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A three-loop check of the "a-maximization" in SQCD with adjoint(s)

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The "a-maximization" was introduced by K. Intriligator and B. Wecht for finding anomalous dimensions of chiral superfields at the IR fixed points of the RG flow. Using known explicit calculations of anomalous dimensions in the perturbation theory of SQCD (with one or two additional adjoint fields), it is checked here at the three-loop level.

The NSVZ β-function in SUSY theory has the form [1]:

$$\frac{da}{d\ln\mu} = \beta(a) = -\frac{a^2}{1-a}\,N,$$
$$N = \left[3\,T_G - \sum_i T_{r_i}\Big(1+\gamma_i(a)\Big)\right], \tag{1}$$
$$a = a_{NSVZ} = \frac{N_c g^2_{NSVZ}}{8\pi^2},$$

where γ_i are the field anomalous dimensions, r_i denotes the representation of the i-th field. Really, in what follows we will consider only two particular (but nontrivial) examples of the $\mathcal{N}=1$ $SU(N_c)$ gauge theory with N_F fundamentals Q and $\overline{Q}$ and one (X) or two (X, Y) adjoint fields, and with zero superpotential.

One adjoint field X. Then:

$$N = \Big[b_o - N_F\gamma_Q(a) - N_c\gamma_X(a)\Big],\ b_o = 2N_c - N_F,$$
$$\mathrm{Tr}_r\Big(T^A T^B\Big) = T_r\,\delta^{AB},\ T_G = T_X = N_c, \tag{2}$$
$$T_Q = T_{\overline{Q}} = \frac{1}{2}.$$

At the IR fixed point:

$$N = 0,\quad a = a^* = \mathrm{const},$$
$$d_i \equiv \Big(1 - \frac{1}{2}\gamma_i(a^*)\Big) = \frac{3}{2}|R_i(a^*)|. \tag{3}$$

The problem is to find the values of all anomalous dimensions $\gamma_i(a^*)$ of chiral superfields, while $N=0$ gives only one constraint. Intriligator and Wecht [2] found a method to find all $\gamma_i(a^*)$ (within the conformal window).

As was shown in [3], in SUSY theories at the conformal IR fixed point the well known central charge a_T entering the trace anomaly, $T^\mu_\mu \sim a_T\,$(Euler), has the form:

$$a_T(R^*_i) = \sum_i \left[3\Big(R^{(f)}_i(a^*)\Big)^3 - R^{(f)}_i(a^*)\right], \tag{4}$$
$$R^*_i = R_i(a^*),$$

where $R^{(f)}_i(a^*)$ is the superconformal R-charge of the fermionic component of the i-th superfield at the IR fixed point. As was obtained in [2], the right values R^*_i maximize $a_T(R_i)$ (the method of "a-maximization", see also [4 – 6]). This allows one to find the values $R_i(a^*)$ and so $\gamma_i(a^*)$, see (3).

The purpose of this short letter is to perform a check of the 'a-maximization" predictions using the known results of explicit calculations of $\gamma_i(a)$ up to a three-loop level in the perturbation theory of SQCD-like theories, see [7 – 9] and refs. therein[2)].

The coupling is small in perturbation theory calculations. This is achieved at $z \equiv b_o/2N_c \ll 1$, i.e. at N_F slightly below $2N_c$, and $N_c \gg 1$. So, the genuine small expansion parameter used everywhere below will be not $a^* \ll 1$ by itself, but rather $z \ll 1$:

$$a^*(z) = a_1 z + a_2 z^2 + a_3 z^3 + O(z^4). \tag{5}$$

We use below some notations from [7 – 9] (the dimensional reduction (DR) scheme is used therein):

$$\Big(T^A T^A\Big)^i_j = C(r)^i_j = C_r\,\delta^i_j,$$
$$C_Q = C_{\overline{Q}} = C_F = \frac{N_c^2-1}{2N_c} \to \frac{N_c}{2},$$
$$C_X = C_G = N_c,\quad T = \frac{1}{D}\sum_i \mathrm{Tr}_{r_i}\Big(C(r_i)^2\Big), \tag{6}$$
$$\Delta = \sum_i C_{r_i}\,T_{r_i},\quad D = N_c^2 - 1,\quad \kappa = 6\,\zeta(3),$$
$$N_F = 2N_c(1-z),$$
$$1/N_c \ll z = b_o/2N_c \ll 1,\ N_c \gg 1.$$

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2)Our definition of γ_i differs by the factor "–2" from those in [7], i.e: $\gamma_{\rm here} = -2\,\gamma_{\rm there}$.

