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Superconducting triplet spin valve

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We study the critical temperature T_c of SFF trilayers (S is a singlet superconductor, F is a ferromagnetic metal), where the long-range triplet superconducting component is generated at noncollinear magnetizations of the F layers. We demonstrate that T_c can be a nonmonotonic function of the angle α between the magnetizations of the two F layers. The minimum is achieved at an intermediate α, lying between the parallel (P, $\alpha = 0$) and antiparallel (AP, $\alpha = \pi$) cases. This implies a possibility of a "triplet" spin-valve effect: at temperatures above the minimum T_c^{Tr} but below T_c^{P} and T_c^{AP}, the system is superconducting only in the vicinity of the collinear orientations. At certain parameters, we predict a reentrant $T_c(\alpha)$ behavior. At the same time, considering only the P and AP orientations, we find that both the "standard" ($T_c^{\mathrm{P}} < T_c^{\mathrm{AP}}$) and "inverse" ($T_c^{\mathrm{P}} > T_c^{\mathrm{AP}}$) switching effects are possible depending on parameters of the system.

In superconducting spin valves with the layer sequence F1/S/F2 the superconducting transition temperature T_c of the system can be controlled by mutual alignment of magnetizations $\mathbf{M}_{1,2}$ of the two ferromagnetic layers F1 and F2. Therefore, at a temperature T fixed inside the range of T_c variation, there is an opportunity for switching the superconductivity on and off by reversing the magnetization direction of the F1 or F2 layer. Model calculations have shown that the transition temperature T_c^{AP} for the antiparallel ($\mathbf{M}_1 \uparrow\downarrow \mathbf{M}_2$) orientation of the F1 and F2 magnetizations should be higher than the transition temperature T_c^{P} for the opposite case ($\mathbf{M}_1 \uparrow\uparrow \mathbf{M}_2$) [1 – 3]. The situation with this order of T_c's (i.e., $T_c^{\mathrm{P}} < T_c^{\mathrm{AP}}$) is commonly referred to as the "standard" switching (see, e.g., [4]), and the switching in this case actually occurs at temperatures T such that $T_c^{\mathrm{P}} < T < T_c^{\mathrm{AP}}$. The basic physical reason for the difference $\Delta T_c = T_c^{\mathrm{AP}} - T_c^{\mathrm{P}} > 0$ is partial compensation of the pair-breaking ferromagnetic exchange field, if the magnetizations of the F1 and F2 layers are aligned antiparallel.

Several experimental groups have published results on superconducting spin valves of the F1/S/F2 type [4 – 13]. The experimental results turned out to be controversial. Some studies of F1/S/F2 structures have shown the standard spin-valve effect [4 – 8] with the maximum shift $\Delta T_c \approx 41$ mK reported for the Ni/Nb/Ni trilayer in [7]. However, some experiments revealed the "inverse" spin-valve effect [9 – 11, 13] with $T_c^{\mathrm{P}} > T_c^{\mathrm{AP}}$ (i.e., $\Delta T_c < 0$). The most advanced calculations within the proximity effect theory, which take into account the triplet components of the superconducting pairing [14], demonstrate only the standard switching [3, 15] with T_c monotonically increasing from the P to AP configuration [3]. Additional physical mechanisms like spin imbalance effect [9, 11] or magnetic domain structure [10, 12] should be recruited to explain the inverse spin-valve effect in the studied F1/S/F2-type structures.

A bit earlier an unconventional spin-valve-like S/F1/F2 structure was theoretically proposed in [16] to control the superconducting T_c in the S layer by mutual alignment of the magnetizations of the two *adjacent* ferromagnetic layers F1 and F2. The authors of [16] argued that $T_c^{\mathrm{P}} < T_c^{\mathrm{AP}}$ in their system because of partial cancelation of the pair-breaking exchange fields just within the magnetic F1/F2 subsystem of the structure, thus predicting the standard switching as in the interleaved F1/S/F2 structure.

