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Coulomb zero bias anomaly for fractal geometry and conductivity of granular systems near the percolation threshold

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A granular system slightly below the percolation threshold is a collection of finite metallic clusters, characterized by wide spectrum of sizes, resistances, and charging energies. Electrons hop from cluster to clusters via short insulating "links" of high resistance. At low temperatures all clusters are Coulomb blockaded and the dc-conductivity σ is exponentially suppressed. At lowest T the leading transport mechanism is variable range cotunneling via largest (critical) clusters, leading to the modified Efros-Shklovsky law. At intermediate temperatures the principal suppression of σ originates from the Coulomb zero bias anomaly occurring, when electron tunnels between adjacent large clusters with large resistances. Such clusters are essentially extended objects and their internal dynamics should be taken into account. In this regime the T-dependence of σ is stretched exponential with a nontrivial index, expressed through the indices of percolation theory. Due to the fractal structure of large clusters the anomaly is strongly enhanced: it arises not only in low dimensions, but also in $d = 3$ case.

Granular materials play important role in modern technology and material science (see [1]). In the recent years nanocomposite granular materials were invented, with characteristic grain size a on the scale of 1–10 nm [2]. For systems with so small grains quantum effects should be essential, in particular the Coulomb blockade effect [3]. Therefore it is important to understand, how the Coulomb blockade is manifested in realistic disordered granular metal.

There are two large families of granular metallic materials. In the systems of the first family conducting grains are randomly embedded in an insulating matrix (Fig.1a), while the systems of the second family are mixtures of conducting and insulating grains (Fig.1b). The percolation [4, 5] is a general geometric phenomenon, generic for systems of both families. Some conducting grains may touch each other [6], establishing a good contact (with dimensionless conductance G), while the conductances g_{ij} between grains i and j which do not touch each other, are much smaller: $g_{ij} \ll G$. If there is a percolation via a network of touching each other conducting grains, then electrons can travel throughout the system hopping from grain to grain only via good contacts G. Otherwise hopping through some bad contacts g is unavoidable.

If all conductances are small (both $G \ll 1$ and $g_{ij} \ll \ll 1$), then the Coulomb blockade effect exists at each metallic grain of the system. It is characterized by charging energy on the scale $E_C^{(0)} \sim e^2/a$. The mechanism of the electronic transport at $T \ll E_C^{(0)}$ in this case is

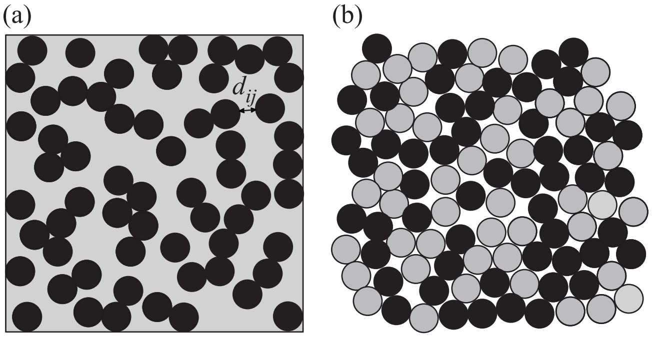

Fig.1. Clusters of conducting grains in granular materials. (a) – Metal grains (shown black) in an insulating matrix (shown grey). Randomly distributed distances d_{ij} between two non-touching conducting grains determine corresponding tunnel conductances g_{ij}. (b) – A mixture of conducting and insulating grains. The "links" (one-grain insulating bridges) between two conducting clusters are supposed to have the same conductances $g_{ij} = g$, while all longer bridges have $g_{ij} \ll g$ and can be neglected

either direct intergrain hopping, described by the Arrhenius law

$$\sigma \propto \exp\{-E_{\rm act}/T\}, \quad T_{\rm ES} \ll T \ll T_{\rm Arr}, \qquad (1)$$

or *variable range cotunneling* (VRC)[7, 8, 9], described by the modified Efros-Shklovskii law [10]

$$\sigma \propto \exp\left\{-\left(E_{\rm ES}/T\right)^{1/2}\right\}, \quad T \ll T_{\rm ES}, \qquad (2)$$

where

$$T_{\rm Arr} \sim E_C^{(0)}, \quad T_{\rm ES} \sim E_C^{(0)}/\mathcal{L}^*, \tag{3}$$

$$E_{\rm act} \sim E_C^{(0)}, \quad E_{\rm ES} \sim \mathcal{L}(T) E_C^{(0)}, \tag{4}$$

$$\mathcal{L}^* \sim \ln(1/\overline{g}), \quad \mathcal{L}(T) \approx \mathcal{L}^* + \ln\left[T_{\rm ES}^2/(T^2+T_{el}^2)\right]. \tag{5}$$

Here $\overline{g}$ is a properly averaged intergrain conductance. The temperature $T_{el} \sim [E_C^{(0)}\delta]^{1/2} \ll T_{\rm ES}$ (δ being the typical level spacing in a grain) corresponds to the crossover from the inelastic cotunneling (at $T > T_{el}$) to the elastic cotunneling (at $T < T_{el}$). The details can be found in [8]. Thus, $\mathcal{L}$ is a moderately large logarithmic factor and the Arrhenius law can only be observed in a restricted intermediate temperature range. In the low-G case the presence or absence of the percolation in the system is only relevant for the value of $\overline{g}$, appearing in $\mathcal{L}^*$ as an argument of the log-function, and, therefore, it is only of a secondary importance.

