Pis'ma v ZhETF, vol. 90, iss. 12, pp. 866 – 868

Roton dipole moment

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Submitted 9 November 2009
Resubmitted 13 November 2009

The roton excitation in the superfluid ^{4}He does not possess a stationary dipole moment. However, a roton has an instantaneous dipole moment, such that at any given moment one can find it in the state either with positive or with negative dipole moment projection on its momentum direction. The instantaneous value of electric dipole moment of roton excitation is evaluated. The result is in reasonable agreement with recent experimental observation of the splitting of microwave resonance absorption line at roton frequency under external electric field.

PACS: 67.10.–j, 67.25.–k

The interaction of electromagnetic radiation with superfluid ^{4}He was the subject of recent intensive investigations undertaken by Kharkov experimental group. Among several new effects there was observed the resonance absorption of microwaves at the frequency corresponding to the roton gap $\Delta(T)$ of the phonon-roton excitation spectrum [1]. As well the inverse effect of generation of electromagnetic radiation with frequency of the roton gap by heat flow in superfluid helium has been detected [2, 3]. The photon momentum $\hbar\mathbf{k}_{ph}$ is many orders less than the roton momentum $\hbar\mathbf{k}_r$, hence, the creation of a single roton by means of single photon absorption is prohibited due to momentum conservation law. This problem can be, however, passed round noting that roton momentum is compensated by momentum of liquid flow [4] $\mathbf{P}_l$ arising in some macroscopic volume V, which is much larger than the volume per atom in liquid helium

$$\mathbf{P}_l = Vmn\mathbf{v} = \hbar\mathbf{k}_{ph} - \hbar\mathbf{k}_r \approx -\hbar\mathbf{k}_r. \qquad (1)$$

Here m is the mass of ^{4}He atom and n is the fluid density The process can happen practically without a change of flow energy

$$E_l = \hbar c k_{ph} - \Delta = Vn\frac{mv^2}{2} = \frac{P_l^2}{2Vnm} \approx \frac{\hbar^2 k_r^2}{2Vnm} \ll \Delta. \qquad (2)$$

An example of such a kind transition between the two states of superflow with the same energy but different momenta was found by Volovik [5] who considered quantum mechanical formation of vortices from the homogeneosly moving superfluid. This case, however, the momentum of the liquid is not conserved because of violation of translational symmetry by inhomogeneity – a hemisphere on the wall of container with liquid helium. The same is probably happens in the experiments under discussion: the inhomogeneities on the walls of the container are responsible for the non-conservation of momentum. They provide the necessary matrix element for the transition between the states with different momentum of liquid [6].

The resonance microwave absorption in liquid ^{4}He at the frequency corresponding to the roton minimum can be interpreted as the evidence of an electric dipole moment of roton excitations. Indeed, the following investigations have demonstrated that the resonance absorption line at roton frequency splits on two lines by the constant electric field [7]. The splitting at small enough fields depends linearly on the field value that corresponds to the roton dipole moment $d \approx 10^{-22}$ CGSE units.

The existence of the roton dipole moment is unnatural from the symmetry point of view. A roton is collective excitation that is described by a compact in space wave function with typical size about few interatomic distances [8]. A roton possess the definite momentum characterized by its modulus and direction. This causes the local space parity violation inside the region occupied by the roton wave pocket. On the other hand the state with definite momentum is characterized by the time reversal breaking such that only the product space and time inversion PT is conserved quantity. On the contrary the polar vector of the dipole moment changes its sign under space inversion and it is not changed under the time reversal. So, the roton and the dipole obey the different symmetry. Hence, the roton cannot possess the stationary dipole moment. We shall demonstrate, however, that the roton dipole moment can be treated as a sort of nonstationary phenomenon.

According to the Feynman [8] conjecture each roton is described by many particle wave function corresponding to dipole distribution of velocity field of ^{4}He atoms.

