Pis'ma v ZhETF, vol. 91, iss. 6, pp. 291 – 296

Observation of wave energy accumulation in the turbulent spectrum of capillary waves on the He-II surface under harmonic pumping

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Submitted 27 November 2009
Resubmitted 10 February 2010

Recent results of the experimental study of capillary turbulence on superfluid helium-4 surface are presented. For the first time wave energy accumulation was observed in the wave turbulent spectrum at high frequencies when the surface was excited by harmonic pumping. We suggest a qualitative explanation of this phenomenon in the frames of weak wave turbulence approach that takes into account discreteness of the eigenfrequencies spectrum of surface oscillations in the cylindrical resonator.

Wave turbulence (WT) is a non-equilibrium state of the weakly nonlinear wave system wherein scales of energy pumping and energy dissipation are well-separated [1]. Examples of this nonlinear phenomenon can be found in diverse fields such as plasma physics, nonlinear optics, ferro-magnetism, and hydrodynamics. In particular, an ensemble of nonlinear capillary waves on the liquid surface is a well-known system in which wave turbulence can be realized. For capillary waves the main processes of the nonlinear interaction are three-wave processes of decay and merging that satisfy the conservation laws of frequency ω and wave-vector $\mathbf{k}$: $\omega_1 + \omega_2 = \omega_3$, $\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3$. The WT theory predicts that direct turbulent cascade is formed in the inertial range between the energy pumping scale and energy dissipation scale with the wave spectrum described by power law function of frequency (Kolmogorov-Zakharov spectrum). In recent years, due to the development of technical equipment and computational tools for data processing, capillary turbulence has been in focus of experimental studies of capillary waves on the surface of water [2 – 4], ethanol [4], silicon oil [5] and mercury [6]. Our previous experiments [7, 8] showed that the use of quantum fluids (liquid ^{4}He and liquid hydrogen) for investigations of capillary turbulence can provide additional advantages with respect to traditional liquids due to their low viscosity. In the present paper we focus on the peculiarities of the shape of the turbulent wave energy distribution at high frequencies observed in experiments with He-II, a superfluid phase of liquid ^{4}He (preliminary results were published in [9]). For the first time we observed that under certain conditions a deviation from the power law spectrum is formed at frequencies of the order of the viscous cut-off scale. We also suggest a qualitative model of this phenomenon that takes into account the influence of discreteness arising due to the finite sizes of the experimental cell.

In the reported experiments with superfluid ^{4}He we used the similar technique as in our previous studies of capillary turbulence on the surface of liquid hydrogen [10]. The measurements were made at temperature $T \approx 1.7$ K (for comparison, the temperature of the normal- to-superfluid state transition for liquid ^{4}He at saturated vapor pressure is equal to $T_\lambda = 2.17$ K). At this temperature superfluid helium has kinematic viscosity $\nu = 8.9 \cdot 10^{-5}\,\mathrm{cm^2/s}$, density $\rho = 0.145\,\mathrm{g/cm^3}$, surface tension $\sigma = 0.32\,\mathrm{din/cm}$ [11]. Liquid helium was condensed into a copper cup of inner diameter $D = 30$ mm and depth 4 mm. At the bottom of the cup a radioactive thin plate was placed. Emitted from the plate β-electrons ionized ^{4}He atoms which led to formation of positively charged ions and electrons. Between the cup and the upper electrode (a copper plate placed in vapor at a distance of 3 mm above the liquid surface) constant DC voltage $U \approx 600$ V of appropriate polarity was applied, so that the liquid surface was charged by positive ions. By applying low-frequency AC voltage with amplitudes from 1 up to 20 V to the cup, in addition to DC voltage, the capillary waves were excited on the charged surface. The advantage of this method of wave pumping is that the surface is excited directly by the driving force, but not through an instability as in the experiments with parametrically excited waves [2, 4, 5] and not by blade-wavemakers as in [6] which probably resulted in any motion in the bulk. Oscillations of the surface were detected by measurements of the power of the laser beam reflected from the surface. The laser beam fell on the surface at sliding angle $\alpha \approx 0.15$ producing an elliptic spot of size 1 mm×7 mm. Typical wavelengths of capillary waves in the experiments were smaller than the spot dimensions. The laser beam reflected from the oscillating surface was focused