In this paper we will study the large-G case: $g_{ij} \ll 1$, but $G \gg 1$. Here the mechanism of transport is very sensitive to the percolation transition. The resistance between two contacting metal grains is so low that the charge easily spreads over *clusters* of connected metal grains (see Fig.1), and it is clusters – not the individual grains – that may either be Coulomb blockaded, or not blockaded. If the fraction of the metal in the system x is larger than the percolation threshold x_c, then the Coulomb blockade of finite clusters is not relevant, since the infinite cluster of conducting grains exists (see [4, 5]), the current goes through this infinite cluster, and electrons do not have to visit finite clusters whatsoever. It is not the case below the percolation threshold, for $x < x_c$, where the infinite cluster does not exist, and electrons have to hop from one conducting cluster to another due to tunneling through high-resistance insulating bridges between them.

Close to the percolation threshold the distribution $N(n)$ of numbers of grains n in a cluster has a long power-law tail: $N(n) \sim n^{-\tau}$ at $1 \ll n \ll n_{\rm cr}$. This tail is cut off only at $n \sim n_{\rm cr}$, where

$$n_{\rm cr} \sim \xi^{d_f} \sim (x_c - x)^{-\nu d_f} \gg 1 \tag{6}$$

is the number of grains in a critical cluster, $\xi \sim (x_c - x)^{-\nu}$ is its radius (measured in the units of a), d_f is the fractal dimension of the infinite percolation cluster (and of any large finite cluster with n in the range $1 \ll n \lesssim n_{\rm cr}$ as well). The values of relevant critical exponents are given in the table. An estimate for typical charging energy for a cluster consisting of n grains was found in [11]:

$$E_C(n) \sim E_C^{(0)} n^{-(\nu+s)/\nu d_f} \tag{7}$$

We will see in what follows that the critical clusters with low charging energy

$$E_C^{(\rm cr)} \sim E_C(n_{\rm cr}) \sim E_C^{(0)}(x_c - x)^{\nu+s} \ll E_C^{(0)} \tag{8}$$

play the key role in low temperature transport at $x < x_c$.

Numerical values of some critical exponents

d	ν	d_f	τ	μ	s	Θ	$\tilde{\Theta}$	φ
$d=2$	4/3	91/48	187/91	1.30	1.30	0.14	0.11	0.33
$d=3$	0.875	2.524	2.32	2.14	0.74	0.38	0.08	0.41

It is natural to expect that the universal character of the percolation transition, generic for most granular systems, should lead to the essential universality of the conductivity mechanism for $x_c - x \ll 1$. One should, however, have in mind that besides the topological disorder (that is responsible for the percolation phenomena), there is yet another disorder: the randomness of "bad" conductances g_{ij}. Indeed, $g_{ij} \propto \exp(-2\kappa d_{ij})$ where d_{ij} is the separation between the two grains and κ is the tunneling decrement of the electronic wave function in the insulator. If typical value of $\kappa d_{ij} \gg 1$, then the dispersion of g is exponentially wide.

In mixtures the distribution of g is approximately discrete: the thickness d_{ij} of the insulating interval between two metal clusters is, roughly, measured in the units of the diameter of the insulating grain d. Then the tunneling conductances of shortest one-grain insulating bridges (links) are $g_1 \propto \exp(-2\kappa d)$; the conductances of two-grain bridges are $g_2 \propto \exp(-4\kappa d) \ll g_1$, and so on. For such a system one can simply ignore all long insulating bridges and take into account only the shortest – the links, ascribing the same conductance $g \equiv g_1$ to all of them. The conduction process in the resulting system resembles the next nearest neighbor (NNN) percolation (see [11] for detailed discussion of this process).

For metal grains, embedded in an insulating continuum, the distribution of d_{ij} is essentially continuous. This fact introduces to the system yet another percolation-like physics, similar to that of the standard hopping conductivity (see, e.g., [12]). In the present paper we do not consider this facet of the problem explicitly, so that, strictly speaking, the consideration below is directly applicable only to mixtures. We expect, however, that the principal universal features of conductivity, based on the universal properties of the clusters distribution near the threshold, will be present also for the systems with continuous insulating matrix.

The temperature dependence of conductivity of a mixture in the intermediate range of temperatures

$$E_C^{(\mathrm{cr})}, E_C^{(\mathrm{m})} \ll T \ll E_C^{(0)}, \tag{9}$$

was already studied in the previous paper [11]. Here

$$E_C^{(\mathrm{m})} \sim E_C^{(0)} G^{-(s+\nu)/[\mu+(2-d)\nu]} \ll E_C^{(0)} \tag{10}$$

is the charging energy of a "marginal cluster", whose classic resistance $R_m \sim 1$. The classic resistance across a fractal cluster of n grains (see Fig.2 and Refs. [5, 11])

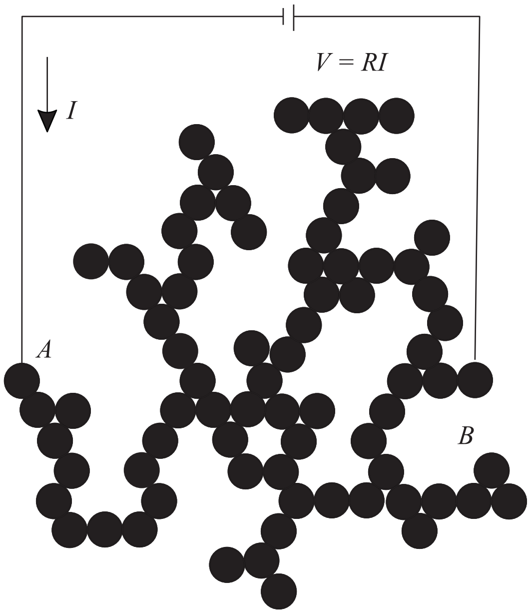

Fig.2. Definition of the classic resistance R across a cluster. External voltage V is applied to points A and B at the periphery of the cluster, then the ratio of V and the current I in the external circuit is the resistance R

