will concentrate on the first case as potentially relevant to our discussion.

The ADM mass and the dilaton charge can be extracted from the expansions at spatial infinity in the Einstein frame:

$$
\begin{aligned}
w_E &= 1 - \frac{2M}{\hat{r}} + O(\hat{r}^{-2}), \\
S &= S_\infty + \frac{2S_\infty D}{\hat{r}} + O(\hat{r}^{-2}).
\end{aligned}
\tag{26}
$$

It turns out that in the limit $\tilde{\alpha} \to \infty$ the mass M remains finite while the dilaton charge D and the asymptotic value of the dilaton S_∞ diverge. Two other physical parameters appropriate to global solutions are the electric and magnetic charges Q and P defined asymptotically by integration of the corresponding fluxes over the large sphere:

$$Q = \frac{g}{\sqrt{S_\infty}}, \qquad P = m\sqrt{S_\infty}. \tag{27}$$

They differ from the charges defined on the horizon by the asymptotic value of the dilaton. In the limit $\tilde{\alpha} \to \infty$ the electric charge Q tends to a constant while the magnetic charge P diverges.

Near the boundaries of the allowed region of $\tilde{\alpha}$, namely for $\tilde{\alpha} \to \tilde{\alpha}_{\min}$ and $\tilde{\alpha} \to \infty$, one observes a peculiar relation between the asymptotic charges and the ADM mass. Recall, that in the EMD theory without curvature corrections the BPS limit corresponds to the following condition [18, 21]:

$$M^2 + D^2 = Q^2 + P^2. \tag{28}$$

When $\tilde{\alpha} \to \infty$ (large curvature corrections), the dilaton charge D and the magnetic charge P diverge while the mass M and the electric charge remain Q constant. In the case of small curvature corrections, $|D|$ and P are small with respect to M and Q. To describe the results of numerical calculations we introduce the ratio

$$C_{BPS} = \frac{Q^2 + P^2}{M^2 + D^2} \tag{29}$$

and explore its variation with growing $\tilde{\alpha}$ from $\tilde{\alpha}_{\min}$ to infinity. The results for two fixed values of the magnetic parameter $u = 1.25$ and $u = 1.39$ are shown in Fig.2. It turns out that in both limits $\tilde{\alpha} = \tilde{\alpha}_{\min}$ and $\tilde{\alpha} \to \infty$ the BPS condition (28) of the supergravity without curvature corrections is fulfilled. Thus the family of curvature corrected black hole solutions is bounded by two quasi-BPS states. The lower limit is the charged black hole with the mass M and the absolute value of the electric charge $|Q|$ of the same order and significantly larger than the dilaton charge $|D|$ and the magnetic charge $|P|$. The upper limit is the black hole with extremely strong magnetic and dilaton fields. With u approaching $\sqrt{2}$, the plot shrinks to the abscissa. The limit $u = \sqrt{2}$ corresponds to the single BPS solution when the range of $\tilde{\alpha}$ shrinks to the point $\tilde{\alpha} = 0$.

The entropy of our black holes can be calculated using the Sen entropy function approach [11, 15]. The result is:

$$\mathrm{S} = \pi\rho_E^2 + 4\pi\tilde{\alpha}. \tag{30}$$

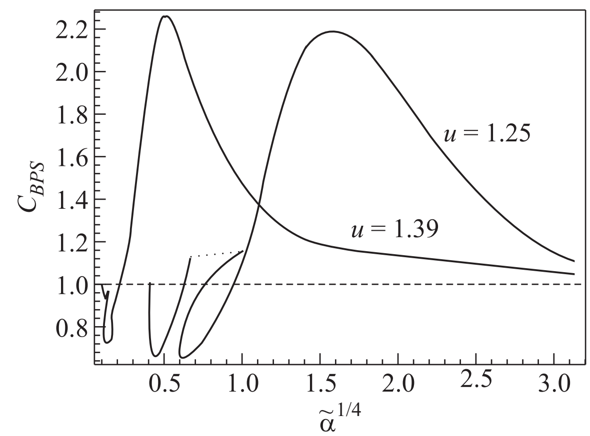

Fig.2. The ratio $C_{BPS} \equiv (Q^2+P^2)/(M^2+D^2)$ as a function of the GB coupling $\tilde{\alpha}$ for $u = 1.25$ and $u = 1.39$. Dotted line shows the interval where regular solutions do not exist

The first term is exactly the Bekenstein-Hawking entropy, while the second term describes curvature corrections. Evaluating $\tilde{\alpha}$ in terms of the expansion parameters we obtain

$$\mathrm{S} = \pi\rho_E^2 \left[1 + \frac{\Psi_0}{u^2 p_0 \Psi_1}\left(2 - u^2\right)\right]. \qquad (31)$$

V. Conclusions. Let us briefly summarize the results obtained. Our aim was to investigate the domain of an effective curvature coupling parameter $\tilde{\alpha}$ for which the system admits the extremal black hole solutions with the horizon geometry $AdS_2 \times S^2$. We have found that if $\tilde{\alpha}$ exceeds some minimal value $\tilde{\alpha}_{\min}$, the black holes exist which are endowed with both electric and magnetic charges. For lower values of $\tilde{\alpha}$ in the allowed region the electric charge is dominant, while for large enough $\tilde{\alpha}$ the solutions are magnetically-dominated. On the boundary $\tilde{\alpha} \to \tilde{\alpha}_{\min}$ and in the asymptotic region $\tilde{\alpha} \to \infty$ the mass and the charges satisfy the same BPS condition, as singular extremal black holes in the theory without curvature corrections. This feature is similar to that observed previously in the Einstein-frame Gauss-Bonnet gravity [9].

We think that the existence of the lower $\tilde{\alpha}$-boundary for solutions in the region $u < \sqrt{2}$ can be traced to the conjectured black hole – string transition. Indeed, the string coupling parameter g_s is large for large $\tilde{\alpha}$ in which case the system admits the black hole solutions. With decreasing g_s one observes disappearance of the black hole solutions while the mass and the charges of the configuration are still finite. This is what is expected for the transition region. One can also argue that in the process of evaporation the mass of the black hole decreases and at some moment the parameters of the solution leave the allowed domain, so one enters into the transition region. However this hypothesis must be investigated in more detail taking into account an actual evolution of charges during the evaporation process.

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