The perturbative series for the anomalous dimensions have the form:

$$\gamma_i^{NSVZ}(a) \equiv \gamma_i(a) = \gamma_i^{(1)} a + \gamma_i^{(2)} a^2 + \gamma_i^{(3)} a^3,$$
$$\gamma_i^{DR}(a_{DR}) \equiv \Gamma_i(a_{\rm DR}) = \Gamma_i^{(1)} a_{\rm DR} + \Gamma_i^{(2)} a_{\rm DR}^2 + \Gamma_i^{(3)} a_{\rm DR}^3. \quad (7)$$

As shown in [9], the couplings $a = a_{NSVZ}$ and a_{DR} are connected (with the three-loop accuracy) as:

$$a_{\rm DR} = a\left[1 + a^2\left(-\frac{T}{2N_c^2} + \frac{b_o}{4N_c}\right)\right] \to$$
$$\to a_{\rm DR}^* = a^*\left[1 - a^{*2}\frac{T}{2N_c^2}\right] + O(a^{*4}). \quad (8)$$

At the IR fixed point:

$$a(\mu/\Lambda \ll 1) \to a^* = \text{const},$$
$$\left[(1-z)\gamma_Q(a^*) + \frac{1}{2}\gamma_X(a^*)\right] = z. \quad (9)$$

At one-loop:

$$\gamma_r^{(1)} = \Gamma_r^{(1)} = 2\frac{C_r}{N_c}, \quad \gamma_Q^{(1)} = 2\frac{C_F}{N_c} \to 1,$$
$$\gamma_X^{(1)} = 2, \to a^*(z) = \frac{z}{2} + O(z^2). \quad (10)$$

At two-loops:

$$\gamma_Q^{(2)} = \Gamma_Q^{(2)} = -\frac{1-2z}{2},$$
$$\gamma_X^{(2)} = \Gamma_X^{(2)} = -2(1-z), \to$$
$$\to a^*(z) = \frac{z}{2} + \frac{7}{16}z^2 + O(z^3). \quad (11)$$

At three-loops [7–9]:

$$\Gamma_r^{(3)} = \kappa\frac{C_r}{N_c}\left[-3 + \frac{\Delta}{N_c^2} + \frac{b_o}{N_c}\right] +$$
$$+ \frac{C_r}{N_c}\left[4\frac{C_r^2}{N_c^2} - 5\frac{T}{N_c^2} - \frac{b_o C_r}{N_c^2} + \frac{5}{2}\frac{b_o}{N_c} - \frac{b_o^2}{2N_c^2}\right], \quad (12)$$

$$\Gamma_Q^{(3)} \to \frac{\kappa}{2}\left(-3 + \frac{\Delta}{N_c^2}\right) + \frac{1}{2}\left[1 - 5\frac{T}{N_c^2}\right],$$
$$\Gamma_X^{(3)} \to \kappa\left(-3 + \frac{\Delta}{N_c^2}\right) + \left[4 - 5\frac{T}{N_c^2}\right], \quad (13)$$

and from $\gamma(a^*) = \Gamma(a_{DR}^*)$, see (8),(10):

$$\gamma_r^{(3)} = \Gamma_r^{(3)} + \frac{C_r}{N_c}\left(-\frac{T}{N_c^2} + \frac{b_o}{2N_c}\right) \to \left[\Gamma_r^{(3)} - \frac{C_r}{N_c}\frac{T}{N_c^2}\right],$$
$$\gamma_Q^{(3)} \to \left[\Gamma_Q^{(3)} - \frac{T}{2N_c^2}\right], \; \gamma_X^{(3)} \to \left[\Gamma_X^{(3)} - \frac{T}{N_c^2}\right]. \quad (14)$$

It is convenient to define:

$$a^* = \overline{a} + \frac{1}{4}\delta a^*, \quad \overline{a} = \frac{z}{2}\left(1 + \frac{7}{8}z\right), \quad \delta a^* = O(z^3). \quad (15)$$

Then one obtains from (9):

$$\delta a^* = 2z - \sum_{i=1}^{3}\left(\overline{a}\right)^i\left[\gamma_X^{(i)} + 2(1-z)\gamma_Q^{(i)}\right] =$$
$$= \frac{z^3}{8}\left(\frac{15}{2} - \gamma_X^{(3)} - 2\gamma_Q^{(3)}\right). \quad (16)$$

So:

$$\gamma_Q(a^*) =$$
$$= \left[\overline{a} - (\frac{1}{2} - z)\,\overline{a}^2 + \frac{15}{8}\overline{a}^3 + \frac{\overline{a}^3}{2}\left(\gamma_Q^{(3)} - \frac{1}{2}\gamma_X^{(3)}\right)\right],$$
$$\gamma_X(a^*) =$$
$$= \left[2\,\overline{a} - 2\,(1-z)\,\overline{a}^2 + \frac{15}{4}\overline{a}^3 - \overline{a}^3\left(\gamma_Q^{(3)} - \frac{1}{2}\gamma_X^{(3)}\right)\right], \quad (17)$$

and finally, see (13)–(15):

$$\gamma_Q(a^*) = \left[\frac{1}{2}z + \frac{5}{16}z^2 + \frac{11}{64}z^3\right] + O(z^4),$$
$$\gamma_X(a^*) = \left[z + \frac{3}{8}z^2 + \frac{9}{32}z^3\right] + O(z^4). \quad (18)$$