is

$$R(n) \sim G^{-1} n^{[\mu+\nu(2-d)]/\nu d_f}, \tag{11}$$

so that the number of grains in the marginal cluster is

$$n_m \sim G^{\nu d_f/[\mu+(2-d)\nu]} \gg 1. \tag{12}$$

The clusters with $n < n_m$ (and, therefore, with $R < 1$) can be treated as structureless *point-like* supergrains characterized by the unique quantity – the charging energy $E_C(n)$. Under the condition (9) that is true for all relevant clusters in the system and the conductivity is described (see [11]) by the formulas

$$\sigma_{\mathrm{ins}}(x,T) \sim \sigma_{\mathrm{ins}}^{(0)}(x)[T/E_C^{(0)}]^{\Theta}, \quad T \ll T_{\mathrm{cross}}(x), \tag{13}$$

$$\sigma_{\mathrm{cross}}(T) \sim \sigma_{\mathrm{cross}}^{(0)}[T/E_C^{(0)}]^{\Theta'}, \quad T \gg T_{\mathrm{cross}}(x), \tag{14}$$

where

$$T_{\mathrm{cross}}(x) \sim E_C^{(0)} \left[(x_c - x)/\Delta_{\mathrm{cross}}\right]^{(\mu+s)/\Theta} \tag{15}$$

is the temperature of the crossover from the "insulator-controlled" conduction mode, where the resistivity is dominated by the insulating links, to the "critical crossover" mode, where the voltage drops occur both on links and on the conducting clusters. Note, that in the latter regime the conductivity σ_{cross} does not depend on x. The critical exponents

$$\Theta = \frac{(d-2)\nu + s - 1}{\nu + s}, \quad \Theta' = \frac{\mu\Theta}{\mu + s} \tag{16}$$

are given in the table. At high temperatures $T \sim E_C^{(0)}$, when the Coulomb blockade effect becomes irrelevant, the expressions (13), (14) match with the known results (see [13]) obtained in the absence of the Coulomb effects:

$$\sigma_{\mathrm{ins}}^{(0)} \sim g(x_c - x)^{-s}, \quad (\Delta_{\mathrm{cross}} \ll x_c - x \ll 1), \tag{17}$$

$$\sigma_{\mathrm{cross}}^{(0)} \sim g^{\mu/(\mu+s)} G^{s/(\mu+s)}, \quad (x_c - x \lesssim \Delta_{\mathrm{cross}}), \tag{18}$$

where the width of the critical crossover domain of concentrations is

$$\Delta_{\mathrm{cross}} = (g/G)^{1/(\mu+s)} \tag{19}$$

The physics behind the results (13), (14) is as follows: In the temperature range (9) it is possible to find conducting paths consisting of *only large* metal clusters connected by links, so that an electron never visits the Coulomb-blockaded small clusters with $n < n_{\mathrm{CB}}(T)$ (and with charging energies $E_C(n) > T$). Provided $n_{\mathrm{CB}}(T) < n_{\mathrm{cr}}$, the NNN-percolation in the system persists despite the fact that all small clusters with $n < n_{\mathrm{CB}}(T)$ are not available for travelling electrons. This fact is by no means trivial, since almost all conducting grains in the system belong to small clusters. The power-law suppression factor $(T/E_C^{(0)})^{\Theta}$ in the conductivity reflects just the reduction of the number of available conducting paths.

In the present paper we will address the following questions:

1. What is changed if $T \ll E_C^{(\mathrm{m})}$, so that the relevant clusters are not point-like zero-dimensional, but essentially extended objects with their internal dynamics? How one should describe the Coulomb blockade phenomena in this case?

2. What is the conduction mechanism and what is the T-dependence of conductivity at very low temperatures, when all clusters are Coulomb blockaded?

The low-temperature tunneling into a finite, but extended conductor is suppressed due to the process of charge spreading which transforms the initial point-like distribution of the tunneling charge into the smooth equipotential distribution. The corresponding suppression factor $\exp(-S_{\rm spr}(T))$ is nothing else, but the Coulomb zero-bias-anomaly (ZBA) factor, which appears in the tunneling probability alongside with the usual Coulomb blockade factor $\exp(-E_C/T)$. At low temperatures, when the sample size L is smaller than the "spreading length" $L(T)$, the activation factor dominates, while at intermediate temperatures (for $L > L(T)$) the principal contribution comes from the ZBA factor. Explicit expressions for $S_{\rm spr}(T)$ (as well as for $L(T)$) are known for diffusive conductors of different geometries: one-, two-, and three-dimensional (see 14–20]); the suppression is exponentially strong only in $1d$ case. In our problem we deal with an unusual case: we have to find the ZBA-factor for tunneling between two large clusters with *fractal geometry*, where the diffusion constant D, the conductivity $\tilde{\sigma}$, and the dielectric constant ε are scale-dependent. We will see that for such a geometry the suppression is exponentially strong in any dimension: in particular, in $2d$ and in $3d$.

