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Four-point vector correlators and AdS/QCD correspondence

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We derive the four-point vector correlators in QCD from AdS/QCD correspondence. It is shown that meson poles are correctly reproduced. The final expression also suggests a nonzero amplitude in the limit of zero virtuality of two longitudinal photons. This fact does not mean that one can produce, absorb or scatter real longitudinal photons.

I. Introduction. It is known that some strongly coupled gauge theories have a dual description. In the recent years several effective models of AdS/QCD duality were suggested. For example, a simple five-dimensional framework in a curved background introduced in [1] appears to be useful for obtaining low-energy quantities. Namely, in this model with only three free parameters one can reproduce lightest meson masses, their decay rates and couplings with a surprisingly good precision. The model is described in the next section.

In the present paper we derive the four-point vector current correlator in QCD in the strong coupling limit within AdS/QCD model introduced in [1].

We introduce an additional dilaton field [2] in order to reproduce Regge behavior of meson masses at large energies. It is shown that our result correctly reproduces meson spectrum at small and at large energies. The expression (8) suggests the final answer for the QCD four-point vector correlator.

We also calculate the four-point vector current correlator with two conserved and two longitudinal currents. The expression (11) shows that the resulting amplitude is finite when virtuality of longitudinal photons tends to zero. This naively suggests that there is a nonzero possibility to emit real longitudinal photons. In the contrary, gauge invariance strongly prohibits the possibility of producing, absorbing and scattering real longitudinal gauge particles in physical processes. Athought the processes involving virtual longitudinal gauge particles exist and give the essential contribution to the full cross-section, this does not explain the contradiction. An analogous problem was already discussed in [3, 4] in the context of processes in massless quantum electrodynamics. It was shown that such a contradiction has a physically sensible explanation. On the one hand, it happens that the amplitude in the limit of longitudinal photon zero virtuality is not smooth. This implies that one can not judge about real longitudinal photon interaction by this limit. On the other hand, although the discussed processes can in principle be observed in deep inelastic collisions, the smallest experimentally detectable virtuality p^2 is of order E/L (E is photon energy and L is the characteristic apparatus size).

An additional Chern-Simons term can be introduced to the theory [5 – 7]. In the case of vector correlator its contribution is zero as it has the form $S_{\mathrm{CS}}[A_{\mathrm{L}}] - S_{\mathrm{CS}}[A_{\mathrm{R}}]$ with $A_{\mathrm{L}} = A_{\mathrm{R}} = V$.

We also derive some particular kinematic limits and show that the result has a symmetric form. Namely, if two external momenta are on-shell, the resulting four-point function is symmetric with respect to the exchange of any other two squared momenta and the s Mandelstamm variable.

In the next section we briefly discuss the model. In the section III the general form of four-point vector correlator is given. In the section IV we consider some particular limits, and the conclusions in the section V complete the paper.

II. The model. The simple holographic five-dimensional model of QCD suggested in [1] describes dual dynamics of left- and right-handed currents corresponding to the $\mathrm{SU_L}(N_f) \times \mathrm{SU_R}(N_f)$ chiral flavor symmetry of QCD. The 5D action

$$S = \int d^4x dz e^{-\Phi} \sqrt{g} \mathrm{Tr} \left[|\mathcal{D}X|^2 + 3|X|^2 - \frac{1}{4g_5^2} \left(F_{\mathrm{L}}^2 + F_{\mathrm{R}}^2 \right) \right] \tag{1}$$

($\mathcal{D}_\mu X = \partial_\mu X - iA_{\mathrm{L}\mu} X + iX A_{\mathrm{R}\mu}$, $g_5 = 12\pi^2/N_f$), written in the anti-de Sitter metric,

$$ds^2 = \frac{1}{z^2} \left(-dz^2 + dx^\mu dx_\mu \right), \tag{2}$$

is defined for three fields. These are the scalar X^a in bi-fundamental representation of $\mathrm{SU_L}(N_f) \times \mathrm{SU_R}(N_f)$ and left and right gauge vector fields $A^a_{\mathrm{L}\mu}$, $A^a_{\mathrm{R}\mu}$ (corresponding to chiral left- and right-handed currents). In these terms, the four-dimensional QCD theory lies on AdS boundary. It is more convenient for us to use vector V_μ^a and axial A_μ^a gauge fields defined by $A_{\rm L} = V + A$, $A_{\rm R} = V - A$. They describe vector and axial currents in 4D theory. See [1] for details.