The S/F1/F2 structures are much less investigated experimentally [8, 17], and the experiments indicate the standard switching effect [16] with the maximal size of about 200 mK. In this Letter we study the critical temperature of a S/F1/F2 trilayer at arbitrary angle between the in-plane magnetizations of the ferromagnetic layers (see Fig.1). We demonstrate that this structure allows not only the standard but also inverse spin-switching effect. Moreover, we show for the first time that the minimal critical temperature T_c^{Tr} of the structure is achieved at a noncollinear alignment of the magnetizations, when the long-range triplet component of the superconducting pairing is generated. Since T_c^{Tr} is lower than both T_c^{P} and T_c^{AP}, this offers a possibility of a "triplet spin-valve effect" never reported before.

1. Model. We consider the S/F1/F2 structure in the dirty limit, which is described by the Usadel equations. Near T_c, the Usadel equations are linearized and contain only the anomalous Green function $\check f$ [14, 18]:

$$\frac{D}{2}\frac{d^2\check f}{dx^2} - |\omega|\check f - \frac{i\operatorname{sgn}\omega}{2}\left\{\hat\tau_0(\mathbf{h}\hat{\boldsymbol\sigma}),\check f\right\} + \Delta\hat\tau_1\hat\sigma_0 = 0. \quad (1)$$

Here, $\check f$ is a 4×4 matrix, $\hat\tau_i$ and $\hat\sigma_i$ are the Pauli matrices in the Nambu-Gor'kov and spin spaces, respectively, D is the diffusion constant, and $\omega = \pi T_c(2n+1)$ with integer n is the Matsubara frequency. The exchange field in the middle F1 layer is along the z direction, $\mathbf{h} = (0,\ 0,\ h)$, while the exchange field in the outer F2 layer is in the yz plane: $\mathbf{h} = (0,\ h\sin\alpha,\ h\cos\alpha)$. The angle α changes between 0 (parallel configuration, P) and π (antiparallel configuration, AP). The order parameter Δ is real-valued in the superconducting layer, while in the ferromagnetic layers it is zero. In general, the diffusion constant D acquires a proper subscript, S or F, when Eq. (1) is applied to the superconducting or ferromagnetic layers, respectively. However, for simplicity we take them equal in this paper, because this assumption does not influence qualitative behavior of $T_c(\alpha)$.

The Green function $\check f$ can be expanded into the following components:

$$\check f = \hat\tau_1\left(f_0\hat\sigma_0 + f_3\hat\sigma_3 + f_2\hat\sigma_2\right), \quad (2)$$

where f_0 is the singlet component, f_3 is the triplet with zero projection on the z axis, and f_2 is the triplet with ±1 projections on z (the latter is present only if $\alpha \neq 0,\pi$). The singlet component is even in frequency (and real-valued), while the triplet ones are odd (and imaginary): $f_0(-\omega) = f_0(\omega)$, $f_3(-\omega) = -f_3(\omega)$, and $f_2(-\omega) = -f_2(\omega)$, which makes it sufficient to consider only positive Matsubara frequencies, $\omega > 0$.

As we show below, the problem of calculating T_c can be reduced to an effective set of equations for the singlet component in the S layer: the set includes the self-consistency equation and the Usadel equation,

$$\Delta\ln\frac{T_{cS}}{T_c} = 2\pi T_c\sum_{\omega>0}\left(\frac{\Delta}{\omega} - f_0\right), \quad (3)$$

$$\frac{D}{2}\frac{d^2 f_0}{dx^2} - \omega f_0 + \Delta = 0, \quad (4)$$

with the boundary conditions

$$\left.\frac{df_0}{dx} = 0\right|_{x=-d_S}, \qquad \left.-\xi\frac{df_0}{dx} = Wf_0\right|_{x=0}. \quad (5)$$

Here T_{cS} and $\xi = \sqrt{D/2\pi T_{cS}}$ are the superconducting transition temperature and coherence length for an isolated S layer, and we assume that the S layer occupies the region $-d_S < x < 0$ (see Fig.1). This is exactly the problem for which the multi-mode solution procedure (as well as the fundamental-solution method) was developed in [19] and then applied to F1/S/F2 spin valves in [3]. We only need to determine the explicit expression for W in Eq. (5), solving the boundary problem for the S/F1/F2 structure.