Let us consider a tunneling of an electron through a link between two clusters with similar numbers of grains $n_1 \sim n_2 \sim n$. Immediately after the intercluster hop the system finds itself under the barrier, with large energy deficit $\sim E_C^{(0)}$, so that the charge density has yet to spread over a large region in a tunneling manner, before the system manages to get from under the barrier (see Fig.3). As a result, the suppression factor has the form

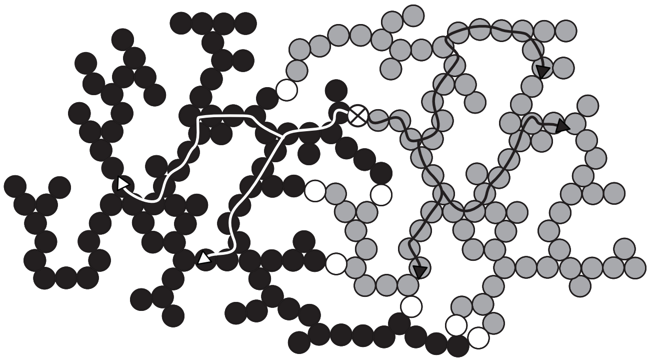

Fig.3. The charge spreading process. After the tunneling of an electron between two clusters (black and grey) through one of the links (marked by a cross) a diffusional spreading of charge occurs in both clusters

$\exp\{-S(T)\}$ with

$$S(T,n) = E_C(n)/T + S_{\rm spr}(T,n), \tag{20}$$

where the under-barrier action $S_{\rm spr}(T,n)$ can be estimated by the semiclassical method. Different variants of such method were proposed in [16, 17], in this paper we will use the variant due to Levitov and Shytov [17].

Strictly speaking, the act of the electron hop between two neighboring large clusters creates an electron-hole pair, and each component of this pair then spreads over its own cluster. However, this separation does not prevent the Coulomb interaction of both components. The electron-hole interaction effectively leads to partial screening of self-interaction in each cloud. If both clouds develop in the same spatial domain, then this screening is strong and leads to a dramatic suppression of the action $S_{\rm spr}$. Exactly this situation arises, when electron and hole clouds proliferate in nearby parallel planes with similar properties. The action in this case is parametrically smaller than that in the STM case, where the hole is immediately evacuated [17].

In our problem we apparently have an intermediate case. The neighboring large clusters to some extent interpenetrate, but by no means they coincide. The relative overlap is probably considerable, but definitely it is not close to a complete coincidence. Obviously, the effect of mutual screening is important for our problem, but it only leads to an effective suppression of self-interaction by a numerical factor, without introducing any new scale or any new small parameter. Thus, we conclude, that neglecting the mutual screening would overestimate the under-barrier action $S_{\rm spr}$ only by a numerical factor of order, say, two. Since we are anyway not able to determine the numerical factor in the action, we will neglect the effect of mutual screening in what follows.

We write the expression of the action $S_{\rm spr}$ in the case, when the process is controlled by finite temperature (not by the external voltage), and neglect the mutual electron-hole screening:

$$S_{\rm spr}(T,n) \sim \sum_{k=0}^{\infty} \frac{2\pi T}{2\pi T(2k+1) + \tilde{D}_q q^2} \times \\ \times \int_{1/L}^{1} \frac{d^d\mathbf{q}}{(2\pi)^d} \frac{U_q}{2\pi T(2k+1) + \tilde{\sigma}_q q^2 U_q + \tilde{D}_q q^2} \tag{21}$$

(see [20] for details). Here L is the size of an electrode, $\tilde{\sigma}$ and $\tilde{D}$ are its conductivity and diffusion constant, and U is the screened Coulomb interaction. The wave-vector q is measured in the units of $1/a$. Now we extend the result (21) to the fractal case with

$$\tilde{\sigma}_q \sim G q^{\mu/\nu}, \quad \tilde{D}_q \sim G E_D^{(0)} q^{d_f - d + \mu/\nu}, \\ U_q \sim E_C^{(0)} q^{1-d+s/\nu}, \quad L \sim n^{1/d_f}, \tag{22}$$

(see [5]). The energy scale $E_D^{(0)} \sim E_C^{(0)}(ap_F)^{-2} \ll E_C^{(0)}$, therefore the diffusion terms $\tilde{D}_q q^2$ can be neglected in both denominators in (21). The sum over k in (21) is dominated by $k\sim 1$ and the integrals over q are dominated by $q\sim L(T)^{-1}$, where

$$L(T)\sim\left(GE_C^{(0)}/T\right)^{\nu/[\mu+s+(3-d)\nu]}. \tag{23}$$

The spreading factor is the dominant one, if $1\ll\ll L(T)\ll L$. Introducing new variable $Q==qL(T)(2k+1)^{-\nu/[\mu+s+(3-d)\nu]}$, and extending the integration to the infinity, we get

$$S_{\rm spr}(T,n)\sim\left(E_C^{(\rm m)}/T\right)^{\varphi}, \tag{24}$$

where the new critical exponent φ is expressed in terms of the universal indices of the percolation theory:

$$\varphi=\frac{\mu+(2-d)\nu}{\mu+s+(3-d)\nu}=\begin{cases}0.33, & (d=2)\\ 0.41, & (d=3)\end{cases}. \tag{25}$$

The situation, when the action (20) is dominated by the activation term, we call the "Coulomb blockade", while the regime, dominated by the spreading term is called the "Coulomb ZBA". The phase diagram for different regimes on the $n-T$ plane is shown in Fig.4. The

