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Curvature effects on magnetic susceptibility of 1D attractive two component fermions

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We develop a bosonization approach for finding magnetic susceptibility of 1D attractive two component Fermi gas at the onset of magnetization taking into account the curvature effects. It is shown that the curvature of free dispersion at the Fermi points couples the spin and charge modes and leads to a linear critical behavior and finite susceptibility for a wide range of models. Possible manifestations of spin-charge coupling in cold atomic gases are also briefly discussed.

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Elementary excitations in 1D interacting electron systems are not conventional quasiparticles carrying both spin and charge, but rather spin and charge waves that propagate with different velocities [1]. This behavior called spin-charge separation has been addressed in a number of experimental studies and demonstrated, in particular, in experiments with quantum wires in semiconductors [2]. There is a growing interest in revealing effects of spin-charge separation in experiments with cold Fermi gases [3], where the 1D regime has been recently achieved [4, 5].

On the other hand, the interaction between spin and charge degrees of freedom can lead to pronounced effects. The spin-charge interaction is seen in exact solutions for integrable systems, for instance in the Fermi Hubbard model for spin-1/2 fermions [6 – 9]. In this case the spin-charge coupling can also be treated by bosonization accounting for the curvature of the spectrum at the Fermi points [9]. In the presence of two gapless modes, this leads to the phenomenon of charge transfer by spin excitations [9].

In this letter we show that the spin-charge coupling drastically changes the critical behavior at the commensurate-incommensurate (C-IC) phase transition for spin-gapped fermions, ensuring a finite susceptibility. This transition occurs when the gap gets closed by a critical magnetic field and the magnetization emerges in the system. In the absence of spin-charge coupling, in particular at half filling, one has a universal square root dependence of magnetization on the field [10, 11] and an infinite susceptibilty. This is a consequence of the fact that solitons appearing in the spin sector above the critical field h_c behave themselves as free fermions. They have a quadratic dispersion, and the soliton density is proportional to the magnetization m. The kinetic energy of solitons is thus proportional to m^3 and minimization of their total energy $E \sim [-(h-h_c)m + \text{const} * m^3]$ in the field $h > h_c$ gives the square root dependence $m \sim \sqrt{h-h_c}$.

Away from half filling the spin-charge interaction, entering the problem through the curvature of the spectrum at the Fermi points, leads to an effective non-local and relatively long-range interaction in the spin sector. The effect of the interaction on the ground state energy is reduced to the change of basic parameters of the spin sector. This provides the appearance of $\sim m^2$ term in the ground state energy, ensuring a linear field dependence of magnetization and a finite susceptibility at the C-IC transition. We develop an effective field theory applicable for a wide range of models. Those include continuum models and extended Fermi Hubbard models with anisotropic interactions [12] and/ or mass (hopping) anisotropy [13]. For the integrable Fermi Hubbard model with only on-site interactions, this type of critical behavior is seen from the Bethe Ansatz solution of Ref. [7]. We give a transparent interpretation of this picture and show how the spin-charge interaction changes the behavior of correlation functions.

We first consider a dilute strong coupling limit for attractively interacting two component fermions and obtain the magnetization across the CIC transition. In the strong coupling limit, spin-↑ and spin-↓ fermions form strongly bound pairs and at low density the interaction between the pairs and uncompensated (for example, spin-up) particles created by the magnetic field can be neglected. Thus in this limit the system represents a mixture of noninteracting hard core bosons (bound pairs) and free fermions (uncompensated spin-$\uparrow$ particles) and the density of the thermodynamic potential is:

$$\Omega=\frac{v_\uparrow}{2}[(\partial_x\phi_\uparrow)^2+(\partial_x\theta_\uparrow)^2]+\frac{v_p}{2}[(\partial_x\phi_p)^2+(\partial_x\theta_p)^2]-\\-\frac{h}{2}\frac{\partial_x\phi_\uparrow}{\sqrt{\pi}}+W\cos\sqrt{4\pi}\phi_\uparrow-\mu\frac{(\partial_x\phi_\uparrow+2\partial_x\phi_p)}{\sqrt{\pi}}. \quad (1)$$

