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Spectrum of bound fermion states on vortices in ^{3}He-B

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We study subgap spectra of fermions localized within vortex cores in ^{3}He-B. We develop an analytical treatment of the low-energy states and consider the characteristic properties of fermion spectra for different types of vortices. Due to the removed spin degeneracy the spectra of all singly quantized vortices consist of two different anomalous branches crossing the Fermi level. For singular o and u vortices the anomalous branches are similar to the standard Caroli-de Gennes-Matricon ones and intersect the Fermi level at zero angular momentum yet with different slopes corresponding to different spin states. On the contrary the spectral branches of nonsingular vortices intersect the Fermi level at finite angular momenta which leads to the appearance of a large number of zero modes, i.e. energy states at the Fermi level. Considering the v, w and uvw vortices with superfluid cores we show that the number of zero modes is proportional to the size of the vortex core.

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1. Introduction. Since the pioneering work of Caroli, de Gennes and Matricon (CdGM) [1] it is well known that quantized vortices in superconductors and Fermi suprfluids have nontrivial internal electronic structure. It consists of low energy fermionic excitations localized within the vortex cores with characteristic interlevel spacing defined as $\Delta_0^2/E_F \ll \Delta_0$, where Δ_0 is the energy gap far from the vortex line and E_F is the Fermi energy. For conventional s-wave superconductors the excitation spectrum of each individual vortex $E(Q)$ of a subgap state varies from $-\Delta_0$ to $+\Delta_0$ as one changes the angular momentum Q defined with respect to the vortex axis.

At small energies $|E| \ll \Delta_0$ the spectrum is a linear function of Q:

$$E(Q) \simeq -Q\omega, \qquad (1)$$

where $\omega \approx \Delta_0/k_\perp\xi$, Δ_0 is the superconducting gap value far from the vortex axis, $k_\perp = \sqrt{k_F^2 - k_z^2}$, k_F is the Fermi momentum, k_z is the momentum projection on the vortex axis, $\xi = \hbar V_F/\Delta_0$ is the coherence length, V_F is the Fermi velocity, and Q is half an odd integer. Under some exotic conditions [2] several vortices can merge and then one obtains a multiquantum vortex with a certain winding number M. The number of anomalous branches per spin projection [3] is equal to the vorticity M. For the states with an even vorticity all the anomalous branches cross the Fermi level at nonzero angular momentum Q_j:

$$E(Q) \sim -(Q \pm Q_j)\Delta_0/k_\perp\xi \qquad (2)$$

where $j = 1...M/2$, $Q_{M/2} \sim k_\perp\xi$. For a vortex with an odd winding number there appears a branch crossing the Fermi level at zero impact parameter.

The quantized vortices in ^{3}He-B have much in common with vortices in ordinary s-wave superconductors. However in multi-component superfluid system ^{3}He axial symmetry allows the nucleation of additional order parameter components inside vortex core. Thus vortices in this system are in general nonsingular, i.e. may have a superfluid core unlike singular vortices in s-wave superconductors which always have a normal core. There exist five types of vortices with different internal core structures in ^{3}He-B: o, u, v, w and uvw vortices [4 – 6]. The o vortex is the most symmetric one, it has no superfluid core and consists of almost pure B-phase without inclusions of other phases. Other vortices break some of the discrete symmetries existing for the most symmetric o vortex. Among them the u vortex is singular while the remain v, w and uvw vortices have superfluid cores.

According to the analysis in the framework of Ginzburg-Landau theory [5 – 7] near the critical temperature only v vortex is stable. The cores of such vortices are occupied by an A phase and a ferromagnetic β phase [4 – 6]. These additional phases correspond to a nonzero total angular momentum projection on the vortex axis and a zero vorticity in the real space. Therefore nuclea of additional phases remain finite at the vortex center. Nucleation of ferromagnetic β phase inside vortex cores

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explains a large spontaneous magnetic moment of vortices revealed in the NMR experiments in rotating ^{3}He-B [8]. The first order phase transition seen in the NMR experiments was associated with the change of the symmetry of the internal core structure [8, 9].