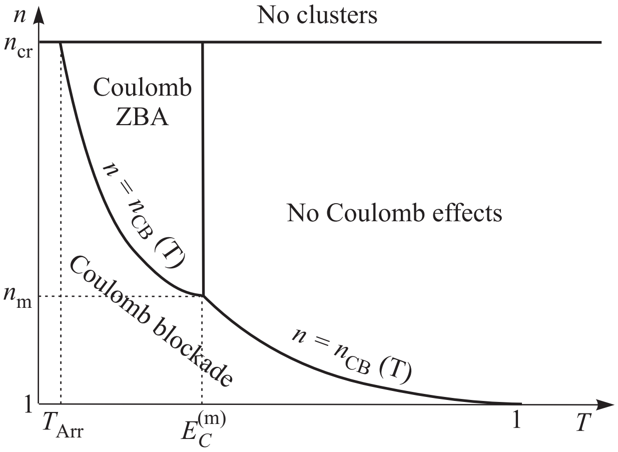

Fig.4. Different regimes for tunneling between two clusters with n grains each on the $n-T$ plane. The most interesting case $n_{\rm cr}>n_{\rm m}$ is shown; in the opposite case, for $n_{\rm cr}<n_{\rm m}$, there are no clusters in the "Coulomb ZBA" domain

crossover line is defined by

$$n_{\rm CB}(T)\sim\begin{cases}(T/E_C^{(0)})^{-\nu d_f/(\nu+s)}, & (T\gg E_C^{(\rm m)})\\ (T/GE_C^{(0)})^{-\nu d_f/[\mu+s+(3-d)\nu]}, & (T\ll E_C^{(\rm m)})\end{cases}.$$

The crossover temperature $T_{\rm Arr}(x)$, below that all clusters in the system are Coulomb blockaded, so that the Arrhenius activated regime of conduction sets on, can be found from the condition $n_{\rm cr}(x)=n_{\rm CB}(T_{\rm Arr})$:

$$\frac{T_{\rm Arr}(x)}{E_C^{(0)}}\sim\begin{cases}(x_c-x)^{s+\nu}, & (x_c-x\gg\Delta_{\rm m}),\\ G(x_c-x)^{\mu+s+(3-d)\nu}, & (x_c-x\ll\Delta_{\rm m}),\end{cases} \tag{26}$$

where

$$\Delta_{\rm m}\equiv x_c-x_m\sim G^{-1/[\mu+(2-d)\nu]} \tag{27}$$

and x_m is defined by the equation $n_{\rm cr}(x_m)=n_m$.

The result (24) is very different from the standard Coulomb ZBA effect in plain (non-fractal) systems, where the stretched exponential law (with power 1/2) appears only in one-dimensional case [17 – 20], while in 2-d there is only a log-squared function of T in the exponent and in 3-d the anomaly is weak. The reason for such a striking difference is the nontrivial scale dependence (22) of both dielectric screening and the conductivity in the fractal system.

In the range $T_{\rm Arr}(x)\ll T\ll E_C^{(\rm m)}$ the NNN-percolation via large non-blockaded clusters (with $n_{\rm CB}(T)<n<n_{\rm cr}$), is still possible. However, the tunneling between these clusters is strongly modified due to the Coulomb ZBA. As a consequence, the expressions for conductivity (13), (14) are modified also: The result (13) for the insulator controlled transport is changed to

$$\frac{\sigma_{\rm ins}(x,T)}{\sigma_{\rm ins}^{(0)}(x)}\sim\left(\frac{T}{E_C^{(0)}}\right)^{\tilde\Theta}B\exp\left\{-c\left(\frac{E_C^{(\rm m)}}{T}\right)^{\varphi}\right\}, \tag{28}$$

where c is some unknown numerical constant, $B==B\left(T/E_C^{(\rm m)}\right)$ is unknown prefactor. Note that the expression on the right hand side of (28) does not depend on x. The result (14) for the conductivity in the critical crossover range is changed to

$$\frac{\sigma_{\rm cross}(T)}{\sigma_{\rm cross}^{(0)}}\sim\left(\frac{T}{E_C^{(0)}}\right)^{\tilde\Theta'}B^{\frac{\mu}{\mu+s}}\exp\left\{-\frac{\mu c}{\mu+s}\left(\frac{E_C^{(\rm m)}}{T}\right)^{\varphi}\right\}, \tag{29}$$

and the expression for the crossover temperature (15) is changed to

$$T_{\rm cross}(x)\sim E_C^{(\rm m)}\ln^{1/\varphi}\left[\Delta^*_{\rm cross}/(x_c-x)\right], \tag{30}$$

$$\Delta^*_{\rm cross}\sim\Delta_{\rm cross}G^{-\frac{(s+\nu)\tilde\Theta}{(\mu+s)[\mu+(2-d)\nu]}}. \tag{31}$$

The modified critical exponents

$$\tilde\Theta=\frac{(d-2)\nu+s-1}{\mu+s+(3-d)\nu},\quad \Theta'=\frac{\mu\tilde\Theta}{\mu+s}. \tag{32}$$

Now we turn to the question about the nature and the temperature dependence of the conductivity at low temperatures $T < T_{\rm Arr}$. Here an electron can not avoid Coulomb blockaded clusters, since practically all clusters are blockaded. At moderately low T electrons travel through the "critical network of critical clusters". This mode of transport is similar to the standard nearest-neighbor-hopping regime in the hopping conductivity (see [12]). The hopping goes via critical clusters, since, on one hand, they have small charging energies $\sim E_C^{\rm (cr)}$, and, on the other hand, they form an NNN-percolation network (i.e., it is possible to travel by hopping from one critical cluster to another through direct links). In this regime the conductivity obeys the Arrhenius law (1) with $E_{\rm act}(x) \sim E_C^{\rm (cr)}$.