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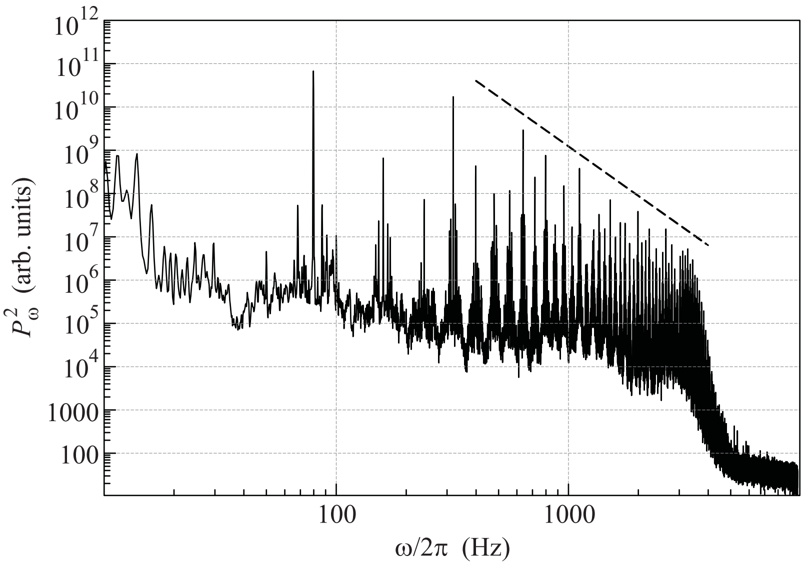

Fig.1. Turbulent spectrum of capillary waves on the He-II surface ($T = 1.7$ K) obtained when the surface was pumped by sinusoidal force with frequency 80 Hz, amplitude of pumping voltage 11 V. The dashed line corresponds to power law $P_\omega^2 \sim \omega^{-3.7}$ predicted by the theory for narrow-band pumping

by the lens on a Si photodiode (Hamamatsu s3590-08). Time variations of the signal from the photodiode were amplified by a SR570 amplifier and then digitized by a 24-bit analogue-to-digital converter Leonardo-II [12] with discretization frequency 102.4 kHz. Obviously, the amplified signal from the diode was proportional to the time variations of the reflected laser beam power $P(t)$. Signal $P(t)$ was recorded to the hard disk into the series of 16 files of approximately 5 seconds long each. From Fast Fourier Transform (FFT) of every data file, power spectrum $|P_\omega|^2$ was computed, and averaging was performed over all the 16 files. As it was shown in our previous works (see, for example, [8, 10]) the reflected laser beam power spectrum $|P_\omega|^2$ was proportional to the spectrum of the correlation function of surface elevation $I_\omega = |\eta_\omega|^2$, hence $|P_\omega|^2 \sim |\eta_\omega|^2$.

In the reported measurements two types of low-frequency driving force were used. In the first type experiments the surface was driven by a sinusoidal force with frequency $\omega_{\rm pump}$ that was equal to one of resonant frequencies of the cell. In the measurements of the second type driving by a noisy force was used. In this case the electrical noise was produced by a programmable noise generator.

Fig.1 shows the turbulent spectrum of capillary waves P_ω^2 obtained in our experiments when the surface was pumped by AC voltage at frequency 80 Hz and amplitude 11 V. Turbulent distribution P_ω^2 looks similar to that observed earlier in our experiments on the surface of liquid hydrogen [7] and in our first experiments on the surface of liquid ^{4}He [8]. The spectrum consists of equidistant harmonics: the first harmonic frequency corresponds to the pumping frequency, while other harmonics are generated due to the nonlinear interactions at frequencies multiple to the pumping frequency. In the inertial range the amplitudes of the harmonics depend on frequency as power law function $P_\omega^2 \sim \omega^{-m}$ with index $m \approx 3.7$ which is in accordance with the theoretical prediction for narrow-band pumping [13]. At frequencies of about 4 kHz the wave amplitudes were reduced significantly due to viscous dissipation and tended to the level of instrumental noise.