As was shown by Volovik [10, 11] in vortices with dissolved core singularity the spectrum of bound fermion states can be substantially modified in contrast to ordinary CdGM spectrum of singular vortices. In particular the presence of other superfluid phases inside vortex core leads to the appearance of large number of zero modes, i.e. the spectral branches crossing the Fermi level. The number of these zero modes can be as high as $E_F/\Delta_0 \sim k_F\xi \gg 1$. Thus a minigap in spectrum of bound fermions, which is a characteristic feature of CdGM spectrum [1] is absent for nonsingular vortices in ^{3}He-B. Zero modes also exist even for a singular and most symmetric o vortex. Although the number of them is much smaller than for nonsingular vortices it can be effectively controlled by external magnetic field [12, 13]. Even in zero magnetic field due to a broken relative spin-orbital symmetry in B-phase of ^{3}He the spin degeneration of the energy spectrum is removed [12]. As a result the CdGM spectral branches acquire spin dependent shift which closes the minigap. Localized fermions which occupy the negative energy states on the spectral branches intersecting zero energy level form a one dimensional Fermi liquid inside vortex core, which can lead to the instability of the vortex core structure [13].

In this Letter we develop a generalization of CdGM theory for the case of vortices in B phase of superfluid ^{3}He. We derive a general expression for spectrum of vortex core quasiparticles in the presence of multiple order parameter components inside vortex core and analyze the spectra of several particular vortex types.

The method that we use is based on the approximate analytical solution of quasiclassical Andreev equation describing the motion of quasiparticles along the trajectories inside vortex core. Earlier this method was applied to study the spectrum of quasiparticles localized within the cores of multiquanta vortices [3, 14]. Generally the Andreev equation for two component wave function $\psi = (U, V)$ along the quasiclassical trajectory has the form

$$-i\tilde{\xi}\hat{\tau}_3\frac{\partial\psi}{\partial s} + \hat{\tau}_1\hat{\Delta}_R\psi - \hat{\tau}_2\hat{\Delta}_{Im}\psi = E\psi, \qquad (3)$$

where $\hat{\tau}_{1,2,3}$ are Pauli matrices of Bogolubov-Nambu spin, $\tilde{\xi}$ is a length scale of the order of coherence length, $\hat{\Delta}_R = (\hat{\Delta} + \hat{\Delta}^+)/2$ and $i\hat{\Delta}_{Im} = (\hat{\Delta} - \hat{\Delta}^+)/2$ are the hermitian and anti-hermitian parts of normalized gap operator, E is energy normalized to the bulk value of gap function Δ_0.

Here we should take into account the spinor structure of quasiparticle wave functions which is essential in ^{3}He. In this case the coefficients $\hat{\Delta}_{R,\mathrm{Im}}$ in Andreev equation (3) are 2×2 matrices in spinor space. Then the matrix Eq. (3) is a system of 4 scalar equations. If matrices $\hat{\Delta}_R$ and $\hat{\Delta}_{\mathrm{Im}}$ commute $[\hat{\Delta}_R, \hat{\Delta}_{\mathrm{Im}}] = 0$ the fourth order Andreev equation can be reduced to 2 equations of the second order. However this can not always be the case. To develop a general perturbation theory we note that if $\hat{\Delta}_{\mathrm{Im}} \equiv 0$ the exact solution of the Andreev Eq. (3) corresponding to $E = 0$ can be obtained in a spinor basis diagonalizing the matrix $\hat{\Delta}_R = \mathrm{diag}(\Delta_{R1}, \Delta_{R2})$. Then we obtain two degenerate solutions $\psi_{1,2} = (1, -i)_\tau f_{1,2}(s)$ corresponding to the zero energy, where $f_j(s) = \Lambda_j \exp\left(-\tilde{\xi}^{-1}\int_0^s \Delta_{Ri}ds\right)$ and Λ_j is an eigen spinor of matrix $\hat{\Delta}_R$. As we will see below in case of a single–quantum vortex the functions $\Delta_{R1,2}(s)$ have asymptotics of different signs $\Delta_{Ri}(+\infty)\Delta_{Ri}(-\infty) < 0$. We assume that $\Delta_{Ri}(+\infty) > 0$ therefore the solutions $f_{1,2}(s)$ decay at $s = \pm\infty$. Using this localized solution as a zero–order approximation for the wave function the spectrum can be found within the first order perturbation theory assuming that $|E| \ll 1$ and $|\hat{\Delta}_{\mathrm{Im}}(s)| \ll 1$. In general $f_{1,2}(s)$ are not the eigen spinors of the operator $\hat{\Delta}_{\mathrm{Im}}$ which therefore couples the ψ_1 and ψ_2 states. Then the standard perturbation theory yields the secular equation

$$\det\begin{pmatrix} S_{11} - E & S_{12} \\ S_{12}^* & S_{22} - E \end{pmatrix} = 0, \qquad (4)$$

where the matrix elements are
$S_{11(22)} = 2\langle f_{1(2)}|\hat{\Delta}_{\mathrm{Im}}|f_{1(2)}\rangle$ and $S_{12} = 2\langle f_1|\hat{\Delta}_{\mathrm{Im}}|f_2\rangle$.