If the temperature is lowered further, then, in the spirit of the Mott's variable range hopping [21, 12] the variable range cotunneling regime (VRC) [7–9] sets on. This regime is characterized by distant hops between *resonant clusters* with small charging energies. The hops are realized as acts of multiple cotunneling (elastic or inelastic) through chains of nonresonant clusters, connecting the resonant ones. Thus, VRC involves two sorts of clusters:

- The terminal ones, where the real charged states can occur. These clusters should be resonant.
- The intermediate clusters, forming chains of virtual intermediate states. These nonresonant clusters should be as large as possible – to minimize the number of intermediate states. Of course, they also should be NNN-connected.

What is the nature of the resonant clusters? While the characteristic scale of the charging energy for a cluster of size n is $E_C(n)$, its particular value for a given cluster is random, due to random form of the clusters, and, most important, due to random electrostatic potentials of the surrounding. The electrostatic interaction of metallic clusters with charges Q_i (measured in the units of electronic charge e) is $H_C\{Q\} = \frac{e^2}{2}\sum_{ij}\left[C^{-1}\right]_{ij}\tilde{Q}_i\tilde{Q}_j$, where $\tilde{Q}_i \equiv (Q_i - q_i)$ and C_{ij} is the matrix of capacitances. The so called "background charges" $-1/2 < q_i < 1/2$ are due to random potentials of stray charges, trapped in the insulator; the set of q_i is chosen so that the ground state corresponds to $Q_i \equiv 0$. When one extra electron (one extra hole) is put on the cluster i, the energy of the system is changed by

$$E_{Ci}^{(\pm)} = \frac{e^2}{2}\left\{\left[C^{-1}\right]_{ii} \mp 2\sum_j \left[C^{-1}\right]_{ij} q_j\right\}. \qquad (33)$$

The resonant clusters are characterized by anomalously small $E_{Ci}^{(+)}$ or $E_{Ci}^{(-)}$. Counter-intuitively, the principal contribution to the low energy density of states comes not from large resonant clusters, but from small resonant clusters. To prove this, let us first neglect the long range Coulomb interaction (i.e., the Coulomb gap effect) for a while, and consider the so called "density of ground states" $\nu_{GS}^{(0)}(E)$ (see [7] for the detailed definition). It is instructive to decompose it into the sum of contributions $\nu_{GS}^{(0)}(E,n)$ of clusters with fixed n. Under the most natural condition of strong charge disorder (when q_i are homogeneously distributed in the interval $-1/2 < q_i < 1/2$) each $\nu_{GS}^{(0)}(E,n)$ is a structureless function with a single scale $E_C(n)$, so that $\nu_{GS}^{(0)}(E,n) \sim 1/E_C(n)$ for $E \lesssim E_C(n)$ and $\nu_{GS}^{(0)}(E,n) \approx 0$ for $E \gg E_C(n)$. As a result, for the sum we obtain

$$\nu_{GS}^{(0)}(E) = \sum_n \nu_{GS}^{(0)}(E,n)N(n) \approx$$
$$\approx \sum_{n=1}^{n(E)} n^{-\tau}/E_C(n) \approx \sum_n n^{-[\tau-(s+\nu)/\nu d_f]} \sim 1. \qquad (34)$$

Here $n(E)$ is defined by $E_C(n) = E$. Since $\tau - (s + \nu)/\nu d_f > 1$, the sum is indeed dominated by small clusters with $n \sim 1$. It is important to stress that, although most clusters with low charging energies are small resonant clusters, the latter do not play any role in the conduction processes at $T > T_{\rm Arr}$. Unlike large clusters, these small resonant clusters are not NNN-connected: they are separated from each other by many nonresonant ones. That is why we did not take them into account in the previous parts of this paper (as well as in [11]).

Taking into account the long range Coulomb interaction modifies the result (34) and leads to the appearance of the soft Coulomb gap [12, 7]:

$$\nu_{GS}(E) \sim |E|^{d-1}(\tilde{\epsilon}/e^2)^d, \qquad (35)$$

but it can not alter the small-cluster nature of the majority of low energy states.

The asymptotic law (35) is valid only for lowest energies $|E| \ll E_C^{\rm (cr)}$, while in the range $E_C^{\rm (cr)} \ll E \ll 1$ the density of states is strongly modified by the fractal character of the system. However, the VRC regime is actual just in the temperature range, where the resonant clusters have energies $E \ll E_C^{\rm (cr)}$, so that the formula (35) is sufficient for our purposes.

The intermediate clusters involved in the acts of multiple cotunneling are the critical ones. In contrast with small resonant clusters, which are always point-like, these clusters can be either point-like (if $n_{\rm cr} < n_{\rm m}$), or extended (if $n_{\rm m} < n_{\rm cr}$). There is a considerable difference in the cotunneling probability between the two cases.