The fields $\partial_x\phi_p$, $\partial_x\phi_\uparrow$ and $\partial_x\theta_p$, $\partial_x\theta_\uparrow$ represent density and current fluctuations for the pairs and uncompensated fermions [1], h is the magnetic field, μ is the chemical potential, and the multiple $\mathcal{N}=(\partial_x\phi_\uparrow+2\partial_x\phi_p)/\sqrt{\pi}$ describes fluctuations of the total number of fermions. The term $W\cos\sqrt{4\pi}\phi_\uparrow$ provides a gap for spin excitations which in the absence of the field can be described as massive fermions. Indeed, using bosonization ruls, it is easy to see that the spin-up part of Eq.(1) can be rewritten as the Hamiltonian of noninteracting massive gapped fermions, where the term $W\cos\sqrt{4\pi}\phi_\uparrow$ emerges [1]. The gap gets closed by a critical magnetic field h_c. At a fixed μ the fields $\partial_x\phi_p,\partial_x\theta_p$ and $\partial_x\phi_\uparrow,\partial_x\theta_\uparrow$ are decoupled and one obtains the usual square root dependence of magnetization on the field [10, 11]: $m\sim\sqrt{h-h_c}$, for $h\to h_c+0$. At a constant number of particles we have a constraint $\langle\mathcal{N}\rangle=0$, which provides coupling between the fields of spin-$\uparrow$ fermions and pairs and modifies the square root dependence to a linear one. At a critical field $h_c=2\Delta$, where 2Δ is equal to the binding energy of the pairs, the low-momentum dispersion relation for spin-$\uparrow$ fermions is $E_\uparrow(k)=\sqrt{v_\uparrow^2k^2+\Delta^2}-\Delta\simeq v_\uparrow^2k^2/2\Delta$, with $v_\uparrow$ being their velocity. The bound pairs disperse linearly with velocity $E_p(k)=v_p|k|$. Taking into account the constraint $\langle\mathcal{N}\rangle=0$ we have the total energy $E=\sum E_\uparrow(k)+\sum E_p(k)-m(h-h_c)==v_pm^2/2\pi-(h-h_c)m+O(m^3)$. Minimazing E we obtain linear dependance $m=(h-h_c)/\pi v_p$ for $h\to h_c+0$ and a finite susceptibility

$$\chi=\partial m/\partial h|_{h_c}=1/\pi v_p. \quad (2)$$

We now turn to the opposite case of weak coupling and derive in the spin and charge basis an asymptotically exact theory near the critical point. Taking into account the curvature κ of the spectrum at the Fermi points [9, 14], the low-energy Euclidean action in the weak coupling limit to the lowest order in κ can be written as [9, 15]:

$$S_E=\int dx d\tau\left\{\sum_{\alpha=c,s}\frac{u_\alpha}{2K_\alpha}[(\partial_x\phi_\alpha)^2+\frac{1}{u_\alpha^2}(\partial_\tau\phi_\alpha)^2]+\right.\\+\frac{g_s}{2\pi}\cos(\sqrt{8\pi}\phi_s)-\frac{h}{v_F}\frac{\partial_x\phi_s}{\sqrt{2\pi}}+\\+\frac{\sqrt{2\pi}\kappa}{K_sK_cv_F}\partial_x\phi_s\partial_\tau\phi_s\partial_\tau\phi_c+\\\left.+\frac{\sqrt{\pi}\kappa}{\sqrt{2}v_F}\partial_x\phi_c\left[(\partial_x\phi_s)^2+(\partial_\tau\phi_s)^2/K_s^2\right]+\cdots\right\}, \quad (3)$$

where $\tau=iv_Ft$ is the Euclidean time, v_F is the Fermi velocity, and the subscripts $_c$ and $_s$ stand for the charge and spin sectors. The field $\partial_x\phi_c$ describe fluctuations of the charge (mass) density, while $\partial_x\phi_s$ stands for fluctuations of the spin density with $\phi_{c,s}=(\phi_\uparrow\pm\phi_\downarrow)/\sqrt{2}$. The action (3) is applicable for a wide range of models for spin-1/2 fermions, including continuum and extended Hubbard models. The coupling constant g_s, Luttinger parameters $K_{c,s}$, and spin/charge velocities $v_{c,s}=u_{s,c}v_F$ depend on the Fourier transforms of the interaction potential at wavevectors $k=0$ and $k=2k_F$ [1]. For spin-gapped fermions which are SU(2) symmetric at $h<h_c$, one has $g_s<0$, $K_s=1+g_s/2$, and the charge sector is gapless. In the weakly interacting regime both K_s and K_c are close to unity. For simplicity, we put $v_c=v_s=v_F$, which does not affect our main results.