Roughly speaking, roton is similar to "a vortex ring of such small radius that only one atom can pass through the center.[9] Outside the ring there is a slow drift of atoms returning for another passage through the ring." The roton momentum is approximately equal to the inverse interatomic distance $\hbar k_r \approx \hbar/a$. An atom passing through the ring center, first acquires this momentum, then it slows down its motion to go around and come back to the initial point where it is accelerated again. The corresponding force which is necessary to get and then to lose such a momentum is

$$\mathbf{f}(t) \approx \frac{\hbar^2 g(t)}{ma^3}\,\hat{\mathbf{k}}_r, \qquad (3)$$

where $g(t) = \sum_{n\geq 1} c_n \sin n\omega t$ is a periodic function of time with period $\tau = 2\pi/\omega \approx ma^2/\hbar$, and $\hat{\mathbf{k}}_r$ is the roton momentum direction. This force acting on given atom from the side of other atoms pushes it through the ring center.

The helium atom in the ground state does not have an electric dipole moment. The uneven motion of helium atom under the action of force given by eqn.(3) causes the deformation of the atom electronic shell. To estimate the dipole moment caused by this deformation let us write the Hamiltonian of two electrons in He atom as

$$\hat{H} = \hat{H}_0 + \mathbf{F}(\mathbf{r}_1 + \mathbf{r}_2), \qquad \mathbf{F} = \mathbf{F}(t) = e\mathbf{E} + \mathbf{f}(t), \quad (4)$$

where the first term $\hat{H}_0$ presents the unpertubed electron Hamiltonian of helium atom and the second term is a potential of perturbation determined by the external electric field and by the force $\mathbf{f}(t)$ introduced above. The vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ gives positions of two electrons relative to the nuclei position. The Hamiltonian properly describes the electronic state of helium atom in usual approximation when the electron mass is much smaller than the mass of atom $m_e \ll m$. The dipole moment is given by the average of $e(\mathbf{r}_1 + \mathbf{r}_2)$ over the ground state wave function. The motion of the atom nuclei is described by separate equation and has no influence on the dipole moment value.

The frequency of perturbation ω is much smaller than the distance between the energies of the ground and the first excited state of He atom: $\hbar\omega \ll (E_1 - E_0)$. Hence, one can prove (see eg [10]) that electrons in this atom are in the quasi stationary state characterized by the wave function

$$\Psi_0'(t) \approx \left[\Psi_0 - \frac{\langle\Psi_0|\mathbf{F}(\mathbf{r}_1 + \mathbf{r}_2)|\Psi_1\rangle}{E_1 - E_0}\Psi_1\right] e^{-\frac{iE_0 t}{\hbar}}. \quad (5)$$

Here, Ψ_0, Ψ_1 are the wave functions of the ground and the first excited states of He atom correspondingly. We neglect here admixture of the higher excited states.

The correction to the electron energy is

$$E_0' - E_0 \approx -\frac{r_{at}^2 F^2}{E_1 - E_0}. \qquad (6)$$

Here, r_{at} is the size of electron wave function of helium atom (hard core radius in the potential of interaction between two He atoms).

The linear in respect of electric field $\mathbf{E}$ term in (6) gives roton dipole moment

$$\mathbf{d} \approx \frac{2er_{at}^2}{E_1 - E_0}\,\mathbf{f}. \qquad (7)$$

This value obeys the proper symmetry properties: being odd in respect of space inversion it is even in respect of time reversal. Its projection on momentum direction is

$$d \approx \frac{2e\hbar^2 r_{at}^2}{ma^3(E_1 - E_0)} g(t). \qquad (8)$$

Hence, as it was expected, the time average of the roton dipole moment is equal to zero. However, a roton possess an instantaneous dipole moment, such that at any given moment one can find it in the state either with positive or with negative projection of dipole moment on its momentum direction. To get the correspondence with the experimental observations one should assume that the time of transition between these two states is much shorter than period τ but still much longer than $\hbar/(E_1 - E_0)$. The latter condition provides the validity of quasistationary approximation has been used.

Substituting the numerical values and taking into account that $r_{at} \approx a$ is about few Angstroms and $(E_1 - E_0) \approx 20\,\text{eV}$ we obtain $d \approx \pm 10^{-22}$ CGSE units. This corresponds to the experimentally determined value of roton dipole moment. The roton dipole moment is temperature independent but increases with pressure.

The author is indebted to A. Rybalko and E. Rudavskii for the kind introduction to the vast field of their experimental results. I am also grateful to L. Melnikovsky and G. Volovik for the useful and stimulating discussions.

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