Under the condition $n_{\rm cr} < n_{\rm m}$ all the critical clusters at low temperature act as effective "supergrains" with characteristic charging energy $E_C^{(\rm cr)}$. The average number of links, connecting two adjacent critical clusters, is $N_{\rm links}(\xi) \sim 1/(x_c - x)$ (see [11]). Therefore the effective conductance between the two clusters is

$$g_{\rm eff} \sim g N_{\rm links}(\xi) \sim g/(x_c - x). \qquad (36)$$

In this paper we consider only the case $g_{\rm eff} \ll 1$. We also concentrate on the case the inelastic cotunneling; a subtle question about the elastic cotunneling and the x-dependence of $T_{el}(x)$ will be discussed in a separate publication. For point-like clusters the method of evaluation of the probability of multiple cotunneling, proposed in [8], can be applied directly. Introducing the characteristic number K of critical clusters, separating two resonant ones, and the width Δ of the Mott strip, we obtain

$$\sigma_{\rm ins} \propto g_{\rm eff}^K \left(\Delta/K E_C^{(\rm cr)}\right)^{2K} \exp\{-\Delta/T\}, \qquad (37)$$

for $T_{el}(x) \ll T \ll T_{\rm Arr}(x)$. Then, having in mind the relation $\Delta \cdot \nu_{GS}(\Delta)(\xi K)^d \sim 1$ between Δ and K, and optimizing the conductivity (37), we arrive at the formula (2) with $E_{\rm ES} \sim \mathcal{L}(T) E_C^{(\rm cr)}$ and

$$\mathcal{L}(T) = \mathcal{L}^* + 2\ln(T_{\rm ES}/T), \quad \mathcal{L}^* \sim \ln(1/g_{\rm eff}). \qquad (38)$$

Thus, for point-like critical clusters $T_{\rm ES} \sim T_{\rm Arr}/\mathcal{L}^*$.

In the case $n_{\rm cr} < n_{\rm m}$ we have to deal with cotunneling through extended objects with large resistance. Such cotunneling was considered in [20] for the case of long diffusive wire; the corresponding exponential factor was found by means of modified Levitov-Shytov semiclassical method. The approach of [20] can be applied also to the extended fractal clusters. Writing the action for the process, where an electron and a hole are simultaneously injected into an extended cluster at its opposite ends, at distance $r \sim \xi$ from each other, we obtain

$$S_{\rm cotun}(T) \sim \sum_{k=0}^{\infty} \frac{2\pi T}{2\pi T(2k+1) + \tilde{D}_q q^2} \times$$
$$\times \int_{1/\xi}^{1} \frac{d^d \mathbf{q}}{(2\pi)^d} \frac{U_q \sin^2(\mathbf{q}\cdot\mathbf{r}/2)}{2\pi T(2k+1) + \tilde{\sigma}_q q^2 U_q + \tilde{D}_q q^2} \sim$$
$$\sim 2R(n_{\rm cr}) \ln(T_{\rm Arr}/T). \qquad (39)$$

The expression (39) differs from (21) only in the factor $\sin^2(\mathbf{q}\cdot\mathbf{r}/2)$. In the "Coulomb ZBA scenario" the integral over q in (21) is cut off at the intrinsic scale $q \sim L^{-1}(T)$, controlled by the temperature and x-independent. On the contrary, in the VRC regime $L(T) \gg \xi$, so that the integral is cut off at $q \sim \xi^{-1}$. As long as we are interested only in the inelastic cotunneling, the diffusion terms $\tilde{D}_q q^2$ again can be neglected in both denominators in (39)

Now, repeating the arguments of the formula (37) with account for the exponential suppression of the cotunneling amplitude, we get again the result (2), but with $E_{\rm ES} \sim \mathcal{L}(T) E_C^{(\rm cr)} R(n_{\rm cr})$, $T_{\rm ES} \sim T_{\rm Arr}/\mathcal{L}^*$ and $\mathcal{L}^* \sim 1/R(n_{\rm cr}) \ln(1/g_{\rm eff})$.

Finally the results for both $n_{\rm cr} < n_{\rm m}$ and $n_{\rm cr} < n_{\rm m}$ can be unified in the following form: the crossover temperatures $T_{\rm Arr}$ and $T_{\rm ES}$, separaiting the domains of validity of the Arrenius law (1) and the Efros-Shklovskii law (2) are strongly reduced in the vicinity of percolation threshold,

$$T_{\rm Arr} \sim E_C^{(\rm cr)}/R_{\rm eff}, \quad T_{\rm ES} \sim T_{\rm Arr}/\mathcal{L}^*, \qquad (40)$$

$$\mathcal{L}^* = 1 + (1/R_{\rm eff}) \ln(1/g_{\rm eff}), \quad R_{\rm eff} \approx 1 + R(n_{\rm cr}), \quad (41)$$

$$R(n_{\rm cr}) \sim G^{-1}(x_c - x)^{-1/[\mu + (2-d)\nu]}, \qquad (42)$$

and $R_{\rm eff}$ has the meaning of the effective dimensionless resistance across the critical cluster.

Note that the gap between $T_{\rm Arr}$ and $T_{\rm ES}$ only exists, if $\mathcal{L}^* \gg 1$. This condition is fulfilled for $x_c - x \gg \Delta_0$, where

$$\Delta_0 = \Delta_{\rm m} [\ln(1/g)]^{-[\mu + (2-d)\nu]}, \qquad (43)$$

while for $x_c - x < \Delta_0$ $\mathcal{L}^* \sim 1$ so that the gap shrinks to zero and there is no room for the Arrhenius law (see Fig.5). The "activation energies" $E_{\rm act}$ and $E_{\rm ES}$ entering (1) and (2) are also reduced:

$$E_{\rm act} \sim E_C^{(\rm cr)}, \quad E_{\rm ES} \sim \mathcal{L}(T) E_C^{(\rm cr)} R_{\rm eff}, \qquad (44)$$

$$\mathcal{L}(T) = \mathcal{L}^* + 2\ln(T_{\rm ES}/T). \qquad (45)$$

In conclusion, we have studied the low-temperature behavior of a granular system near the percolation threshold. The peculiarities of this behavior stem from the strong dispersion and fractal properties of the conducting clusters. As in the system away from the percolation threshold, the transport mechanism at lowest temperatures is variable range cotunneling, and the corresponding temperature dependence of σ is the Efros-Shklovsky law. However, the parameters entering this law are dramatically renormalized in the vicinity of the threshold. In particular, the onset of the VRC regime

