Pis'ma v ZhETF, vol. 90, iss. 11, pp. 830 – 833 © 2009 December 10

Remote preparation of N photon GHZ polarization entangled states within a network

Y. Xia[1)], J. Song[+], Y. Ning, P.-M. Lu, H.-S. Song[+]

Department of Physics, Fuzhou University, 350002 Fuzhou, China

[+]*School of Physics and Optoelectronic Technology, Dalian University of Technology, 116024 Dalian, China*

Submitted 14 October 2009

We propose a new linear optical protocol for remote state preparation (RSP) between two parties under control of a number of controllers in terms of optical elements. The proposed setup involves simple linear optical elements, a N-photon polarization entangled state, and photon detectors, witch have been widely used in experiment. The realization of this protocol is appealing due to the fact that quantum state of light is robust against the decoherence and photons are ideal carriers for transmitting quantum information over long distances.

PACS: 03.65.Ud, 03.67.Hk

A principal goal of quantum information theory is understanding the resources necessary and sufficient for intact transmission of quantum states. Using the theory of quantum mechanics in the field of information in the recent years has produced many interesting developments, for example, quantum teleportation, proposed by Bennett et al. [1], provides a scheme whereby the unknown state of a qubit can be completely transferred from one place to another through a previously entangled two-bit channel. This process is a critical foundation for quantum communication and large scale quantum computers. Different from quantum teleportation, recently, Lo [2], Pati [3] and Bennett et al. [4] have presented an interesting new method to transmit pure known quantum state using a prior shared entanglement and some classical communication when the sender knows completely the transmitted state. This communication protocol is called remote state preparation (RSP). The main difference between RSP and teleportation are in that, (1) in RSP, the sender Alice knows the state that she wants the receiver Bob to prepare, in particular, Alice need not own the state, but only know the information about the state, while in teleportation Alice must own the teleportted state, but she need not know the state; (2) in RSP, the required resource can be traded off between classical communication cost and entanglement cost while in quantum teleportation, two bits of forward classical communication and one ebit of entanglement (an EPR pair) per teleportted qubit are both necessary and sufficient, and neither resource can be traded off against the other [4, 5].

RSP has attracted much attention [2 – 12] in theories and experiments using many kinds of methods in recent years. A variety of theoretical suggestions and experimental efforts have been made in this realm. For example, Pati [3] has showed that for some special ensemble of qubits chosen from the equatorial or polar great circles on the Bloch sphere, RSP requires only one classical bit from Alice to Bob, unlike the case in standard teleportation where two classical bits are needed. Moreover, Lo [2] and Bennett et al. [4] have investigated the classical information cost in the protocol for RSP of general states and considered the trade-off between the required entanglement and classical communication. They showed that in the presence of a large amount of previously shared entanglement, the asymptotic classical communicatiton cost of RSP for general states is only one bit per qubit. In addition, some authors have also investigated the RSP protocols via using different quantum channels such as partial EPR pairs [7] and three-particle Greenberger-Horne-Zeilinger (GHZ) entangled state [8]. Zeng et al. [9] have proposed a protocol for RSP in higher dimension and the parallelizable manifold S^{n-1}. Zhang et al. [11] have proposed a RSP protocol using Control-NOT (CNOT) gate. Compared to the previous protocols, both classical communication cost and required quantum entanglement in this protocol are remarkably reduced. Very recently, Liu et al. [12] demonstrate an experiment for remote preparation of arbitrary photon polarization states. In this protocol, with local operations, polarization measurement, and one way classical communication, any states lying on and inside the Poincaré sphere can be remotely prepared.

[1)]e-mail: xia-208@163.com

In this paper, motivated by the protocol [12] of Liu et al., we propose a protocol to remote preparation of N photons polarization GHZ entangled state within a network consisting of an arbitrary $N \geq 3$ remote parties named P_1, P_2, $\ldots$, P_N: a known photon polarization GHZ entangled state needs to be remote preparation between any two parties under control of all the remaining parties. The realization of this protocol is appealing due to the fact that quantum state of light is robust against the decoherence and photons are ideal carriers for transmitting quantum information over long distances. For convenience, in the revised version we will quote some descriptions and notations in the original Liu et al.'s protocol [12].