Compared to the standard action which is quadratic in currents and spin-charge separated, Eq.(3) has extra (cubic) terms [14, 9, 15] accounting for the curvature of the free spectrum at the Fermi points. It couples the spin and charge sectors and is proportional to $\kappa\equiv\partial^2E(k)/2\partial k^2|_{k_F}$. Dots in Eq.(3) stand for higher order terms that we neglected and for cubic terms within the charge sector which we omitted as irrelevant modifications of the linearly dispersing charge mode. The cubic terms of Eq.(3) describe a long-range interaction between spin solitons through the charge sector in the second order of perturbation theory. We will show that the effect of this interaction can be reduced to modifications of the basic parameters of the spin sector. As a result the ground state energy shifts proportionally to the soliton density in the square.

For finding the susceptibility at a given number of particles we have to impose a constraint: $\langle\partial_x\phi_c\rangle=0$, which allows us to integrate out the charge modes. We calculate the ground state energy at the onset of magnetization, confining ourselves to the terms proportional to m^2. For extracting these terms we write: $\partial_x\phi_s==:\partial_x\phi_s:+\sqrt{2\pi}m$, with the symbol :: standing for the normal ordering with respect to the k_F corresponding to $m=0$[1]. This ammounts to separation of $\partial_x\phi_s$ into its mean part and fluctuations at $h>h_{cr}$. Then, after integrating out charge degrees of freedom, the Euclidean action is $S_{eff} = S_s^0 + S_\kappa$, where:

$$S_s^0 = \frac{1}{2K_s}\int \Big[(:\partial_\tau\phi_s:)^2 + (:\partial_x\phi_s:)^2 + 2\pi m^2 + + \frac{g_s K_s}{\pi}\cos\left(\sqrt{8\pi}:\phi_s:+4\pi m x\right)\Big]\, d\tau dx, \quad (4)$$

and it does not give rise to an m^2 contribution in the ground state energy[10, 11]. Retaining only contributions proportional to m^2, the term S_κ originating from the spin-charge interaction is given by:

$$S_\kappa = -\frac{2m^2\kappa^2\pi^2}{v_F^2}\times \int \sum_{i,j=0,1}^{i\neq j}\big[\partial^2_{x_i y_i}G_c(\mathbf{x},\mathbf{y}):\partial_{x_i}\phi_s(\mathbf{x})::\partial_{y_i}\phi_s(\mathbf{y}): - - \partial^2_{x_i y_j}G_c(\mathbf{x},\mathbf{y}):\partial_{x_i}\phi_s(\mathbf{x})::\partial_{y_j}\phi_s(\mathbf{y}):\big]\, d\mathbf{x}d\mathbf{y}. \quad (5)$$

Here $\mathbf{x} = \{x,\tau\} \equiv \{x_0, x_1\}$, and $\mathbf{y} = \{y,\tau'\} \equiv \{y_0, y_1\}$, and the propagator for the charge sector is $G_c(\mathbf{x},\mathbf{y}) = -K_c/4\pi \ln((x-y)^2/a^2 + (\tau-\tau')^2/a^2 + 1)$, where a is a short distance cut-off.

The ground state energy, in which we intend to extract the m^2 contribution, is given by:

$$E_0 = -\frac{v_F}{\int d\tau}\ln\left\{\int D\phi_s e^{-S_s^0}(1 - S_\kappa + O(m^4))\right\}, \quad (6)$$

where we have written $e^{-S_\kappa} = 1 - S_\kappa + O(m^4)$. As we see, Eq. (6) involves the calculation of the expectation value of S_κ (5) in the vacuum of the sine- Gordon theory at $m \to 0$.