In general the accuracy of the first order perturbation correction should be determined by the factor $O(\hat{\Delta}^2_{\mathrm{Im}})$, where $|\hat{\Delta}_{\mathrm{Im}}| \ll 1$ is a small parameter. However in a particular case of Eq.(3) the second order correction to the zero energy level is exactly zero and therefore the accuracy of Eq.(4) is much better: $O(\hat{\Delta}^3_{\mathrm{Im}})$. To prove this result we assume for simplicity that $[\hat{\Delta}_R, \hat{\Delta}_{\mathrm{Im}}] = 0$ so that $S_{12} = 0$. Then if the eigen function $\psi = (U, V)_\tau$ of Eq.(3) with $\hat{\Delta}_{\mathrm{Im}} = 0$ corresponds to the energy ε_n the other function $\tilde{\psi} = (-V, U)_\tau$ corresponds to the energy $-\varepsilon_n$. Therefore it is easy to check that the contribution from negative energy levels to the second order perturbation of the energy level $E = 0$ exactly compensates the contribution from the positive levels. The proof modification to the general case $[\hat{\Delta}_R, \hat{\Delta}_{\mathrm{Im}}] \neq 0$ is straightforward.

2. Basic formulas. Our further consideration is based on the Bogoulubov-Nambu equation for the quasiparticles near the Fermi level. From the beginning we assume the system to be homogeneous in z direction which coincides with the vortex axis. Then we obtain two-dimensional Bogoulubov-Nambu equations with the effective Fermi energy $E_\perp = E_F - \hbar^2 k_z^2/2m$ and the Fermi momentum in xy plane $k_\perp = \sqrt{k_F^2 - k_z^2}$:

$$\hat{H}_0\psi + \hat{\tau}_1\hat{\Delta}_R\psi - \hat{\tau}_2\hat{\Delta}_{\rm Im}\psi = E\psi, \qquad (5)$$

where $\hat{H}_0 = \hat{\tau}_3(\hat{\mathbf{p}}^2 - \hbar^2 k_\perp^2)/2m$, and $\hat{\mathbf{p}} = -i\hbar\nabla$. Further we will assume that the gap function and energy are normalized to the bulk value of the energy gap Δ_0.

Generally the gap function in ^{3}He-B can be parameterized as follows: $\hat{\Delta} = -i\hat{\sigma}_y(\hat{\sigma}\cdot\mathbf{d})$, where $\mathbf{d}$ is a vector in 3D space and $\hat{\sigma}_{x,y,z}$ are Pauli matrices in conventional spin space. Being proportional to the wave function of Cooper pairs in isotropic liquid ^{3}He the gap function can be presented as a superposition: $\hat{\Delta} = \sum_{\mu,\nu} C_{\mu\nu}\{e^{i(M-\mu-\nu)\varphi}, \hat{\Delta}_{\mu\nu}\}$, where φ is a polar angle in xy plane, M is vorticity, $\hat{\Delta}_{\mu\nu}$ is a Cooper pair wave function with definite angular momentum $\nu = -1, 0, 1$ and spin $\mu = -1, 0, 1$ projections on the z axis and $\{\hat{A}\hat{B}\} = (\hat{A}\hat{B} + \hat{B}\hat{A})/2$ is an anticommutator.

The order parameter distribution should be axisymmetric with the generator of rotation symmetry around z axis [6] $\hat{Q} = \hat{L}_z + \hat{S}_z - M\hat{I}$, where $\hat{L}_z$ and $\hat{S}_z$ are the projections of internal angular momentum and spin of Cooper pairs onto the z axis and M is a total vorticity. Thus for all order parameter components the condition $\mu + \nu = M$ should be satisfied. For singly quantized vortices $M = 1$ there can exist five basic components of the order parameter. Among them are $C_{1,-1}$, $C_{-1,1}$ and C_{00} which correspond to the main B phase, $C_{0,1} = C_A$ and $C_{1,0} = C_\beta$ which correspond to the additional A and β phases localized inside vortex core. The additional A phase has a zero spin projection ($\mu = 0$) and unit projection of orbital momentum ($\nu = 1$) on the z axis while β phase has $\mu = 1$ and $\nu = 0$. The components of gap function $\hat{\Delta}_{\mu\nu}$ are characterized by $\mathbf{d}$ vector as follows [6] $\mathbf{d} = \lambda^\mu(\lambda^\nu\cdot\mathbf{q})$, where $\mathbf{q} = \mathbf{k}/k_F$ and $\lambda^{\pm1} = (\mathbf{x_0} \pm i\mathbf{y_0})$, $\lambda^0 = \mathbf{z_0}$. Correspondingly in B phase we have $\mathbf{d} = \mathbf{q}$, in A phase $\mathbf{d} = \mathbf{z_0}(q_x + iq_y)$ and in β phase $\mathbf{d} = q_z(\mathbf{x_0} + i\mathbf{y_0})$. Far from the vortex core at $r \gg \xi_v$ only B superfluid phase exists so that $C_{1,-1} = C_{-1,1} = C_{00} = 1$ and $C_{A,\beta} = 0$. The vortex type is determined by the behaviour of amplitudes $C_{\mu\nu}$ at smaller distances $r \sim \xi_v$ and there exist five types of vortices [6].