When the amplitude of pumping was decreased to 10 V, the shape of the turbulent spectrum was changed (Fig.2). The high-frequency edge of the turbulent cascade shifted towards low frequencies which was in accordance with our previous experiments [8, 14]. But a new phenomenon was observed, namely formation of a well pronounced local maximum in the wave amplitudes at frequencies of the order of high-frequency edge of turbulent cascade ω_b (shown by the dashed curve in Fig.2). With further reduction of the pumping amplitude, the local maximum shifted towards low frequencies. Eventually, at very low pumping amplitudes the spectrum consisted only of a few harmonics, and no local maximum was found.

Formation of a local maximum was seen in a number of experiments with harmonic pumping. The common feature observed in the experiments was that the local maximum was formed near the high-frequency edge

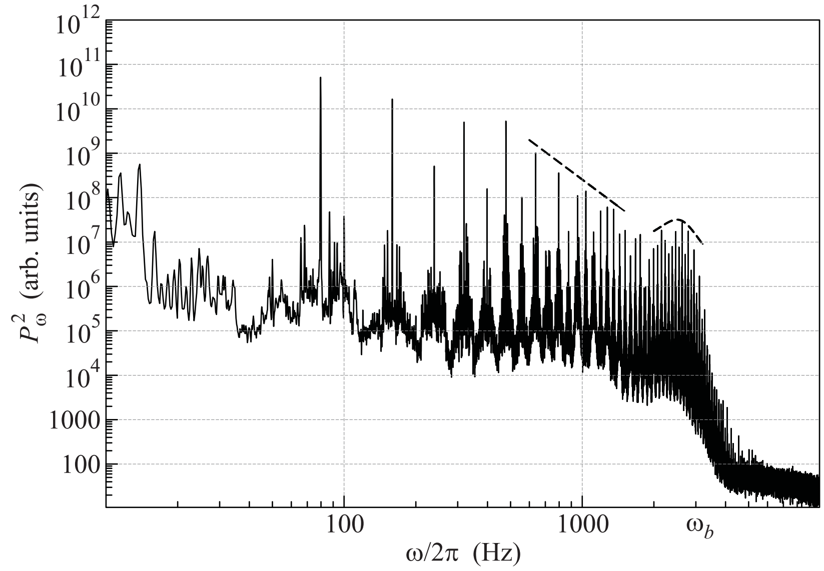

Fig.2. Turbulent spectrum after reduction of pumping amplitude. Frequency of pumping 80 Hz, amplitude of pumping voltage 10 V. At frequencies of about 2.5 kHz a local maximum is observed (emphasized by dashed curve). The dashed straight line corresponds to $P_\omega^2 \sim \omega^{-3.7}$