Without loss of generality suppose that P_1, P_N, and P_3, P_4,...,P_{N-1}, are the sender, receiver and controllers, respectively. Before performing the network RSP, the parties should share an N-photon entangled polarization state of the form

$$|\Phi\rangle_{ABC} = \frac{1}{\sqrt{2}}(|H\rangle^{\otimes N} + |V\rangle^{\otimes N})_{12\ldots N}, \qquad (1)$$

where photons 1, 2, $\ldots$, N are in possession of P_1, P_2, $\ldots$, P_N, respectively.

Let us assume the sender P_1 wishes to help the receiver P_N remotely prepare polarization the state

$$|\psi^T\rangle = \alpha|H\rangle^{\otimes N} + \beta e^{i\phi}|V\rangle^{\otimes N}, \qquad (2)$$

with α, β and ϕ are real and $|\alpha|^2 + |\beta|^2 = 1$. It is reasonable to assume that $\alpha, \beta \geq 0$ with $\phi \in [0, 2\pi)$. $|H\rangle$ denotes the horizontal polarization state and $|V\rangle$ denotes the vertical polarization state.

Now we turn to an experimental setup for the network RSP. The kinds of operations to be performed by the sender P_1 and a controller P_n ($n = 2, 3, \ldots, N - 1$) are different. As for the controllers P_n ($n = 2, 3, \ldots, N - 1$), each of them proceeds as follows: In order to help the sender P_1 to remotely prepare the quantum state of Eq.(2), the controller (P_2) first sends his/her photon 2 through a half-wave plates (HWP). After HWP, the state of the channel in Eq.(1) changes into

$$|\Phi\rangle_{ABC} = \frac{1}{2}\{|H\rangle_2(|H\rangle^{\otimes N-1} + |V\rangle^{\otimes N-1})_{134\ldots N} + |V\rangle_2(|H\rangle^{\otimes N-1} - |V\rangle^{\otimes N-1})_{134\ldots N}\}. \qquad (3)$$

Then P_2 sends the photon 2 pass through a PBS, which transmits horizontal polarization and reflects vertical ones. At the outputs of PBS we obtain

$$|\Phi\rangle_{ABC} = \frac{1}{2}\{|H\rangle_2^{a'}(|H\rangle^{\otimes N-1} + |V\rangle^{\otimes N-1})_{134\ldots N} + |V\rangle_2^{b'}(|H\rangle^{\otimes N-1} - |V\rangle^{\otimes N-1})_{134\ldots N}\}, \qquad (4)$$

where a' and b' are output paths of PBS, as are shown in Fig.1. In order to realize the network RSP, P_2 performs

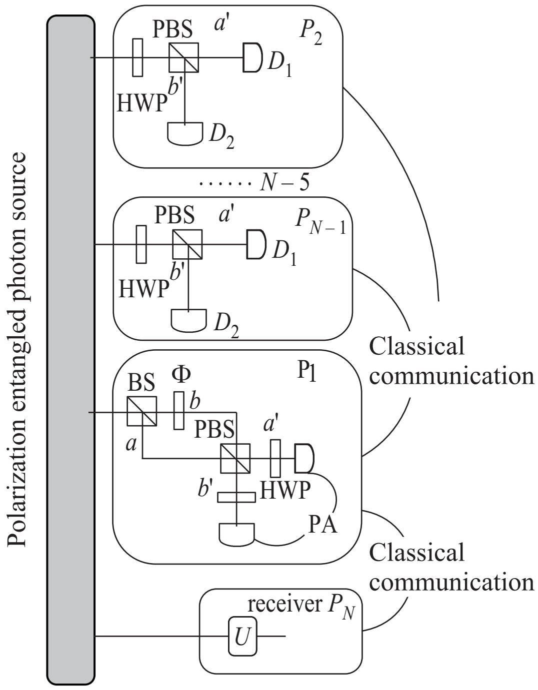

Fig.1. Schematic diagram for RSP of N photon polarization GHZ entangled states within a network. Beam splitter's (BS) reflection and transmission coefficients are α and β, respectively. Polarization analyzer (PA) are single photon detectors. HWP denotes the Half-wave plates, PBS denotes the polarization beam spliter, and $D_{1(2)}$ are conventional photon detectors

a measurement with the conventional photon detectors $D_{1(2)}$ (see Fig.1). The conventional photon detector can only distinguish between the presence and absence of photons, and no information on the exact number of photons can be obtained. P_2 will inform the sender P_1 and the receiver P_N of his/her measurement result V via a classical communication. If his/her measurement result is D_1 register a photon, $C = 0$; if the measurement result is D_2 register a photon, $C = 1$. The communication costs is 1 cbit since there are two possible results. After that, All the other controller repeat this process for the photons $3, 4, \ldots, N-1$, respectively, and inform the sender P_1 and the receiver P_N of their measurement results C_n ($n = 3, 4, \ldots, N - 1$) via classical communication. The total communication costs are $N - 2$ cbits.