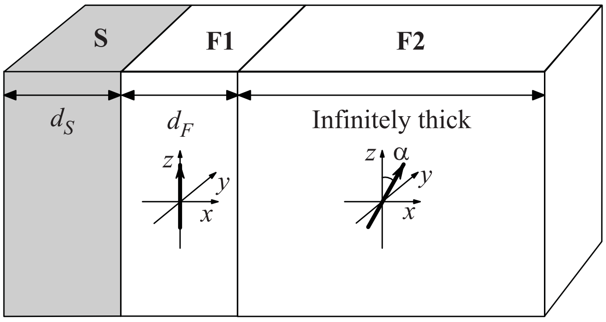

Fig.1. S/F1/F2 trilayer. The S/F1 interface corresponds to $x = 0$. The thick arrows in the F layers denote the exchange fields $\mathbf{h}$ lying in the (y,z) plane. The angle between the in-plane exchange fields is α

2. Solution of the model. To simplify derivations, while keeping the essential physics, we consider the middle ferromagnetic layer F1 of arbitrary thickness ($0 < x < d_F$) but the outer ferromagnetic layer F2 being semi-infinite ($d_F < x < \infty$). The Usadel equation (1) generates the following characteristic wave vectors:

$$k_\omega = \sqrt{\frac{2\omega}{D}}, \quad k_h = \sqrt{\frac{h}{D}}, \quad \tilde k_h = \sqrt{k_\omega^2 + 2ik_h^2}. \quad (6)$$

Only k_ω appears in the solution for the S layer, while the F-layers' solutions are described by k_ω, $\tilde k_h$, and $\tilde k_h^*$. Since the exchange energy is usually larger than the superconducting energy scale, $h \gg T_c$, the k_ω mode in the ferromagnetic layers (arising at noncollinear magnetizations) represents the *long-range* triplet component [14], which plays the key role in the present study.

In the S layer the solution of Eq. (1) is:

$$\begin{pmatrix} f_0(x)\\ f_3(x)\\ f_2(x)\end{pmatrix} = \begin{pmatrix} f_0(x)\\ 0\\ 0\end{pmatrix} + \begin{pmatrix} 0\\ A\\ B\end{pmatrix}\frac{\cosh\left(k_\omega(x+d_S)\right)}{\cosh\left(k_\omega d_S\right)}. \quad (7)$$

The singlet component $f_0(x)$ in the S layer cannot be written explicitly, since it is self-consistently related to the (unknown) order parameter $\Delta(x)$ by Eqs. (3), (4). Our strategy now is to obtain the effective boundary conditions (5) for $f_0(x)$, eliminating all other components in the three layers.

In the middle F1 layer the solution of Eq. (1) reads:

$$\begin{pmatrix} f_0(x) \\ f_3(x) \\ f_2(x) \end{pmatrix} = C_1 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cosh\left(k_\omega x\right) + S_1 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \sinh\left(k_\omega x\right) +$$

$$+ C_2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \cosh\left(\tilde{k}_h x\right) + C_3 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \cosh\left(\tilde{k}_h^* x\right) +$$

$$+ S_2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \sinh\left(\tilde{k}_h x\right) + S_3 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \sinh\left(\tilde{k}_h^* x\right). \quad (8)$$

Finally, the solution in the semi-infinite, outer F2 layer is built only from descending modes:

$$\begin{pmatrix} f_0(x) \\ f_3(x) \\ f_2(x) \end{pmatrix} = E_1 \begin{pmatrix} 0 \\ -\sin\alpha \\ \cos\alpha \end{pmatrix} \exp\left(-k_\omega (x - d_F)\right) +$$

$$+ E_2 \begin{pmatrix} 1 \\ \cos\alpha \\ \sin\alpha \end{pmatrix} \exp\left(-\tilde{k}_h (x - d_F)\right) +$$

$$+ E_3 \begin{pmatrix} -1 \\ \cos\alpha \\ \sin\alpha \end{pmatrix} \exp\left(-\tilde{k}_h^* (x - d_F)\right). \quad (9)$$