Vortices of o and u types are singular so that only the superfluid components of B phase $C_{1,-1}$, $C_{-1,1}$ and C_{00} are nonzero. These amplitudes are real for the most symmetric o vortex and complex ones for u vortex with conserved parity $P_1 = P$ but broken $P_3 = TO_x^\pi$ discrete symmetry. Here T is time inversion and O_x^π is a rotation by the angle π around the axis x perpendicular to the vortex axis z. The gap function which describes singular B phase vortices can be presented in the form

$$\hat{\Delta}_B = -i\hat{\sigma}_y\{(\hat{\sigma}\cdot\mathbf{d_B}), e^{i\varphi}\}, \qquad (6)$$

where $\mathbf{d_B} = (B_+q_x - iB_-q_y, B_+q_y + iB_-q_x, C_{00}q_z)$, $B_\pm = (C_{1,-1} \pm C_{-1,1})/2$. Generally $B_\pm = B_\pm(r)$ are arbitrary complex functions with asymptotics $B_+(\infty) = 1$ and $B_-(\infty) = 0$ so that $\mathbf{d_B}(\infty) = \mathbf{q}$.

Nonsingular v, w and uvw vortices have superfluid cores with the inclusion of A and β phases:

$$\hat{\Delta}_A = C_A q_\perp e^{i\theta_p}\hat{\sigma}_x, \qquad (7)$$

$$\hat{\Delta}_\beta = C_\beta q_z\left(1 - \hat{\sigma}_z\right), \qquad (8)$$

where $q_\perp = k_\perp/k_F$. The functions $C_{A,\beta} = C_{A,\beta}(r)$ describing the spatial distributions of additional A and β phases inside vortex core are finite at $r = 0$ and vanish outside the core at $r \gg \xi_v$. The v and w vortices are characterized by real B phase amplitudes. If $C_{A,\beta}$ are also real then we have a v vortex with conserved $P_2 = PTO_x^\pi$ symmetry. The case when $\mathrm{Re}(C_{A,\beta}) = 0$, $\mathrm{Im}(C_{A,\beta}) \neq 0$ corresponds to w vortex with conserved $P_3 = TO_x^\pi$ symmetry. The less symmetric uvw vortex with all discrete symmetries P_1, P_2, P_3 broken has complex amplitudes of B, A and β phases.

Within the quasiclassical approximation Eq.(5) can be reduced to 4 equations of the first order along linear trajectories, i.e. the straight lines along the direction of Fermi momentum $\mathbf{q} = (\cos\theta_p, \sin\theta_p)$ (for a detailed review of this transformation see e.g. Ref.[14]). Each trajectory is specified by the angle θ_p and the impact parameter $b = k_F\mathbf{z_0}\cdot(\mathbf{q}\times\mathbf{r})$. Introducing the coordinate along trajectory $s = (\mathbf{q}\cdot\mathbf{r})$ we arrive at the quasiclassical equation $\hat{H}\psi = E\psi$ for the wave function $\psi(s, \theta_p)$. The quasiclassical hamiltonian is

$$\hat{H} = -i\xi q_\perp\hat{\tau}_3\frac{\partial}{\partial s} + \hat{\tau}_1\hat{\Delta}_R - \hat{\tau}_2\hat{\Delta}_{\rm Im}. \qquad (9)$$

The impact parameter of quasiclassical trajectories is proportional to the projection of angular momentum Q of quasiparticles on the z axis: $b = -Q/k_\perp$. However the hamiltonian (9) does not commute with the corresponding operator $\hat{L}_z = -i\partial/\partial\theta_p$ since in general it is not conserved in ^{3}He. Still due to the axial symmetry of vortices the total momentum $\hat{L}_z + \hat{S}_z$ is conserved. Therefore the angular and coordinate variables θ_p and s in the quasiclassical hamiltonian (9) can be separated. Let us introduce the new functions

