Pis'ma v ZhETF, vol. 89, iss. 7, pp. 423 – 424

The vortex instability at the Quantum Hall Conditions

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Submitted 4 March 2009

It was shown that the ground states obtained by the projection on the states of the lowest Landau level are unstable due to the formation of the isolated vortex decreasing the energy at the partial Ll fillings.This statement is valid due to the different dependence on the sample size of the various terms in the free energy like the free energy of the rotating liquid irrelevant to the details of the microscopical structure and the interaction.

PACS: 73.43.–f

The experimental discovery of Integer Quantum Hall Effect (IQHE) by K.v Klitzing (1980) and Fractional Quantum Hall Effect (FQHE) by Tsui, Stormer and Gossard (1982) was one of the most outstanding achievements in condensed matter physics of the last century.

Despite the fact that more than twenty years have elapsed since the experimental discovery of quantum Hall Effect (QHE), the theory of this phenomenon is far from being complete (see reviews. This is primarily true for FQHE. Most theoretical works are based on the projection of the ground state wave function on the states in the lowest Landau level assuming extremely high magnetic field (see e.g. [1]). The most successfull phenomenological treatment was done by Jains model of the "composite" fermions [2] wich supply electrons with the additional "magnetic" flux. In a recent work ([3] it was suggested that this additional flux can be supplied by the vortices forming vortex lattices. In this paper I show that the free energy for the states without a macroscopical current is unstable in the external magnetic field due to the formation of the isolated vortices.

The change of the free energy of the charged system in the given external magnetic field is (see e.g.[4])

$$\delta F = -\frac{1}{c}\int \mathbf{A}\delta\mathbf{j}dV,$$

where $\mathbf{A}$ is the external vector-potential, $\delta\mathbf{j}$ is the change of the current and the integration is over the volume of the sample. That corresponds exactly to 2DES, where the external magnetic field can not be essentially changed by a weak 2D current. Thus this equation can be integrated to give

$$F = E - \frac{1}{c}\int \mathbf{A}\mathbf{j}d^2r, \qquad (1)$$

where E is the internal energy.

The standard assumption in the theory of 2DES in a strong magnetic field was the possibility to construct the ground state by the projection on the states in the lowest Landau level(e.g. [1]). In that case the average electron current vanish on the distances of the order magnetic length.

It is possible to calculate the free energy change (1) due to the formation of the isolated vortex. It is convenient to use the axial gauge where the external vector-potential is

$$\mathbf{A}(\mathbf{r}) = \frac{1}{2}rB\mathbf{e}_\phi,$$

where $\mathbf{e}_\phi$ is the unit vector in the azimuthal direction. The effective vector-potential is the sum of the external-vector potential and the additional vector-potential of the vortex

$$\delta\mathbf{A} = \frac{c\hbar}{e}\frac{K}{r}\mathbf{e}_\phi,$$

which is the change due to the formation of the vortex at the origin. The corresponding operator of the electrical current reads

$$\hat{\mathbf{j}} = \frac{\hbar e}{2Ml_B}\Big\{\psi^+\left(-\frac{i\partial}{\rho\partial\phi} + \frac{\rho}{2}\mathbf{e}_\phi - \frac{K}{\rho}\mathbf{e}_\phi\right)\psi + \\ + \left(\frac{i\partial}{\rho\partial\phi}\psi^+ + \frac{\rho}{2}\mathbf{e}_\phi\psi^+ - \frac{K}{\rho}\mathbf{e}_\phi\psi^+\right)\psi\Big\}, \qquad (2)$$

where $\rho = r/l_B$. We shall measure all energies in the unit of $\hbar^2/2Ml_B^2$.

We assume the lowest Ll partially filled and the projected form of the fermi operators is

$$\psi = \sum_m \exp(-im\phi))R_m(\rho)c_m,$$

$$\psi^+ = \sum_m \exp(im\phi)R_m(\rho)c_m^+,$$

where c_m, c_m^+ are fermi operators and

$$R_m(\rho) = \rho^m\exp(-\rho^2/4)N_m^{-1/2},$$

where $N_m = 2^m l_B^{2(m+1)}m!$. It is easy to show that the total azimuthal current through any ray $\phi = const$ vanish at $\delta\mathbf{A}=0$. The second term with the magnetic moment in eq. (1) reads

$$F_2=2\pi\int_0^R rdr\left[\rho\sum_{m>0}R_m^2\left(\frac{m}{\rho}-\frac{\rho}{2}-|\frac{K}{\rho}\right)<c_m^+c_m>\right].$$

The angular brackets denote quantum mechanical average.

We consider large distances from the positions of the vortex where the perturbation of the basic state is small and this expression can be calculated up to the first order as the average over supposed projected ground state.The first term in brackets is linear in the sample size and can be neglected compare to the term due to the formation of the vortex proportional to the sample area.

$$F_2=-K\int_0^R n_e 2\pi r dr, \qquad (3)$$

where n_e is the average ectron density and the integral gives the total number of electrons in the sample. It is essential that the main contribution comes from the large distances where the states are distorted quate weakly therefore the ineraction and microscopical structure are not changed.

The calculation of the internal energy can proceed along the same lines. The change of the internal energy due to vortex formation is given by the kinetic energy term which reads

$$E'=\int_0^R\frac{2K}{\rho}\sum_{m>0}\left(\frac{\rho}{2}-\frac{m}{\rho}\right)R_m^2<c_m^+c_m>2\pi r dr+$$
$$+\int_0^R\frac{K^2}{\rho^2}\sum_{m>0}R_m^2<c_m^+c_m>2\pi r dr. \qquad (4)$$

In the same order of the perturbation theory that gives logarithmic dependence on the sample size. It is evident for the last tem. The first term gives also the logarithmic contribution at large distances. The finite value of E' is obtained by some cut in the vortex core on small distances where the electron density must be reduced.

Thus the vortex formation gives the gain in the energy of the large enough samples when the negative magnetic moment term in the free energy exceeds the logarithmic increase of the internal energy in eq.(1). We see that the supposed ground state projected on the lowest Ll is unstable to the vortex formation. This statement is independent of the details of the microscopic structure or the interaction and is valid only due to the different size dependence of the internal energy and the magnetic moment term of 2DES for the states wthout macroscopical currens in a close analogy to the case of the rotating liquid [5].

The regularization is essential to obtain this result. There are two possibilities for the regularization known from the theory of superfluid He_3 [6]. The simplest are singular vortices with the hard core defined by Coulomb interaction and the atomic structure of the underlying semiconductor of the heterostructure. That gives the estimate of the order of the atomic Bohr radius for the core size. The other possibility is a soft core with the size defined by the extension of the phase space. In 2DES that is either electron spin or the isospin connected with the next level of the size quantisation for the electron motion in the perpendicular to 2D plain direction. That corresponds to so called Skyrmion texture [7].That gives the core of the order of the magnetic length. Unfortunately there is no translation group for the lattices with the soft core vortices and the analysis of this kind of electron liquid is difficult. The author express his gratitude to L.P.Pitaevski for the usefull discussions. The work is supported by RFFR grant, the program "Quantum Macrophysics" of the RAS Presidium and the grant for the support of the scientific Scools by the President of RF.

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