We will use the simplest, perfect-transparency boundary conditions at the S/F1 and F1/F2 interfaces (the case $\gamma = 1$ and $\gamma_B = 0$ in the notations of [20]):

$$f_i\big|_{\text{left}} = f_i\big|_{\text{right}}, \quad \left.\frac{df_i}{dx}\right|_{\text{left}} = \left.\frac{df_i}{dx}\right|_{\text{right}}. \quad (10)$$

Altogether there are 12 boundary conditions at the two interfaces (S/F1 and F1/F2). We are mainly interested in one of them, determining the derivative of the singlet component on the S side of the S/F1 interface ($x = 0$):

$$\left.\frac{df_0}{dx}\right|_{x=0} = 2\,\mathrm{Re}(\tilde{k}_h S_2). \quad (11)$$

The remaining 11 boundary conditions form a system of 11 linear equations for 11 coefficients entering Eqs. (7)-(9). The solution of this system is nonzero due to $f_0(0)$ coming from Eq. (7) and entering the "right-hand side" of the system. Finding the S_2 coefficient [which is proportional to $f_0(0)$], we substitute it into Eq. (11) and thus explicitly find W entering the effective boundary conditions (5).

3. Analysis of the solution. After reducing the problem to Eqs. (3)-(5), all the information about the two F layers is contained in the single real-valued function W. This function makes $f_0(x)$ bend at the S/F1 interface, hence the larger W, the stronger T_c is suppressed.

The explicit expression for $W(\alpha)$ is very cumbersome and we do not write it here. However, certain analytical development (as well as complete numerical analysis) is possible. For the analytical consideration, we make an additional assumption of $T_c \ll h$, which implies $k_\omega \ll k_h$. For the collinear cases ($\alpha = 0$ and $\alpha = \pi$) we then find $W(0) = 2k_h\xi$ and

$$W(0) - W(\pi) = 2k_h\xi \frac{\sqrt{2}\sin(2k_h d_F + \pi/4) - e^{-2k_h d_F}}{\sinh(2k_h d_F) + \cos(2k_h d_F)}, \quad (12)$$

which oscillates as a function of d_F, changing its sign (see Fig.2). Thus, as a result of interference in the middle F1 layer, we can either have the standard spin-switching effect with $T_c^{\mathrm{P}} < T_c^{\mathrm{AP}}$ [when the pair-breaking at the P alignment is stronger than at the AP alignment of magnetizations, i.e., at $W(0) - W(\pi) > 0$ as in the range $2k_h d_F < 3\pi/4$ in Fig.2] or the inverse spin-switching effect with $T_c^{\mathrm{P}} > T_c^{\mathrm{AP}}$ [at $W(0) - W(\pi) < 0$ as in the range $3\pi/4 < 2k_h d_F < 7\pi/4$ in Fig.2]. Note that the amplitude of the inverse effect is notably smaller

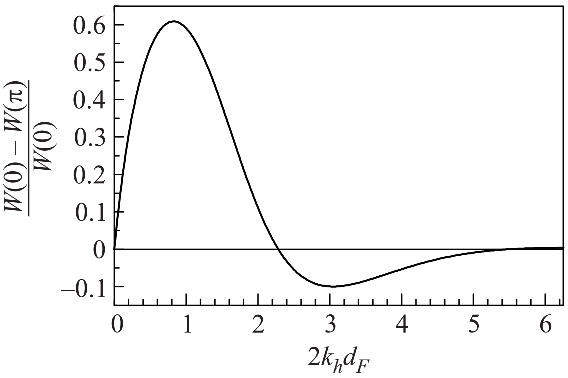

Fig.2. Dependence of $W(0) - W(\pi)$, Eq. (12), on the thickness d_F of the F1 layer. Positive values of this oscillating function correspond to stronger suppression of superconductivity at the P alignment (the standard switching effect), while negative values correspond to stronger suppression of superconductivity at the AP alignment (the inverse switching effect)

compared with the standard one. The analytical calculation of the second derivatives of $W(\alpha)$ at $\alpha = 0$ and π (the first ones are zero) shows that under the above assumption, both the collinear alignments represent local minima of $W(\alpha)$. This means that $T_c(\alpha)$ *decreases* as the configuration deviates from the P or AP alignment. Therefore, $T_c(\alpha)$ is nonmonotonic, and the minimal T_c must be achieved at some noncollinear configuration of magnetizations at $\alpha \neq 0, \pi$.