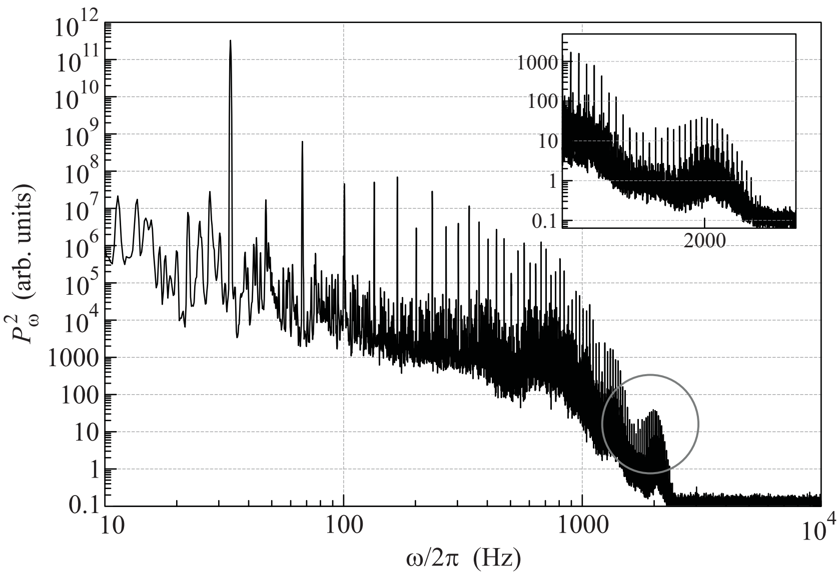

Fig.3. Turbulent spectrum in the case of pumping at $\omega_{\rm pump}/2\pi = 34$ Hz. Formation of a maximum at the spectrum is clearly seen on the main plot (circled) and in the inset on an enlarged scale

of the turbulent cascade although the shape of the local maximum and its exact position depended on the pumping frequency as well as on the pumping amplitude. For example, when the surface was pumped by the sinusoidal force at 34 Hz, the wave spectrum with a distinct local maximum was obtained (Fig.3). However, the local maximum was located in the dissipation region of turbulent distribution rather than in the inertial range as it is shown in Fig.2.

In the case of noisy pumping, we never observed a local maximum. To illustrate this, in Fig.4 we present a turbulent distribution obtained when the surface was pumped by a noisy signal in the frequency range 60–130 Hz. The amplitude of pumping force was chosen so that the high-frequency edge of inertial interval was located approximately at those frequencies at which it was seen in Figures 2,3. The spectrum differs dramatically from the spectra observed in the experiments with

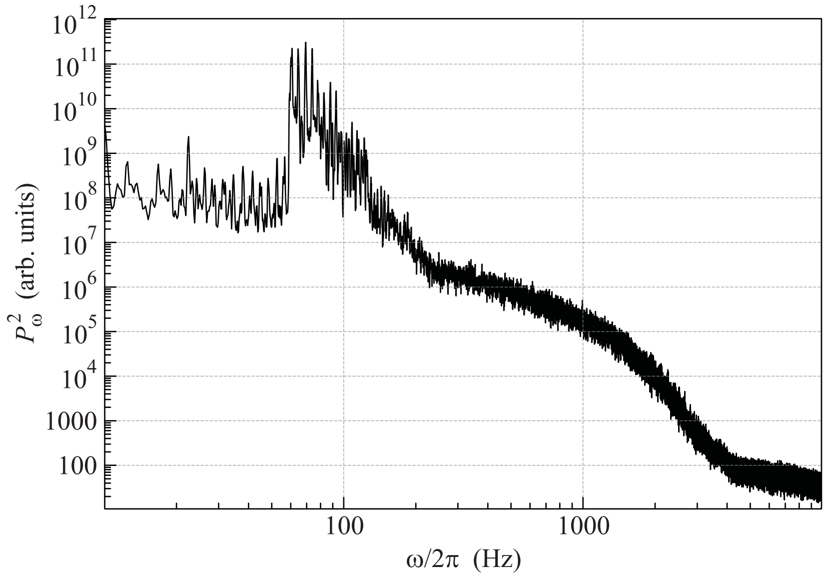

Fig.4. Turbulent spectrum of capillary waves in the case of noisy pumping at 60–130 Hz

harmonic pumping: it is continuous and the spectrum shape at high frequencies is more smooth.