The model at hand must include in some way a parameter of dimension of mass responsible to chiral symmetry breaking. It is its presence that allows one to reproduce low energy physics. We consider two simplest ways to introduce this parameter. The hard-wall model [1] corresponds to the special case when the dilaton field Φ is zero and the AdS_5 space is cut at some "infrared" point $z = z_{\rm IR}$ ($z_{\rm IR} = 1/323$ MeV). The second case corresponds to the so called soft-wall model when the external dilaton field $\Phi = \Lambda^2 z^2$ ($\Lambda \sim 300$ MeV) gives the scale parameter for the theory. It provides the correct meson Regge trajectories [2]. We derive all expressions in the general way so that they can be applied for both models. Some of their differences are compared in section IV.

Throughout this paper the gauge $A_{{\rm L}z} = A_{{\rm R}z} = 0$ is used and μ, ν, α, ... stand for 4-dimensional indices.

Finally, it is necessary to impose boundary conditions for the fields at hand. We put $A_{{\rm L}\mu} = A_{{\rm R}\mu} = 0$ at $z = 0$ and appropriate boundary conditions at large z. For the hard-wall model we take $\partial_z F_{\rm L}(z = z_{\rm IR}) = \partial_z F_{\rm R}(z = z_{\rm IR}) = 0$; for the soft-wall model one should take the natural boundary conditions at $z = \infty$ to make the action finite. See below about the X field boundary conditions.

Let us write the classical quadratic equations of motion for left and right vector fields ($A_{\rm L}$ or $A_{\rm R}$):

$$\left(\partial_z \frac{e^{-\Phi(z)}}{z}\partial_z - \frac{e^{-\Phi(z)}}{z}\partial^2\right) A_\nu^a + \frac{e^{-\Phi(z)}}{z}\partial_\nu\partial_\mu A_\mu^a = 0. \tag{3}$$

In this way, using the appropriate boundary conditions, the solution to the equation (3) correctly reproduces meson spectrum. (Whereas the hard-wall model reproduces correctly only low-lying meson spectrum.) Namely, we introduce vector field propagator (in the 4D momentum representation)

$$G_{\mu\nu}(k, z, z') = \\ = -\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) G_{k^2}(z, z') + \frac{k_\mu k_\nu}{k^2} G_0(z, z'), \tag{4}$$

$$\left[\partial_z \frac{e^{-\Phi(z)}}{z}\partial_z + k^2 \frac{e^{-\Phi(z)}}{z}\right] G_{k^2}(z, z') = i\delta(z - z'). \tag{5}$$

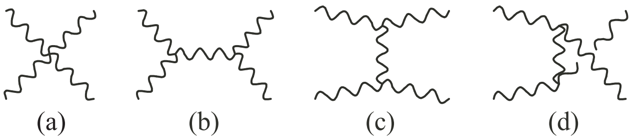
Here k_μ is a 4D momentum. Note that longitudinal part of $G_{\mu\nu}(k, z, z')$ does not depend on k^2. The k^2 poles of $G_{k^2}(z, z')$ correspond to meson masses in this holographic model and are in good agreement with experimental data.