P_1 carries out the following calculation:

$$C_{23\ldots N-1} = C_2 \oplus C_3 \oplus \cdots \oplus C_{N-1}, \qquad (5)$$

where the $\oplus$ denotes an addition mod 2. If $C_{23\ldots N-1} = 0$, the channel Eq.(1) will be transformed into

$$|\Phi\rangle_{1N} = |HH\rangle_{1N} + |VV\rangle_{1N}; \qquad (6)$$

if $C_{23\ldots N-1} = 1$, the channel Eq.(1) will be will be transformed into

$$|\Phi\rangle_{1N} = |HH\rangle_{1N} - |VV\rangle_{1N}, \tag{7}$$

which can be transformed into Eq.(6) by applying a $\pi/2$-phase shift P to change the sign of the polarization state $|V\rangle$. Then, after the operations of all controllers P_2, P_3, $\cdots$, P_{N-1}, the channel between P_1 and P_N is that

$$|\Phi\rangle_{1N} = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle)_{1N}. \tag{8}$$

In the following, we will show P_1 how to help P_N remotely prepare the quantum state of Eq.(2). In order to help P_N to remotely prepare the quantum state of Eq.(2), P_1 put a polarization independent beam splitter (BS) on the line (see Fig.1). The BS's reflection and transmission coefficients are α and β, respectively. After photon 1 passing through the BS, the state of the channel Eq.(8) changes into

$$|\Phi\rangle_{1N} = \frac{1}{\sqrt{2}}(\alpha|H_a H\rangle_{1N} + \beta|H_b H\rangle_{1N} + \\ + \alpha|V_a V\rangle_{1N} + \beta|V_b V\rangle_{1N}). \tag{9}$$

The subscripts a, b represent the possible paths of photon 1. P_1 puts a phase modulator one line b to produce a relative phase shift of ϕ, and lets the both paths (a and b) cross for interference on another PBS. After passing through the PBS we obtain

$$|\Phi\rangle_{1N} = \frac{1}{\sqrt{2}}(\alpha|H_{a'} H\rangle + \\ + \beta e^{i\phi}|H_{b'} H\rangle + \alpha|V_{b'} V\rangle + \beta e^{i\phi}|V_{a'} V\rangle)_{1N}, \tag{10}$$

where a' and b' are output paths of PBS, as is shown in Fig.1. The following half-wave plates (HWP), whose action is given by transformation $|H\rangle \to \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$ and $|V\rangle \to \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$, is set to change the polarization state into

$$|\Phi\rangle_{1N} = \frac{1}{2}\{|H\rangle_{b'1}(\alpha|V\rangle + \beta e^{i\phi}|H\rangle)_N - \\ - |V\rangle_{b'1}(\alpha|V\rangle - \beta e^{i\phi}|H\rangle)_N + \\ + |H\rangle_{a'1}(\alpha|H\rangle + \beta e^{i\phi}|V\rangle)_N + \\ + |V\rangle_{a'1}(\alpha|H\rangle - \beta e^{i\phi}|V\rangle)_N\}. \tag{11}$$

P_1 performs a measurement on photon 1 with the polarization analyser (PA). Then he/she informs P_N of his/her measurement results through a classical information (2 bits). According to P_1's classical communication, P_N take corresponding operations on his/her photon N, see Table. Through adjusting the parameters α, β, and ϕ, arbitrary states on the Poincaré sphere can be

Corresponding relations among P_1's measurement results and P_2's operations.