The formation of the local maximum can be interpreted as wave energy accumulation at the high-frequency edge of the turbulent cascade where transition from the nonlinear wave energy transfer to viscous damping occurs. In the theoretical work [15] it is shown that viscous dissipation (which leads to finiteness of the inertial range) can in principle result in the energy accumulation due to the bottleneck effect. However, it is not clear how the dependence of formation of the local maximum on pumping amplitude observed in our measurements can be explained by the influence of viscous dissipation. On the other hand, it was pointed out by Kartashova [16, 17] that there are peculiarities in weakly nonlinear wave interactions in resonators due to discreteness of wavenumber space. Theory of discrete wave turbulence was developed later in a number of works, but of particular interest for us is the work [18] where the model of "frozen" turbulence was proposed. From this model it follows that, at small amplitudes of pumping, discreteness of wavenumber space can result in an oscillatory wave spectrum. We suppose that the local maximum is also caused by the discreteness of Fourier space. However, in the mentioned above work [18] it was considered the case of waves on the surface with square boundary, when wavenumber space is two-dimensional. Otherwise, in our experiments the boundary geometry is cylindrical, surface oscillations should be described by Bessel functions, and wavenumber space is one-dimensional. It is known that the form of basin boundary strongly influences on the density of resonant wavenumbers [17]. Therefore, we performed the following alternative estimations to understand the influence of the discreteness on spectrum in our experiments. We suppose that the phenomenon is caused by detuning of harmonics frequencies (shown in Figures 1,2,3) and the resonant frequencies of the cell. For surface waves in a cylindrical resonator with diameter D the eigenvalues of wavenumber k are the roots of boundary condition equation $J_1(kD/2) = 0$, where $J_1(x)$ is a Bessel function of the first order. When k is large, the asymptotic form of the Bessel function can be used, and the resonance wavenumbers are equidistant with step $\Delta k \approx 2\pi/D$. Hence, the distance between two neighbour resonance frequencies should grow with increase of frequency as

$$\Delta\omega = \frac{\partial\omega}{\partial k}\Delta k \approx \frac{\partial\omega}{\partial k}\frac{2\pi}{D} = \frac{3\pi}{D}\left(\frac{\sigma}{\rho}\right)^{1/3}\omega^{1/3}. \quad (1)$$

Thus, in the case of capillary waves the resonant frequencies are not equidistant which is in contrast to the frequencies of harmonics that are multiple to the pumping frequency ω_{pump} (it should be mentioned, that in our experiments $\Delta\omega < \omega_{\text{pump}}$).

Obviously, such detuning is important at frequencies where broadening of resonance $\delta\omega$ is smaller than the distance between two neighbour resonances $\Delta\omega$ (see Fig.5)

$$\delta\omega/\Delta\omega < 1. \quad (2)$$

Resonance broadening $\delta\omega$ can be presented as a sum of viscous broadening $\delta\omega_\nu$ and nonlinear broadening $\delta\omega_{nl}$,

$$\delta\omega = \delta\omega_\nu + \delta\omega_{nl}. \quad (3)$$

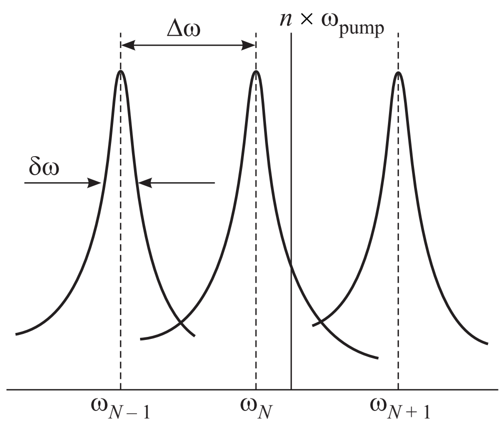

Fig.5. When broadening of resonance $\delta\omega$ is smaller than distance between two neighbour resonances $\Delta\omega$ some harmonics are out of the resonances of the surface oscillations in the experimental cell

Viscous resonance broadening corresponds to viscous damping [19]

$$\delta\omega_\nu = 4\nu k_\omega^2 = 4\nu\left(\frac{\rho}{\sigma}\right)^{2/3}\omega^{4/3}, \tag{4}$$

and relates to characteristic time of viscous dissipation τ_ν as $\delta\omega_\nu = \tau_\nu^{-1}$. Nonlinear resonance broadening corresponds to nonlinear transfer of wave energy and is determined from characteristic time of nonlinear interaction τ_{nl} as $\delta\omega_{nl} = \tau_{nl}^{-1}$. We suppose that for the case of harmonic pumping nonlinear time $\tau_{nl} \sim \omega^{1/6}$ [20] and consequently $\delta\omega_{nl} \sim \omega^{-1/6}$. With increase of the amplitude of pumping A the nonlinearity should increase, too, therefore it can be assumed that

$$\delta\omega_{nl} \approx \epsilon(A)\cdot\omega^{-1/6}, \tag{5}$$

where $\epsilon(A)$ is the increasing function of driving amplitude A.