The vacuum in this theory corresponds to $A_{\rm L} = A_{\rm R} = 0$. Let us tell some words about the X field boundary conditions [1]. In the model without a dilaton field the solution on a vacuum X state is

$$X = \frac{1}{2}Mz + \frac{1}{2}\Sigma z^3, \tag{6}$$

where the matrix M corresponds to the quark mass matrix and Σ can be viewed as a quark condensate, $\Sigma^{\alpha\beta} = \langle \bar{q}^\alpha q^\beta \rangle$. In the model with a dilaton field the expression (6) must be considered as a boundary condition at UV (small z). We choose the simplest possible case when both matrices are proportional to unit matrix, $M = m_q\mathbf{1}$, $\Sigma = \sigma\mathbf{1}$. In this way, the X field itself is proportional to the unit matrix and *drops out from our computations.*

As the AdS/QCD duality suggests, connected Green's functions in QCD can be generated by differentiating the classical five-dimensional action (1) with respect to the sources (UV boundary conditions on $A_{\rm L}(z, x)$ and $A_{\rm R}(z, x)$). The classical five-dimensional action can be easily derived by summing the tree Feynman diagrams. Particularly, to obtain the four-point vector Green's function we need to sum over four graphs (see Figure). In order to make the expression finite



Four-point Feynman graphs as viewed from a 5D theory. Free legs correspond to "bulk-to-boundary propagator"

at the UV boundary (small z) we introduce a cutoff $z = \epsilon \to 0$ and follow the renormalization procedure [8]. Note again that the field X does not interact with the vector field V_μ^a as their interaction refers to the commutator $[V_\mu, X]$ with the vacuum field X which is proportional to the unit matrix. In the derivation of four-point function of axial vector fields one needs to include an extra diagram representing an intermediate X field particle exchange.

III. Vector four-point functions. A. *Conserved currents four-point function.* Let us discuss the first vector four-point function – the holographic dual of correlator of four conserved vector currents in 4D theory, i.e. when there are no sources for the axial field $A_\mu^a(x, z)$ and the source for the vector field is $V_\mu^a(x, \epsilon) = v_\mu^a(x)$,

$\partial_\mu v^a_\mu(x) = 0$. The answer can be easily obtained by summarizing the Feynman graphs (Figure):

$$S_5(\perp,\perp,\perp,\perp) = \frac{1}{4!} w^{abcd}_{\alpha\beta\gamma\delta}(\perp,\perp,\perp,\perp)\; v^a_{1\alpha} v^b_{2\beta} v^c_{3\gamma} v^d_{4\delta}, \tag{7}$$

where $v^a_{i\alpha} \equiv v^a_\alpha(k_i)$ is a Fourier transformation of $v^a_\mu(x)$, $k_\mu v^a_\mu(k) = 0$. It is not difficult to understand the meaning of $w^{abcd}_{\alpha\beta\gamma\delta}$. As it is already mentioned, the differentiation of (7) with respect to the sources $v^a_\mu(k)$ gives the result of calculation of 4D vector four-point function from its 5-dimensional perspective. The explicit expression is

$$\begin{aligned} w^{abcd}_{\alpha\beta\gamma\delta}(\perp,\perp,\perp,\perp) = \frac{1}{g_5^2}\Big\{ &-\left[\int \frac{dz}{z} e^{-\Phi(z)} V_1(z)V_2(z)V_3(z)V_4(z)\right] \times \\ \times \Big[f^{abe} f^{cde}\left(g^{\alpha\gamma}g^{\beta\delta} - g^{\alpha\delta}g^{\beta\gamma}\right) &+ f^{ace} f^{bde}\left(g^{\alpha\beta}g^{\gamma\delta} - g^{\alpha\delta}g^{\beta\gamma}\right) + f^{ade} f^{bce}\left(g^{\alpha\beta}g^{\gamma\delta} - g^{\alpha\gamma}g^{\beta\delta}\right)\Big] + \\ &+ 4i\left[\int \frac{dz}{z} e^{-\Phi(z)} \frac{dz'}{z'} e^{-\Phi(z')} V_1(z)V_2(z) G_{(k_1+k_2)^2}(z,z')\, V_3(z')V_4(z')\right] f^{abe} f^{ecd} \times \\ &\times \Big[g^{\alpha\gamma}k_1^\beta k_3^\delta - g^{\beta\gamma}k_2^\alpha k_3^\delta + g^{\beta\delta}k_2^\alpha k_4^\gamma - g^{\alpha\delta}k_1^\beta k_4^\gamma + g^{\alpha\beta}\left(k_1^\gamma k_2^\delta - k_1^\delta k_2^\gamma\right) + \\ &+ g^{\gamma\delta}\left(k_3^\alpha k_4^\beta - k_3^\beta k_4^\alpha\right) + \frac{1}{4} g^{\alpha\beta}g^{\gamma\delta}(k_1-k_2)^\perp (k_3-k_4)^\perp \Big] + \\ &+ i\left[\int \frac{dz}{z} e^{-\Phi(z)} \frac{dz'}{z'} e^{-\Phi(z')} V_1(z)V_2(z) G_0(z,z')\, V_3(z')V_4(z')\right] f^{abe} f^{ecd} g^{\alpha\beta} g^{\gamma\delta} \frac{\left(k_1^2 - k_2^2\right)\left(k_3^2 - k_4^2\right)}{\left(k_1+k_2\right)^2} + \\ &+ \Big[\text{two transpositions}\Big]\Big\}. \end{aligned} \tag{8}$$