For $h = h_{cr} + 0$ the vacuum of effective theory contains infinitesimally small density of solitons. However, after separating vacuum average from $\partial_x\phi_s$ at $h = h_{cr} + 0$, the m^2 contribution in Eq. (6) can be extracted using the vacuum at $h = h_{cr} - 0$ due to the relation:

$$\langle:\partial_{x_i}\phi_s::\partial_{y_j}\phi_s:\rangle_{h_{cr}+0} = \langle\partial_{x_i}\phi_s\partial_{y_j}\phi_s\rangle_{h_{cr}-0} + O(m). \quad (7)$$

Eq. (7) can be established from mapping the sine-Gordon model onto the massive Thirring model. Then at $K_s = 1/2$, where the spin sector is equivalent to free massive relativistic fermions, one easily gets Eq. (7). For $K_s \neq 1/2$ one finds that Eq. (7) holds in any order of perturbation theory in the Thirring coupling constant.

On the other hand, for $h < h_{cr}$ the magnetic field does not change the states of the system and only shifts the antisoliton and soliton energies by $\sim \pm h$ so that the energy of a soliton-antisoliton pair remains the same. Since $\partial_{x_i}\phi_s(\mathbf{x})$ has nonzero matrix elements only between the states which can differ from each other by a certain number of soliton-antisoliton pairs [16], the correlation function $\langle\partial_{x_i}\phi_s\partial_{y_j}\phi_s\rangle$ for $h = h_{cr} - 0$ is the same as at $h = 0$. At $h = 0$, due to the Euclidean invariance we have: $\langle\partial_{x_i}\phi_s\partial_{y_j}\phi_s\rangle = \partial_{x_i}\partial_{y_j}G_s(r)$, where $r = \sqrt{(x-y)^2 + (\tau-\tau')^2}$ is the radial variable. The expectation value $\langle S_\kappa\rangle$ is given by Eq. (5), with the products $\partial_{x_i}\phi_s(\mathbf{x})\partial_{y_j}\phi_s(\mathbf{y})$ replaced by the corresponding correlation functions. Then, after integrating over the angular variable, the non-local terms of the first line of Eq. (5) and those of the second line cancel each other. This is a consequence of the Eucidean (Lorentz in real space) symmetry of correlation functions. Accordingly, the expectation value of the integrand in Eq. (5) reduces to:

$$\frac{\delta(\mathbf{x}-\mathbf{y})}{2K_c^{-1}}\sum_i <\partial_{x_i}\phi_s(\mathbf{x})\partial_{y_i}\phi_s(\mathbf{y})>.$$

This means that the m^2 contribution to E_0 (6) can be obtained by using a simplified effective action:

$$S_{eff} = S_s^0 - \frac{m^2\kappa^2\pi^2K_c}{v_F^2}\int dx d\tau\left[(\partial_x\phi_s)^2 + (\partial_\tau\phi_s)^2\right]. \quad (8)$$

One thus sees that the effect boils down to the renormalization of the Luttinger parameter of the spin sector (increase of K_s) with m^2:

$$K_s \to K_s\left(1 + 2m^2\kappa^2\pi^2/v_F^2\right). \quad (9)$$

Note that for spin-gapped fermions which are $SU(2)$ symmetric at $h < h_c$, equation (9) encodes breaking of the $SU(2)$ symmetry.

From the rescaling of the Luttinger parameter determined by Eq. (9) we obtain the following m^2 contribution to the ground state energy:

$$\Delta E_0(m^2) = \frac{\partial E_0}{\partial K_s}\Delta K_s = \frac{2K_s m^2\kappa^2\pi^2}{v_F^2}\frac{\partial E_0}{\partial K_s}. \quad (10)$$

For the inverse susceptibility Eq. (10) then yields:

$$v_F^2\chi^{-1} = 4K_s\kappa^2\pi^2\partial\mathcal{E}_0/\partial K_s, \quad (11)$$

where $\mathcal{E}_0$ stands for the ground state energy density of the sine-Gordon model. For $K_s \to 1$ we can follow the RG procedure [17, 18] in order to extract the leading contribution to the ground state energy density of the sine-Gordon model. In the one-loop approach we have $\mathcal{E}_0 = -\lambda\Delta^2/v_F$, where Δ is the soliton mass (gap in the excitation spectrum), and λ is a positive factor which we will fix later for the $SU(2)$ symmetric sine-Gordon case. One-loop RG estimate of the soliton mass is [17–19]:

$$\Delta \simeq E_F \begin{cases} \exp\{-\frac{\arctan\sqrt{g_s^2/(2-2K_s)^2-1}}{\sqrt{g_s^2-(2-2K_s)^2}}\}; \ \frac{|g_s|}{(2-2K_s)} \geq 1 \\ \exp\{-\frac{\operatorname{arctanh}\sqrt{1-g_s^2/(2-2K_s)^2}}{\sqrt{(2-2K_s)^2-g_s^2}}\}; \ \frac{|g_s|}{(2-2K_s)} \leq 1 \end{cases} . \quad (12)$$