Thus, by substituting (1), (3)–(5) in (2), we obtain that detuning is important at frequencies where

$$\frac{4\nu(\rho/\sigma)^{2/3}\omega^{4/3} + \epsilon(A)\cdot\omega^{-1/6}}{(3\pi/D)(\sigma/\rho)^{1/3}\omega^{1/3}} < 1. \tag{6}$$

At cut-off viscous frequency ω_b, where damping of the turbulent cascade is observed, the nonlinear energy transfer is changed by viscous dissipation. Therefore, it can be assumed that the characteristic viscous time is close to the characteristic nonlinear time at ω_b, $\tau_\nu(\omega_b) \approx \tau_{nl}(\omega_b)$. Thus, although function $\epsilon(A)$ is unknown, it can be estimated from the condition that at the cut-off frequency ω_b nonlinear broadening $\delta\omega_{nl} = \tau_{nl}^{-1}$ and viscous broadening $\delta\omega_\nu = \tau_\nu^{-1}$ are equal,

$$\delta\omega_\nu(\omega_b) \approx \delta\omega_{nl}(\omega_b). \tag{7}$$

From our estimations it follows that in our experiments condition (6) is satisfied in the frequency domain near cut-off viscous frequency ω_b (both below and above ω_b, see Fig.6). In this frequency domain the detuning

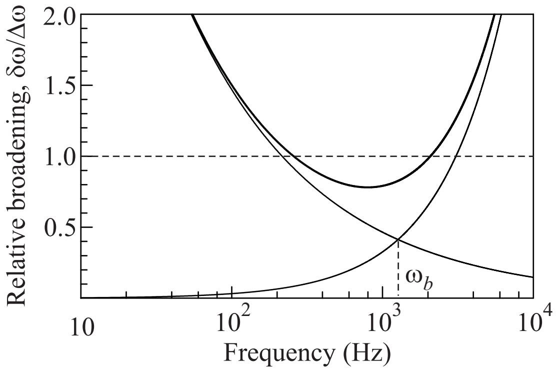

Fig.6. Results of the estimation of relative broadening $\delta\omega/\Delta\omega$. The ascending thin curve represents viscous relative broadening $\delta\omega_\nu/\Delta\omega$, the descending thin curve – nonlinear relative broadening $\delta\omega_{nl}/\Delta\omega$, the thick curve with minimum – total relative broadening $\delta\omega/\Delta\omega$

of the harmonic frequencies and the resonant frequencies becomes essential and determines specific shape of turbulent distribution P_ω^2. However, based upon simple qualitative reasoning it is impossible to predict exact shape and position of the local maximum. Thus, it is needed detailed theoretical studies and numerical simulations to understand mechanism of formation of the local maximum.

The model proposed implies that the following conditions should be satisfied for observation of energy accumulation in the case of capillary turbulence: low viscous broadening (low viscosity of liquid), low nonlinear broadening (harmonic pumping with a moderate amplitude), and large distance between the resonant frequencies (a relatively small size of the cell). In our experiments these conditions are fulfilled. It is interesting to note that the use of superfluid ^{4}He allowed us to register the energy accumulation in turbulent cascade due to its very low viscosity in the comparison with classical liquids.

We are grateful to L. P. Mezhov-Deglin, G. V. Kolmakov, E. A. Kuznetsov for fruitful discussions and A. V. Lokhov for experimental assistance. The work was supported by RFBR grants # 09-02-01146, # 07-02-00728 and the programs of the Presidium of RAS "Quantum physics of condensed matter" and "Fundamental problems of nonlinear dynamics".

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