Here f^{abc} – structure constants of $\mathrm{SU}(N_f)$ and $V_i(z) \equiv V(k_i^2, z)$ are the "bulk-to-boundary propagators" which satisfy (5) with zero right hand side and subject to the boundary conditions $V(k^2,\epsilon) = 1$ (and appropriate zero conditions at $z = z_{\mathrm{IR}}$ or $z = \infty$).

The transverse part of the last term in the fifth line is taken with respect to $k_1 + k_2 = -k_3 - k_4$, i.e.

$$\begin{aligned} &(k_1 - k_2)^\perp (k_3 - k_4)^\perp = \\ &= (k_1 - k_2)(k_3 - k_4) + \frac{\left(k_1^2 - k_2^2\right)\left(k_3^2 - k_4^2\right)}{\left(k_1 + k_2\right)^2}. \end{aligned} \tag{9}$$

Let us describe the meaning of these terms. The first term is obtained just from the 4-vertex interaction of gauge fields (the graph **a** on Fiure), the second is the interaction via the virtual transverse vector particle and the third one is obtained from the interaction with longitudinal vector particle. The three terms of (8) are actually obtained from the diagrams **a** and **b** on Figure, and the answer needs to be symmetrized with respect to the permutations of V in the second and third terms. That is why we sum over two transpositions in (8) (diagrams **c** and **d** on Figure).

The integral in the square brackets in the first term of (8) has a UV divergency when $\epsilon \to 0$. Indeed, at small z $\Phi(z) \approx 0$, $V_i(z) \approx 1$ and we have $\int_\epsilon \frac{dz}{z} \sim \ln\epsilon$. This UV divergency can be eliminated by adding a boundary conterterm to the action [8]. In the particular case this means that replacing $\epsilon \to \mu$, where μ is the UV regulator, is sufficient. The second and the third terms in (8) are finite, and after renormalization the whole answer is UV finite.

B. *Two conserved and two longitudinal currents.* In the same way one can obtain the expression for the four-point vector current correlator in the case of two conserved and two longitudinal currents. This means that we consider the third, nonphysical polarization of $v^a_\mu(k)$ (the third solution of $k_\mu v^a_\mu(k) = 0$) and take the limit $k^2 \to 0$. Thus, the expression written below can be obtained from (8) by taking $v^a_\mu(k_3)$ and $v^a_\mu(k_4)$ as the third polarizations and taking the limit $k^2_{3,4} \to 0$.