P_1's measurement results	P_2's operations
$\lvert H\rangle_{b'}$	I
$\lvert V\rangle_{b'}$	σ_z
$\lvert H\rangle_{a'}$	σ_x
$\lvert V\rangle_{a'}$	$i\sigma_y$

obtained. After above operations, the state of photon N become

$$|\psi\rangle_N = \alpha|H\rangle_N + \beta e^{i\phi}|V\rangle_N. \tag{12}$$

P_N introduces $N-1$ ancillary photons $b_1, b_2, \cdots, b_{N-1}$ in the initial state $|HH\cdots H\rangle_{b_1 b_2 \cdots b_{N-1}}$ and let photons $(b_1, b_2, \cdots, b_{N-1})$ pass through a half-wave plate (HWP), respectively (see Fig.2). The HWP's action is

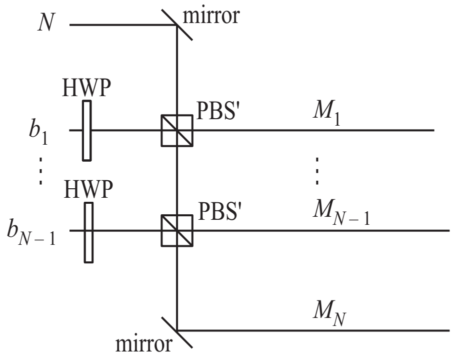

Fig.2. Schematic demonstration for reconstructing the N-photon polarization GHZ entangled state

given by transformation $|H\rangle \to (1/\sqrt{2})(|H\rangle + |V\rangle)$ and $|V\rangle \to (1/\sqrt{2})(|H\rangle - |V\rangle)$. After the HWP,

$$|\Phi\rangle_{b_1\cdots b_{N-1}} = |HH\cdots H\rangle_{b_1\cdots b_{N-1}} \to \\ \to (1/\sqrt{2})^{N-1}(|H\rangle + |V\rangle)_{b_1} \cdots (|H\rangle + |V\rangle)_{b_{N-1}}. \tag{13}$$

Then, photon (N and $b_1, b_2, \cdots, b_{N-1}$) will meet PBS$'$ (see Fig.2), which always transmits $|H\rangle$ polarizing photons and reflects $|V\rangle$ polarizing photons (all PBS in the paper work in this way). Thus at the outputs of PBS$'$, the joint state will become

$$|\Phi\rangle_{Nb_1\cdots b_{N-1}} = (1/\sqrt{2})^{N-1}(\alpha|H\rangle + \beta e^{i\phi}|V\rangle)_N \times \\ \times (|H\rangle + |V\rangle)_{b_1} \cdots (|H\rangle + |V\rangle)_{b_{N-1}} \xrightarrow{(N-1)PBS'} \\ \xrightarrow{(N-1)PBS'} (1/\sqrt{2})^{N-1}[(\alpha|H\rangle_{M_2}|H\rangle_{M_1} + \\ + \alpha|H\rangle_{M_2}|V\rangle_{M_2} + \beta e^{i\phi}|V\rangle_{M_1}|H\rangle_{M_1} + \\ + \beta e^{i\phi}|V\rangle_{M_1}|V\rangle_{M_2})_{Nb_1} \otimes \cdots \\ \cdots \otimes (|H\rangle_{M_{N-1}} + |V\rangle_{M_N})b_{N-1}]. \tag{14}$$

We consider only those terms that contain one photon in each of the modes $M_1, M_2, \ldots, M_N$, hence we have discarded the bunching outcomes in Eq.(14). Then Eq.(14) changes into

$$|\Phi\rangle'_{Nb_1\cdots b_{N-1}} = \\ = (\frac{1}{\sqrt{2}})^{N-1}[\alpha|H\rangle_{M_1}|H\rangle_{M_2}\otimes\cdots\otimes|H\rangle_{M_N} + \\ + \beta e^{i\phi}|V\rangle_{M_1}|V\rangle_{M_2}\otimes\cdots\otimes|V\rangle_{M_N}]_{Nb_1\cdots b_{N-1}}, \quad (15)$$

with the probability $P = (50\%)^{N-1}$. This is the state which P_1 wants to help P_N to prepare remotely. The RSP has been successfully completed.