Now, let us compare with the predictions of the “a-maximization” [2]. These have the form [2] ($x = N_c/N_F = 1/(2-2z)$):

$$3R_Q(a^*) = \frac{x\left(6x + 3 - \sqrt{20x^2 - 1}\right) - 3}{2x^2 - 1},$$
$$3R_X(a^*) = \frac{10}{3 + \sqrt{20x^2 - 1}}. \quad (19)$$

Expanding these expressions in powers of z, one obtains:

$$3R_Q(a^*) = \left[2 - \frac{1}{2}z - \frac{5}{16}z^2 - \frac{11}{64}z^3 - \frac{109}{1024}z^4\right],$$
$$3R_X(a^*) = \left[2 - z - \frac{3}{8}z^2 - \frac{9}{32}z^3 - \frac{67}{512}z^4\right]. \quad (20)$$

Finally, using: $\gamma_i(a^*) = 2 - 3\,R_i(a^*)$, see (3), one finds that the “a -maximization” prediction agrees with the perturbation theory within the three-loop accuracy.

Attempts to use the results from [7–9] for a check of the “a - maximization” predictions at two- and three-loop levels in SQCD with one adjoint have been made previously in [4 – 6]. They were successful only in part, i.e. only the highest power terms $O(C_r^i)$ in $\gamma_r^{(i=2,3)} = A_r^{(i)}\,(C_r)^i + O(C_r^{i-1})$ have been checked. The reason is that the authors of [4 – 6] used expansions in powers of the coupling $a^* \ll 1$, but a^* does not enter explicitly the answers (19) for R_i. To make a complete check, one has to use the genuine expansion parameter $z = b_o/2N_c \ll 1$.

Two adjoint fields $\boldsymbol{X}, \boldsymbol{Y}$. In this theory (the anomalous dimensions $\gamma_X = \gamma_Y$):

$$b_o = N_c - N_F, \quad N = \Big[b_o - N_F\gamma_Q^* - 2N_c\gamma_X^*\Big] = 0,$$
$$N_F = N_c(1-2z), \quad \frac{1}{N_c} \ll z \ll 1, \tag{21}$$

and instead of (9) one has now

$$\Big(\frac{1}{2} - z\Big)\gamma_Q^* + \gamma_X^* = z. \tag{22}$$

Proceeding in the same way as above, one obtains

$$a^* \equiv \overline{a} + \frac{2}{5}\delta a^*, \quad \overline{a} = \frac{2}{5}z\Big(1 + \frac{19}{25}z\Big),$$
$$\delta a^* = \frac{4}{125}z^3\Big(\frac{58}{5} - 2\gamma_X^{(3)} - \gamma_Q^{(3)}\Big), \tag{23}$$

where $\gamma_Q^{(3)}$ and $\gamma_X^{(3)}$ are given in (13), (14). The anomalous dimensions $\gamma_Q^{(3)}$ and $\gamma_X^{(3)}$ are obtained now as

$$\gamma_Q^* = \Big[\Big(\overline{a} + \frac{2}{5}\delta a^*\Big) + (z - \frac{1}{2})\overline{a}^2 + \gamma_Q^{(3)}\overline{a}^3\Big] =$$
$$= \Big[\frac{2}{5}z + \frac{28}{125}z^2 + \frac{344}{3125}z^3\Big] + O(z^4), \tag{24}$$

$$\gamma_X^* = \Big[2\Big(\overline{a} + \frac{2}{5}\delta a^*\Big) + (2z - 2)\overline{a}^2 + \gamma_X^{(3)}\overline{a}^3\Big] =$$
$$= \Big[\frac{4}{5}z + \frac{36}{125}z^2 + \frac{528}{3125}z^3\Big] + O(z^4).$$

The "a-maximization" [2] predicts in this case the values of R^*-charges as:

$$3R_Q^* = \Big[3 + \frac{3x - 2x\sqrt{26x^2-1}}{8x^2-1}\Big],$$
$$3R_X^* = \frac{1}{2}\Big[3 - \frac{3 - 2\sqrt{26x^2-1}}{8x^2-1}\Big], \tag{25}$$

with $x = N_c/N_F = 1/(1-2z)$. Finally, expanding (25) in powers of z, one obtains:

$$3R_Q^* = \Big[2 - \frac{2}{5}z - \frac{28}{125}z^2 - \frac{344}{3125}z^3 - \frac{928}{15625}z^4\Big],$$
$$3R_X^* = \Big[2 - \frac{4}{5}z - \frac{36}{125}z^2 - \frac{528}{3125}z^3 - \frac{1256}{15625}z^4\Big]. \tag{26}$$

So, because $\gamma_i^* = 2 - 3R_i^*$, this agrees with (24), within the three-loop accuracy.

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