Finally, from Eq. (11) in the vicinity of the $SU(2)$ separatrix of the sine- Gordon RG flow, on which the model is $SU(2)$ symmetric, we obtain:

$$\chi^{-1} = \frac{4\lambda K_s \kappa^2 \pi^2 \Delta^2}{3(1-K_s)^2 v_F^3} = \frac{16\lambda K_s \kappa^2 \pi^2 \Delta^2}{3 v_F^3} \ln^2 \frac{\Delta}{E_F} \quad (13)$$

up to subleading contributions.

Equation (13) is valid for a wide class of generic models, including those with the spin anisotropy. Strictly speaking, the Hamiltonian (3) requires small g_s and K_s close to unity. Nevertheless, one can think of extending our results to K_s away from unity, in particular to the Luther-Emery point $K_s \to 1/2$. Then, it is straightforward to evaluate χ^{-1} by mapping the spin sector onto free massive fermions, which gives: $\chi^{-1} \propto \kappa^2 \partial \mathcal{E}_0/\partial K_s \propto \kappa^2 \left\langle (\partial_t \phi_s)^2/v_F^2 - (\partial_x \phi_s)^2 \right\rangle \propto$ $\propto (\kappa \Delta \ln \Delta/E_F)^2$.

The most important result is that the susceptibility at the commensurate-incommensurate phase transition stays finite if the curvature is finite: $\kappa \neq 0$. At the onset of magnetization the susceptibility is finite also for free fermions, where $\Delta = 0$ and $\chi \sim v_F^{-1}$. However, as we see from Eq.(13), in the limit of $\Delta \to 0$ the susceptibility diverges. This was previously observed for the Hubbard model [7] and attributed to a singular character of the zero interaction point.

In the case of integrable Fermi Hubbard model with only on-site attractive interaction $U < 0$, one has $1 - K_s \simeq |U|/2\pi v_F$ and the result of Eq. (13) is similar to the Bethe Ansatz calculation in the weak coupling limit [7]: $\chi^{-1}(|U \to 0) = 8\kappa^2\pi^3\Delta^2/v_F U^2$. This implies that the factor λ is equal to $3/2\pi$ on the $SU(2)$ line.

For strong coupling the Bethe Ansatz inverse susceptibility is given by $\chi^{-1}(|U| \to \infty) = 2\pi^2\nu(1-\nu)^2/|U|$ [7], which at a low filling factor ν tends to our strong coupling result (2), with $v_p = 2\pi\nu/|U|$.

We now analyze the behavior of pair and single fermion correlation functions at the onset of magnetization ($h > h_{cr}$ and $m \to 0$) [20]. For the Hubbard model, using explicitly the dressed charge matrix [21], in the presence of two gapless modes one obtains an effective Hamiltonian density[8, 22, 23]:

$$\mathcal{H}_{eff} = \sum_{\beta=\pm} \frac{v_\beta}{2} \left[(\partial_x \phi_\beta)^2/K_\beta + K_\beta (\partial_x \theta_\beta)^2 \right]. \quad (14)$$

The fields $\phi_\pm$ and $\theta_\pm$ are related to the spin and charge fields through the spin-charge mixing parameter ξ:

$$\phi_+ = \phi_c - \xi\phi_s, \, \theta_+ = \theta_c, \quad \phi_- = \phi_s, \, \theta_- = \theta_s + \xi\theta_c, \quad (15)$$

and $v_\pm$, $K_\pm$ are the Bethe Ansatz velocities and Luttinger parameters for the $\pm$ sectors. For $m \to 0$ we have $v_- \propto m \to 0$, $K_- \to 1/2$ at any U and ν [24]. In the case of half filling ($\nu = 1$) one has $K_+ = 1$, $\xi = 0$ for all $|U|$, and there is an exact spin-charge separation so that the fields $\phi_\pm, \theta_\pm$ coincide with $\phi_{c,s}, \theta_{c,s}$. For $\nu < 1$ one has