$$\tilde{U} = e^{i\hat{\sigma}_z\theta_p/2}\hat{M}_0^+ U, \qquad (10)$$

$$\tilde{V} = e^{i\hat{\sigma}_z \theta_p/2} V, \quad (11)$$

where $\hat{M}_0 = -ie^{i\theta_p}\hat{\sigma}_y(\hat{\sigma}\cdot\mathbf{q})$. It is easy to check that the resulting gap operators (12), (14), (15) after this transformation do not depend on the angle θ_p. For the singular part of gap function we obtain

$$\hat{\Delta}_B = \frac{s-ib}{r}\left[C_B + \delta\hat{\Delta}_{B1}\right], \quad (12)$$

where $r=\sqrt{s^2+b^2}$, $C_B = q_\perp^2 B_+ + q_z^2 C_{00}$ and

$$\delta\hat{\Delta}_{B1} = q_\perp\left[B_-\left(q_z\hat{\sigma}_x - q_\perp\hat{\sigma}_z\right) + i(B_+ - C_{00})q_z\hat{\sigma}_y\right]. \quad (13)$$

The expressions for the gap function describing the additional A and β phases inside vortex core read

$$\hat{\Delta}_A = C_A\left(q_z q_\perp - iq_\perp^2\hat{\sigma}_y\right). \quad (14)$$

and

$$\hat{\Delta}_\beta = C_\beta q_z\left(1-\hat{\sigma}_z\right)\left(q_\perp + q_z\hat{\sigma}_x\right). \quad (15)$$

Then one can search the solution in the factorized form: $\psi(s,\theta_p) = \psi(s)\exp(i(Q+1/2)\theta_p)$. Note that from Eqs.(10), (11) it follows that the values of azimuthal quantum number Q should be integer to pertain the unambiguity of initial wave function $\psi=(U,V)$.

3. Spectrum of vortex core states. At first let us consider the quasiparticle spectrum of singular o and u vortices when $C_{A,\beta}=0$. In general case $C_{1,-1}\neq C_{-1,1}\neq C_{0,0}$ the gap function is given by Eqs.(12), (13). We will assume the simplifying condition to be fulfilled $|B_-|, |B_+ - C_{00}| \ll 1$ which is justified by Ginzburg-Landau calculations [6]. Then we can take into account only the hermitian part of the operator $(\delta\hat{\Delta}_{B1})_R = (\delta\hat{\Delta}_{B1} + \delta\hat{\Delta}_{B1}^+)/2$ in Eq.(12) since the anti hermitian part gives the contribution to the energy spectrum of the higher order in small parameter $O(|B_-|, |B_+ - C_{00}|)$. In this case the zero order solution of Andreev Eq.(3) is spin degenerate $\psi_{1,2}(s) = \psi_0(s) = (1,-i)_\tau f_0(s)$.

Diagonalizing the gap function $\hat{\Delta}_B$ by spin and using the Eq.(4) we obtain the energy spectrum in the following form

$$E(Q,q_z,\chi) = -\omega_\pm Q, \quad (16)$$

with $\omega_\pm = \mathrm{Re}\langle C_B/k_\perp r\rangle_0 + \chi F$, where $F^2 = k_F^{-2}\times[\mathrm{Re}^2\langle B_-/r\rangle_0 + q_z^2\mathrm{Im}^2\langle(B_+ - C_{00})/r\rangle_0]$ and $\chi=\pm1$ corresponds to different spin states. For brevity we have denoted $\langle X\rangle_0 = 2\langle f_0|X|f_0\rangle$. For the o vortex with $\mathrm{Im}(B_+ - C_{00}) = 0$ the difference between ω_+ and ω_- is determined by the asymmetry of amplitudes $C_{1,-1}$ and $C_{-1,1}$. For the u vortex the condition is less restrictive since even in case $C_{1,-1} = C_{-1,1}$ but $C_{-1,1}\neq C_{00}$ we can obtain that $\omega_\pm$ are different.

Now we proceed with the analysis of the quasiparticle spectra for nonsingular vortices. Here we focus on the influence of the additional order parameter components and therefore assume the most simple form of singular part of gap function (12) with $C_{1,-1} = C_{-1,1} = C_{00}$ and consequently $\delta\Delta_{B1}=0$. At first we consider only the influence of A phase and put $C_\beta = 0$. In this case we have

$$(\hat{\Delta}_A)_R = \mathrm{Re}(C_A)q_z q_\perp + \mathrm{Im}(C_A)q_\perp^2\hat{\sigma}_y, \quad (17)$$

$$(\hat{\Delta}_A)_{\mathrm{Im}} = \left[\mathrm{Im}(C_A)q_\perp q_z - \hat{\sigma}_y q_\perp^2\mathrm{Re}(C_A)\right]. \quad (18)$$