Till now, we have proposed a protocol to remotely prepare a N photon polarization GHZ entangled state within a network using linear optical elements. To realize the RSP protocol, we need the help of the controllers P_n ($n \in (2,3,\cdots,N-1)$). Without the help of the controllers, the sender P_1 and the receiver P_N can not share the two-photon quantum channel, so the RSP protocol failed. That is, if P_1 and P_N want to realize the RSP protocol, P_n take the responsibility to decide whether or not and when the task should be done. For some reasons, one of the controllers P_n does not agree on the task, P_1 and P_N are unable to start it. That is, P_1 and P_N could go only when all the controllers agree. In this point of view, the RSP protocol is safer than its ordinary counterpart. We now give a brief discussion on the experimental feasibility of protocol with the current experimental technology. 1) In this protocol, what we used consists of linear optical elements [13], which have been widely used to entangled photons [14]. In particular, the similar optical setups have been used to successfully prepare W (GHZ) states of photons in experiment [15] ([16]). 2) The protocol presented here requires the conventional photon detectors and the polarization analyser. For conventional photon detectors, it can only to distinguish the vacuum and nonvacuum Fock number states, and no information on the exact number of photons can be obtained. Experimental techniques for single-photon detection have made tremendous progress [17]. A photon detector based on a visible light photon counter has been reported, which can distinguish between single-photon incidence and two-photon incidence with high quantum efficiency, good time resolution, and low bit-error rate [18]. For polarization analyser, it has been used to realize RSP of photons in experiment [19]. Therefore, our protocol might be realized in near future. But, some technical difficulties with the present protocol should be pointed out. (1) To realize the RSP protocol, N-party should share an N-photon GHZ polarization-entangled state Eq.(1), but, in practice, it is difficulty to realize N-party share an N-photon GHZ polarization-entangled state. This problem needs further investigation. (2) In terms of practical applicability, our demonstration of the protocol still has some limitations. For example, different attenuations are introduced due to the lack of variable beam splitter, which reduce the probability of success in practice.

In summary, we have presented a protocol for remote state preparation within a network using linear optical elements such as beam splitters, phase shifters, and photo detectors. The realization of this protocol is appealing due to the fact that quantum state of light is robust against the decoherence and photons are ideal carriers for transmitting quantum information over long distances. This protocol is an improvement of Liu et al.'s protocol [12]. However, there are some advantages in this protocol. For example, (1) the RSP protocol could realized only when all controller agree. In this point of view, the present RSP protocol is safer than its ordinary counterpart. (2) The state to be remote prepared is generalized from single-photon to N-photon. We believe that with the existing technology it may be possible to implement the RSP protocol with ease.

1. C.H. Bennett, G. Brassard, C. Crepeau et al., Phys. Rev. Lett. **70**, 1895 (1993).
2. H.K. Lo, Phys. Rev. A **62**, 012313 (2000).
3. A.K. Pati, Phys. Rev. A **63**, 014302 (2001).
4. C.H. Bennett, D.P. DiVincenzo, P.W. Shor et al., Phys. Rev. Lett. **87**, 077902 (2001).
5. M.Y. Ye, Y.S. Zhang, and G.C. Guo, Phys. Rev. A **69**, 022310 (2004).
6. C.S. Yu, H.S. Song, and Y.H. Wang, Phys. Rev. A **73**, 022340 (2006).
7. B.S. Shi, J. Opt. B **4**, 380 (2002).
8. J.M. Liu and Y.Z. Wang, Phys. Lett. A **316**, 159 (2003).
9. B. Zeng and P. Zhang, Phys. Rev. A **65**, 022316 (2002).
10. X.H. Peng, X.W. Zhu, X.M. Fang et al., Phys. Lett. A **306**, 271 (2003).
11. Z.J. Zhang, Y.M. Liu, and D. Wang, quant-ph/07082536v1.
12. W.T. Liu, W. Wu, B.Q. Ou et al., Phys. Rev. A **76**, 022308 (2007).
13. Y. Xia, J. Song, and H.S. Song, Opt. Commun. **281**, 4946 (2008); Q.B. Fan, Chin. Phys. Lett. **25** (2008) 20; X.H. Li, F.G. Deng, and H.Y. Zhou, Appl. Phys. Lett. **91**, 144101 (2007).
14. Y. Xia, J. Song, and H.S. Song, Appl. Phys. Lett. **92**, 021128 (2008).
15. M. Eibl, N. Kiesel, M. Bourennane et al., Phys. Rev. Lett. **92**, 077901 (2004).
16. J.W. Pan, D. Bouwmeester, M. Daniell et al., Nature (London) **403**, 515 (2000).
17. D.F.V. James and P.G. Kwiat, Phys. Rev. Lett. **89**, 183601(1-4) (2002).
18. J. Kim, S. Takeuchi, and Y. Yamamoto, Appl. Phys. Lett. **74**, 902 (1999).
19. G.L. Long and Y. Sun, Phys. Rev. A **64**, 014303(1-4) (2001).