The analytical results obtained at $k_\omega \ll k_h$ are illustrated and extended by numerical calculations at arbitrary relation between k_ω and k_h. Figure 3 shows dependence of the transition temperature T_c on the angle α between the magnetizations. We see that at small thicknesses d_F of the middle ferromagnetic layer F1, the switching effect is standard, while at larger d_F the effect is inverse ($T_c^{\mathrm{P}} > T_c^{\mathrm{AP}}$). Moreover, when the F1 layer thickness is around a half of the coherence length ξ, the minimal critical temperature T_c^{Tr} at noncollinear orientations is significantly lower that both T_c^{P} and T_c^{AP} – this case corresponds to the triplet spin-valve effect. Note that depending on the parameters of the system, the minimum of $T_c(\alpha)$, predicted analytically, can shift to a close vicinity of either $\alpha = 0$ or $\alpha = \pi$, becoming shallow and indistinguishable.

Fig.4 demonstrates the possibility of reentrant $T_c(\alpha)$ dependence. In this situation the triplet spin-valve effect takes place even at $T = 0$.

4. Discussion. The physical interpretation of the triplet spin-valve effect can be given as follows: at the collinear configurations, both the singlet component f_0 and the zero-projection triplet component f_3 of the pairing function are short-ranged (with the characteristic penetration depth of the order of k_h^{-1}), so that at $k_h^{-1} \ll d_F$ the middle F1 layer plays a role of a shield separating the S layer from the ferromagnetic half-space F2. When the angle between magnetizations declines from the collinear configurations, the long-range triplet component f_2 of the pairing function is generated [14]. Then, the S layer becomes effectively coupled by this long-range triplet component to the semi-infinite ferromagnetic F2 layer. The pair-breaking in the S layer enhances, giving rise to more effective suppression of superconducting T_c. In other words, we can say that the T_c suppression is due to "leakage" of Cooper pairs into the ferromagnetic part. In this language, the generation of the long-range triplet component opens up an additional channel for this "leakage", hence T_c is suppressed stronger.

In order to supply the qualitative picture by more quantitative details, we can find the amplitudes of different components at the S/F1 interface in the limit of $k_\omega \ll k_h$ and large d_F. This can be done analytically from the boundary conditions which produce the linear system of equations for the coefficients entering Eqs. (7)-(9). We find that in the limit of $d_F \gg k_\omega^{-1}, k_h^{-1}$, the amplitudes of the long-range triplet components near the S/F1 interface (which are given by C_1, S_1, and B) are suppressed by the factor $e^{-k_\omega d_F - k_h d_F}$ which has a clear physical interpretation. The long-range components are generated from the short-range ones at the F1/F2 interface (i.e., at $x = d_F$), where electrons "feel" inhomogeneous magnetization. Therefore, the long-range contribution at the S/F1 interface is obtained as a result of a "wave" that goes from the S/F1 interface as a short-range component with the wave vector k_h and returns after reflection at the F1/F2 interface as a long-range component with the wave vector k_ω. At the same time, the self-consistency equation (3) that determines T_c, contains only the singlet short-range component. Therefore, the influence of the long-range components on T_c is indirect: the long-range components influence T_c only through their influence on the singlet component. We find that while the difference between W (that encodes the information about the suppression of T_c) for the AP and P cases is suppressed as $e^{-2k_h d_F}$ (the short-range components go from the S/F1 to F1/F2 interface and back), the changes in W due to noncollinear magnetizations contain the same exponential, $e^{-2k_h d_F}$. Of course, the influence of the long-range triplet components is contained in prefactors but no long-ranged exponential (with k_ω instead of k_h) appears in W, because W still originates from the short-range components.