The matrix coefficients $\hat{\Delta}_{R,\mathrm{Im}}$ in Andreev equation (3) are diagonalized simultaneously in spinor basis $f_{1,2}\sim(1,\pm i)_\sigma$ so that $\hat{\sigma}_y f_{1,2} = \pm f_{1,2}$. Then Eq.(4) yields the following energy spectrum

$$E(Q,q_z,\chi) = -\omega_j Q + \alpha_j q_\perp q_z + \chi\gamma_j q_\perp^2, \quad (19)$$

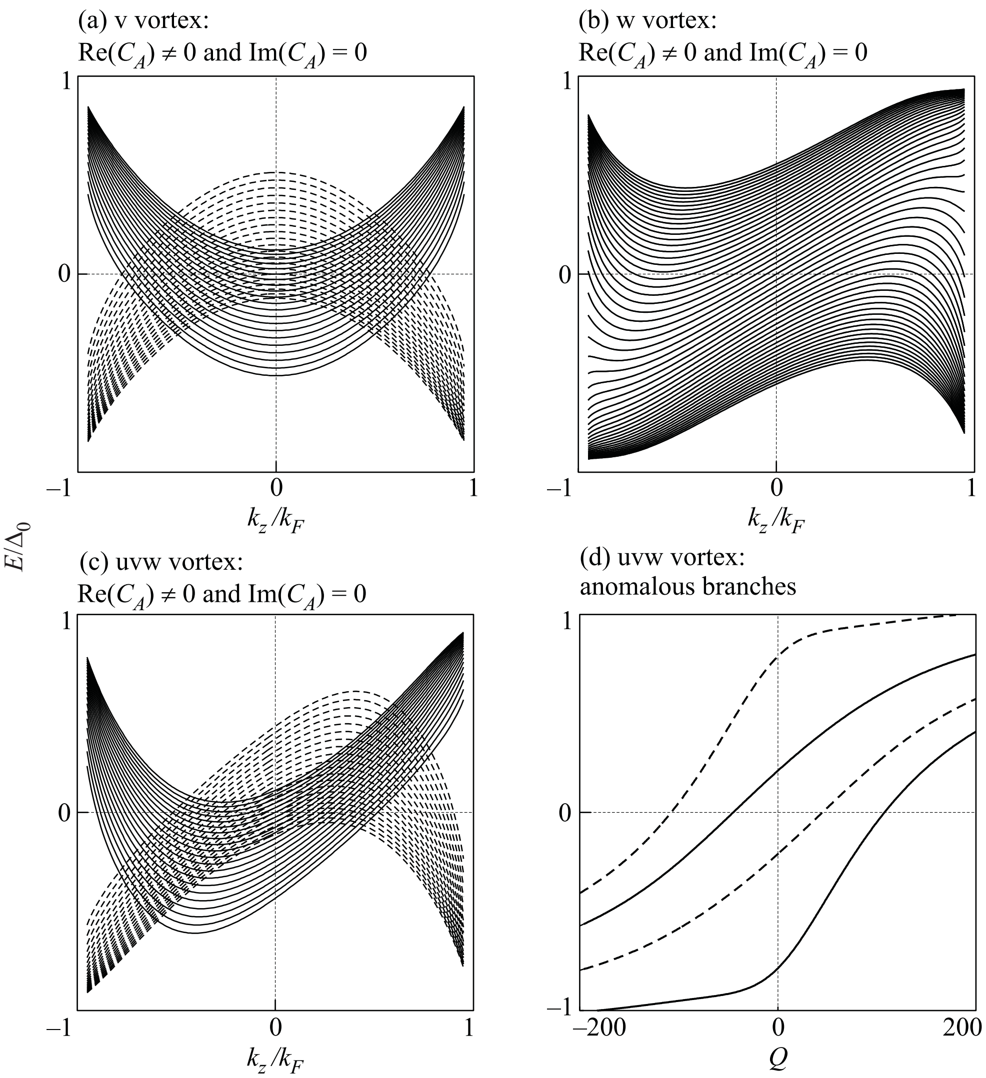
where $j=1,2$ corresponds to $\chi=1,-1$, $\omega_j = 2\langle f_j|C_B/k_\perp r|f_j\rangle$ and $\alpha_j,\gamma_j = 2\mathrm{Re},\mathrm{Im}\langle f_j|C_A|f_j\rangle$. For v vortex with $\mathrm{Im}(C_A)=0$ and w vortex with $\mathrm{Re}(C_A) = 0$ it is easy to check that $(\omega,\alpha,\gamma)_1 = (\omega,\alpha,\gamma)_2$ in Eq.(19) which means following symmetry of v vortex spectrum $E(Q,q_z,\chi) = -E(-Q,q_z,-\chi)$. Thus the spectrum consists of two symmetrical anomalous branches having in particular equal slopes as functions of Q at the Fermi level $E=0$. The spectrum of w vortex is spin divergent since $\gamma_j = 0$ in Eq.(19). The situation is more complicated for the uvw vortex with $\mathrm{Re},\mathrm{Im}(C_A)\neq 0$. In this case $(\omega,\alpha,\gamma)_1\neq(\omega,\alpha,\gamma)_2$ and the two anomalous branches are not symmetrical. Nevertheless there is a relation $(\omega,\alpha,\gamma)_1(q_z) = (\omega,\alpha,\gamma)_2(-q_z)$ which provides spectrum symmetry $E(Q,q_z,\chi) = -E(-Q,-q_z,-\chi)$ corresponding to the CPT invariance of Bogolubov-Nambu hamiltonian.