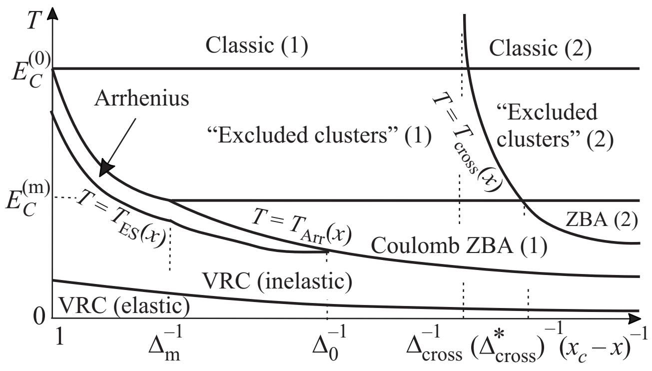

Fig.5. Phase diagram of different regimes for the conductivity. "Classic (1)" – classic T-independent transport, controlled by the links (see Eq(17)); "Classic (2)" – the same in the critical crossover region (see Eq(18)); "Excluded clusters (1)" – transport via large (non-blockaded) NNN-connected clusters, the T-dependence is a power law (see Eq.(13)); "Excluded clusters (2)" – the same in the critical crossover region (see Eq.(14)); "Coulomb ZBA (1)" – transport is controlled by the Coulomb zero-bias anomaly, the T-dependence is stretched exponential (see Eq.(28)); "ZBA (2)" – the same in the critical crossover region (see Eq.(29)); "Arrhenius" – direct hopping between Coulomb blockaded critical clusters, the T-dependence is simple activation (see Eq.(1)); "VRC" – variable range cotunneling (elastic or inelastic) between small resonant clusters via chains of critical non-resonant clusters, the T-dependence is the modified Efros-Shklovskii law (see Eq.(2))

is shifted to lower temperatures. At higher temperatures the VRC is replaced by alternative physical mechanisms. One of them – the NNN-percolation with excluded small (Coulomb blockaded) clusters, was studied in [11], the other – the Coulomb zero-bias anomaly scenario is first discussed in the present paper. The Coulomb interaction suppresses the probability of tunneling between large metallic clusters in the "zero bias anomaly" manner. Due to the fractal structure of clusters the temperature dependence of the Coulomb ZBA factor is described by stretched exponential law (28) with nontrivial index φ.

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1. *Disorder and Granular Media*, Eds. D. Bideau and A. Hansen, North-Holland, Amsterdam, 1993.
2. *Nanocomposite Science and Technology*, Eds. P.M. Ajayan, L.S. Schadler, and P.V. Braun, Wiley-VCH, Weinheim, 2003.
3. *Single Electron Tunnelling*, Eds. H. Grabert and M.H. Devoret, Plenum Press, New York and London, 1992.
4. D. Stauffer and A. Aharony, *Introduction to Percolation Theory*, Taylor and Fransis, London, 1994.
5. A. Bunde and S. Havlin, *Percolation I*, in *Fractals and Disordered Systems*, Eds. A. Bunde and S. Havlin, Springer, Berlin, 1996.
6. The term "touching" does not necessarily mean direct metal-metal contact. In some systems a thin insulating (e.g., oxide) layer still separates two touching grains, but its thickness is much smaller than for nontouching grains.
7. J. Zhang and B.I. Shklovskii, Phys. Rev. B **70**, 115317 (2004).
8. M.V. Feigelman and A.S. Ioselevich, JETP Letters **81**, 341 (2005).
9. I.S. Beloborodov, A.V. Lopatin, and V.M. Vinokur, Phys. Rev. B **72**, 125121 (2005).
10. A.L. Efros and B.I. Shklovskii, J. Phys. C **8**, L49 (1975).
11. A.S. Ioselevich and D.S. Lyubshin, JETP Letters, **90**, 746 (2009).
12. B.I. Shklovskii and A.L. Efros, *Electronic Properties of Doped Semiconductors*, Springer, Berlin, 1984.
13. A.L. Efros and B.I. Shklovskii, Phys. Stat. Sol. b**76**, 475 (1976).
14. B.L. Altshuler and A.G. Aronov, in *Electron-electron interactions in disordered solids*, Eds. A.L. Efros and M. Pollak, North-Holland, Amsterdam, 1985.
15. A.M. Finkelshtein, JETP **57**, 97 (1983); A.M. Finkelstein, JETP **59**, 213, (1984).
16. Yu.V. Nazarov, JETP **68**, 561 (1990); Yu.V. Nazarov, Phys. Rev. B **43**, 6220, (1991); G.-L. Ingold and Yu.V. Nazarov, *Charge tunneling rates in ultrasmall junctions*, Chapter 2 in Ref.[3], p. 21.
17. L.S. Levitov and A.V. Shytov, JETP Letters **66**, 214 (1997).
18. E.G. Mishchenko, A.V. Andreev and L.I. Glazman, Phys. Rev. Lett. **87**, 246801 (2001).
19. R. Egger and A.O. Gogolin, Phys. Rev. Lett. **87**, 066401 (2001).
20. M.V. Feigelman and A.S.Ioselevich, JETP Letters **88**, 882 (2008).
21. N.F. Mott, J. Non-Cryst. Solids **1**, 1 (1968).