In conclusion, we have considered a mesoscopic S/F1/F2 structure composed of a superconducting layer S, a ferromagnetic layer of arbitrary thickness F1, and a ferromagnetic half-space F2. We have demonstrated that the structure exhibits different relations between the critical temperatures in the parallel and antiparallel configuration: both the standard ($T_c^{\mathrm{P}} < T_c^{\mathrm{AP}}$) and inverse ($T_c^{\mathrm{P}} > T_c^{\mathrm{AP}}$) switching can be realized depending on the system's parameters. At the same time, our main result is that T_c^{Tr} at noncollinear magnetizations is lower than both T_c^{P} and T_c^{AP}, which makes this system a triplet spin valve. Possible experimental observation of a nonmonotonic (like curve "3" in Fig.3 or curves "1" and "3" in Fig.4) or even reentrant (like curves "2" and "4" in Fig.4) behavior of $T_c(\alpha)$ could be a signature of existence of the long-range triplet superconducting correlations [14] in SF hybrid structures.

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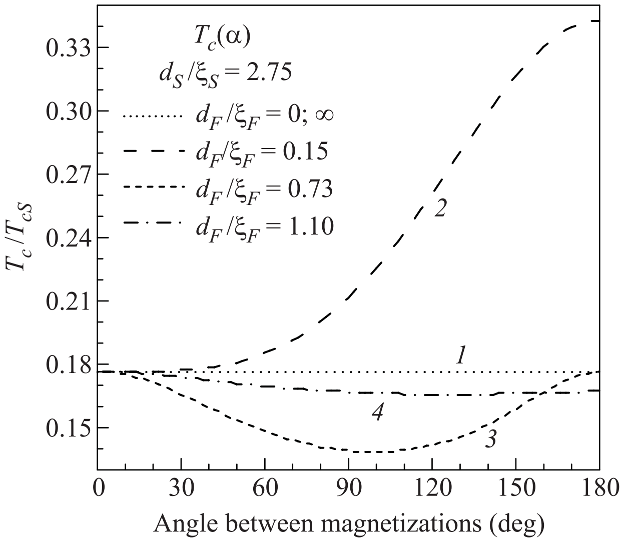

Fig.3. Critical temperature T_c vs. the misalignment angle α for various thicknesses of the F1 layer. We took $h/\pi T_{cS} = 6.8$; all other parameters are shown in the figure. In the cases $d_F = 0$ and $d_F = \infty$, which are physically equivalent (curve *1*), T_c does not depend on α. Curves *2* and *4* correspond to the standard and inverse switching effects, respectively. Curve *3* demonstrates the triplet spin-valve effect. The coherence lengths ξ_S and ξ_F were taken equal (denoted by ξ in the text) in order to present our main results in the simplest possible case. At $\alpha = 0$ all the curves coincide, since in this case the F part of the system is physically equivalent to a single half-infinite F layer

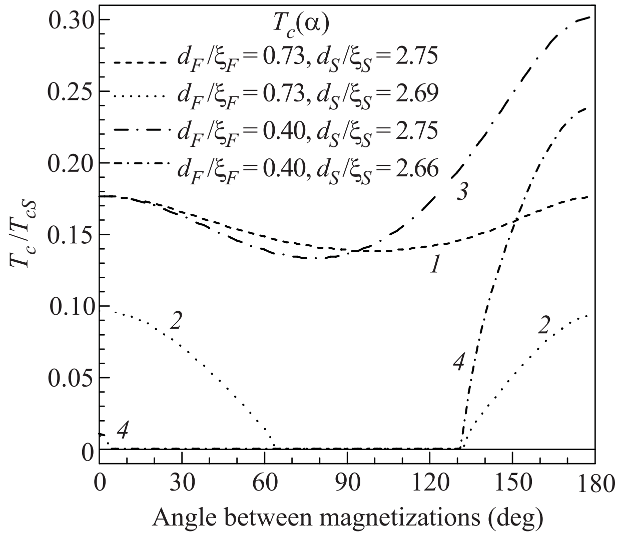

Fig.4. $T_c(\alpha)$ at various d_F and d_S. Curve *1* coincides with curve *3* in Fig.3. Curves *2* and *4* demonstrate the reentrant behavior, in which case the triplet spin-valve effect takes place even at $T = 0$

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