In Figure (a), (b), (c) we plot several spectral branches for v, w and uvw vortices. We take the model dependencies of the amplitudes of B and A components in the following form:

$$C_B(r) = r/\sqrt{r^2+\xi_v^2} \quad (20)$$

$$C_A(r) = e^{i\kappa}\xi_v/\sqrt{r^2+\xi_v^2}, \quad (21)$$

where κ is an arbitrary phase and ξ_v is a characteristic size so that at $r\gg\xi_v$ the asymptotic behaviour is



(a)–(c): Several spectral branches corresponding to different spin projections $\chi = 1$ (solid lines) and $\chi = -1$ (dash lines). (d): Anomalous branches of *uvw* vortex spectrum. Solid and dash lines correspond to the opposite q_z values

$C_B(r) \to 1$ and $C_A(r) \to 0$. In Figure (d) the two asymmetrical anomalous branches are shown for the *uvw* vortex. Note that the CPT invariance is retained with the help of anomalous branches corresponding to the opposite q_z values which are shown in Figure (d) with solid and dash lines.

In general case when both A and β phases are present the spectrum of v vortex still consists of two symmetrical branches. When $C_{A,\beta}$ are real the spin structure of zero order wave functions is $f_{1,2} \sim (q_z, q_\perp \mp 1)_\sigma$ so that the hermitian part of gap operator is diagonal in this basis:

$$(\hat{\Delta}_A + \hat{\Delta}_\beta)_R = (C_A + C_\beta) q_z q_\perp + \chi q_z C_\beta, \qquad (22)$$

where $\chi = \pm 1$ corresponds to $f_{1,2}$. The anti-hermitian part is

$$i(\hat{\Delta}_A + \hat{\Delta}_\beta)_{\mathrm{Im}} = -i\hat{\sigma}_y \left(q_\perp^2 C_A - q_z^2 C_\beta \right). \qquad (23)$$

The spectrum given by Eq.(4) differs from Eq.(19) in the absence of β phase:

$$E(Q, q_z, \chi) = -\omega_+ Q + \chi \sqrt{\omega_-^2 Q^2 + |S_{12}|^2}, \qquad (24)$$

where $\omega_\pm = (\omega_1 \pm \omega_2)/2$ and $S_{12} = 2\langle f_1 | (\hat{\Delta}_A + \hat{\Delta}_\beta)_{\mathrm{Im}} | f_2 \rangle$. However it is easy to check that spectral branches are even functions of q_z therefore the spectrum consists of two symmetrical anomalous branches as before.

An analogous procedure for the w vortex with $\mathrm{Re}(C_{A,\beta}) = 0$ yields the spectrum consisting of two spin splitted anomalous branches

$$E(Q, q_z, \chi) = -\omega Q + \gamma + \chi |S_{12}|, \qquad (25)$$

where $\omega = \omega_1 = \omega_2$ and $\gamma = 2\langle f_1|(\hat{\Delta}_A + \hat{\Delta}_\beta)_{\rm Im}|f_1\rangle$. In this case $|S_{12}|(q_z)$ is even and $\gamma(q_z)$ is odd function of q_z. Thus the presence of β phase removes the spin degeneracy of the w vortex spectrum. Two spin splitted anomalous branches are not symmetrical, since $E(Q,q_z,\chi) \neq -E(-Q,q_z,-\chi)$ similarly to the case of uvw vortex.

The spectra of bound fermion states of nonsingular vortices given by Eqs.(19), (24), (25) consist of two anomalous branches crossing the Fermi level at some points $Q = Q_{1,2} \neq 0$. Such situation is also realized for the spectrum of doubly quantized vortices in ordinary s-wave superconductor [see Eq.(2)]. An important consequence of this fact is an existence of zero modes, i.e. spectral branches crossing the Fermi level as one can see in Figure. Further we discuss zero modes in more detail.