$$K_+ = 1 + \frac{|U|}{2\pi v_F}; \ \xi = \sqrt{\frac{8 v_F}{|U|}} \cos\left(\frac{\pi\nu}{2}\right) \exp\left(-\frac{\pi v_F}{|U|}\right) \quad (16)$$

at $|U| \to 0$, with the Fermi velocity $v_F = 2\sin\pi\nu/2$, and $K_+ = 2$, $\xi = 1-\nu$, $v_+ = 2\pi\nu/|U|$ for $|U| \to \infty$. So, the parameter of spin-charge mixing, ξ, ranges from 0 to $(1-\nu)$ and monotonically increases with $|U|$. The effective Hamiltonian (14) is obtained through the Bethe Ansatz calculation and for the inverse susceptibility it naturally gives the exact result [7]: $\chi^{-1} = 2\pi v_+ \xi^2/K_+$.

Asymptotic behavior of correlation functions for the Hubbard model with a repulsive on-site interaction, in the presence of two gapless modes (and in the presence of magnetic field), was obtained by Frahm and Korepin [6]. Critical exponents for the general case have been obtained from a numerical solution of the coupled Bethe Ansatz integral equations for the dressed charge matrix. The effective Hamiltonian (14) was constructed by Penc and Sólyom in such a way that it reproduces the Bethe Ansatz behavior of correlation functions [8]. This procedure of obtaining an effective Hamiltonian was retranslated to the case of attractive Hubbard model by using the particle hole transformation [23].

The limit of $m \to 0$ allows us to derive analytical expressions for the critical exponents of the correlation functions and make a number of physical conclusions. For the pair correlation function from Eq. (14) we obtain:

$$\langle \psi_\uparrow^\dagger(x) \psi_\downarrow^\dagger(x) \psi_\downarrow(0) \psi_\uparrow(0) \rangle \propto \frac{\cos 2\pi m x}{x^{1/2 + 1/K_+}}; \ x \to \infty, \quad (17)$$

whereas for $h < h_{cr}$ it is $\sim x^{-1/K_+}$. There is a universal jump of 0.5 in the critical exponent, the result that is expected from the theory based on spin-charge separation. However, for the single fermion Green function we find:

$$\langle \psi_{\uparrow(\downarrow)}^\dagger(x) \psi_{\uparrow(\downarrow)}(0) \rangle \propto \frac{\cos k_{F\uparrow(\downarrow)} x}{x^{\nu_{\uparrow(\downarrow)}}}; \ x \to \infty, \quad (18)$$

where $k_{F\uparrow(\downarrow)}$ is the Fermi momentum of spin-up (down) fermions given by the free value. The critical exponent of the majority (spin-up) component is $\nu_\uparrow = 1/2 + +K_+/4+(1+\xi)^2/8+(1-\xi)^2/4K_+$, and for the spin-down component we obtain $\nu_\downarrow = \nu_\uparrow+(1/K_+-1/2)\xi > \nu_\uparrow$. The presence of an additional spin-charge mixing term $\sim \xi$ in the critical exponent of the single fermion Green function suggests that $\nu_\uparrow < \nu_\downarrow$ even in the limit of $m \to 0$, which is a clear signature of spin-charge coupling. Persistence of spin-charge coupling down to $m \to 0$ limit was recently observed numerically [25]. The difference $\nu_\downarrow - \nu_\uparrow$, at a given magnetization m, increases with $|U|$ for weak coupling, reaches its maximum in the regime of intermediate coupling, and then decreases with increasing $|U|$ in the strong coupling regime. Thus, the effect of spin-charge mixing is the most pronounced at an intermediate coupling strength.

In conclusion, we showed that the curvature couples spin and charge modes for $m \to 0$ and changes critical properties of 1D spin gapped fermions at the onset of magnetization. Our findings emphasize the importance of spin-charge coupling in 1D gapped systems, and experiments with cold atoms can shed new light on this problem. Two-component Fermi gas in a 1D optical lattice is well suited for revealing spin-charge separation or observing spin-charge coupling, especially in a box potential where ν is coordinate independent.

Periodic modulations of the box size can only excite in-phase oscillations of the two components (charge oscillations), and they will not excite out-of-phase oscillations (spin modes) at half filling where exact spin-charge separation holds. In contrast, for a significantly smaller filling factor the excitation of these modes will be provided by spin-charge coupling.

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