4. Zero modes. As it was shown in Ref.[11] the number of zero modes N_0 strongly depends on the vortex core size ξ_v. Now with the help of Eq.(19) we will analyze this dependence $N_0(\xi_v)$ for all three types of nonsingular vortices in a model situation when the β phase is absent $C_\beta = 0$. Let us consider the distributions (20), (21) of B and A order parameter components inside vortex core. Such choice of functions $C_{A,B}(r)$ leads to a simplification of the analysis. Indeed in this case we have $\alpha_j/\omega_j = q_\perp(k_F\xi_v)\sin\kappa$ and $\gamma_j/\omega_j = q_\perp(k_F\xi_v)\cos\kappa$. Then the right hand side (r.h.s.) of the following equation for zero modes does not depend on Q:

$$Q/k_F\xi_v = q_\perp^2\,(q_z\sin\kappa + \chi q_\perp\cos\kappa)\,. \qquad (26)$$

It is easy to see that for each $Q \neq 0$ the number of intersections with Fermi level is two or zero. Thus the number of zero modes is $N_0 = 4(Q_{\min} - Q_{\max})$, where $Q_{\min}/k_F\xi_v$ and $Q_{\max}/k_F\xi_v$ are minimum and maximum of the r.h.s. of Eq.(26). The additional factor of 2 is gained from the summation over the two spin states. Then it is obvious that the number of zero modes is $N_0 = \lambda(k_F\xi_v)$, where $\lambda \sim 1$ is a constant coefficient.

5. Summary. To summarize we have studied the spectra of bound fermion states localized within vortex cores for different types of vortices in ^{3}He-B. In contrast to vortices in ordinary s-wave superconductor the spectra of singly quantized vortices in ^{3}He-B in general consist of two anomalous branches corresponding to the different spin structure of quasiparticle states. This results from the removing of spin degeneracy of a standard Caroli-de Gennes – Matricon spectrum. The structure of two anomalous branches is determined by the vortex type.

The spectrum of singular o and u vortices given by Eq.(16) is the most similar to the CdGM one (1). However as distinct from the latter it consists of two anomalous branches with different slopes. As it follows from Ginzburg-Landau calculations the asymmetry between pairing amplitudes $C_{1,-1}$, $C_{-1,1}$ and C_{00} within vortex core is small, therefore the slope difference should also be small: $|\omega_+ - \omega_-| \ll \omega_\pm$.

The spectra of nonsingular v, w and uvw vortices consist of two spin splitted anomalous branches which intersect the Fermi level at finite values of angular momenta $Q_{1,2} \neq 0$. In case of v vortex the spectral branches are even functions of q_z within the same spin subband $E(Q,q_z) = E(Q,-q_z)$ which makes the spectrum analogous to that of the doubly quantized vortex in s-wave superconditor (see Figure (a)).

For w and uvw vortices the spectrum can be a general function of q_z as it is shown in Figure (b), (c). Note that the "skew" of spectral branches of w and uvw vortices is produced by the second term in Eq.(19) which is very similar to the Doppler shift of the energy $\epsilon_d = V_s k_z$ which would appear due to the superflow $V_s = \alpha_j k_F q_\perp^2$ along the vortex axis. However there is no real superflow in the situation that we consider. Still the order parameter symmetry in w and uvw vortices allows the appearance of a spontaneous superflow along the vortex axis [6] and the effective Doppler shift term in the energy spectrum.

In contrast to the v vortex the spectra of w and uvw vortices (25), (19) consist of two asymmetrical anomalous branches (see Figure (d)). In particular it means that the slopes of two anomalous branches dE/dQ can be different at the Fermi level $E = 0$. As distinct from the case of singular o and u vortices the difference in slopes contains no small parameter and therefore can be of significant value.

Since the anomalous branches in spectra of nonsingular vortices intersect the Fermi level at finite angular momenta $Q_{1,2} \neq 0$ there exists a large number of zero modes, i.e. the energy states exactly at the Fermi level. We have calculated the number of zero modes for all three types of nonsingular vortices assuming a model situation when only the additional A phase is present. In a qualitative agreement with the results of work [11] the number of zero modes was shown to be of the order $N_0 \sim k_F\xi_v$, where ξ_v is a size of vortex core.

The significant modification of the spectra of bound fermions as compared to the CdGM case should result in various ramifications of the vortex dynamics which is governed by the kinetics of vortex core quasiparticles [15]. With the help of the analytical results for the spectra obtained in this paper it should be possible to explore the dynamics of nonsingular vortices in ^{3}He-B.

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