We denote conserved currents $v^a_\mu(k)$ like in the previous subsection, $V^a_\mu(k,\epsilon) = v^a_\mu(k)$. For longitudinal currents $V^a_\mu(k,\epsilon) = k_\mu v^a(k)$. The corresponding five-dimensional holographic action can be written as follows

$$S_5(\perp,\perp,\|,\|) = \frac{1}{4} w^{abcd}_{\alpha\beta}(\perp,\perp,\|,\|)\; v^a_{1\alpha} v^b_{2\beta} v^c_3 v^d_4. \tag{10}$$

As in the previous subsection, $w^{abcd}_{\alpha\beta}$ represents the holographic dual of four-point vector correlator in QCD,

$$w^{abcd}_{\alpha\beta}(\perp,\perp,\|,\|)=\frac{1}{g_5^2}\left\{-\left[\int\frac{dz}{z}e^{-\Phi(z)}V_1(z)V_2(z)\right]k_3^\alpha k_4^\beta\times\right.$$
$$\times\left[f^{abe}f^{cde}\left(g^{\alpha\gamma}g^{\beta\delta}-g^{\alpha\delta}g^{\beta\gamma}\right)+f^{ace}f^{bde}\left(g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\delta}g^{\beta\gamma}\right)+f^{ade}f^{bce}\left(g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\gamma}g^{\beta\delta}\right)\right]+$$
$$+2i\left[\int\frac{dz}{z}e^{-\Phi(z)}\frac{dz'}{z'}e^{-\Phi(z')}V_1(z)V_2(z)\,G_{(k_1+k_2)^2}\left(z,z'\right)\right]f^{abe}f^{ecd}\times$$
$$\times\left(k_1+k_2\right)^2\left[k_3^\alpha k_4^\beta-k_3^\beta k_4^\alpha+\frac{1}{4}g^{\alpha\beta}\left(k_1-k_2\right)^\perp\left(k_3-k_4\right)^\perp\right]+$$
$$+i\left[\int\frac{dz}{z}e^{-\Phi(z)}\frac{dz'}{z'}e^{-\Phi(z')}V_1(z)G_{(k_1+k_3)^2}\left(z,z'\right)V_2(z')\right]f^{ace}f^{edb}\times$$
$$\times\left(g^{\alpha\beta}-\frac{k_3^\alpha k_4^\beta}{\left(k_1+k_3\right)^2}\right)\left[\left(k_1+k_3\right)^2-k_1^2\right]\left[\left(k_1+k_3\right)^2-k_2^2\right]+$$
$$+i\left[\int\frac{dz}{z}e^{-\Phi(z)}\frac{dz'}{z'}e^{-\Phi(z')}V_1(z)G_0\left(z,z'\right)V_2(z')\right]f^{ace}f^{edb}\cdot k_3^\alpha k_4^\beta\frac{k_1^2k_2^2}{\left(k_1+k_3\right)^2}+$$
$$\left.+\Big[\text{one transposition }3\leftrightarrow 4\Big]\right\}. \tag{11}$$

One should remember that "bulk-to-boundary" propagator for longitudinal particle is equal to unity: $V_\|(k^2,z)=1$. This is the reason why only two V are present in z integrals in (11). Thus, the first term is obtained from the graph **a** on Figure, the second term represents the graph **b**. The third and fourth terms are obtained from the graph **c**. The transposition of k_3 and k_4 gives the graph **d**. Note that longitudinal intermediate particle also contribute to the final expression.

Thereby, formulas (8) and (11) represent the holographic QCD predictions for the four-point vector currents correlators. Apart from color and Lorentz structure they contain nontrivial external momenta dependencies represented as integrals over the fifth coordinate in the AdS space. It is instructive to analyze these dependencies in some particular cases. For both formulas (8) and (11) we consider two cases: all external photons are real, two of them are virtual.

IV. Particular limits. The z-dependent integrals in (8) and (11) represent nontrivial, non-perturbative contributions to the correlators. Apparently, they are expressed as one-dimensional tree Feynman diagrams. There are two types of such integrals: contact integrals (the first lines in (8) and (11)) and terms representing one-particle exchange. Contact terms diverge and should be regularized [8]. One-particle exchange terms are finite. It is possible to calculate some particular limits of one-particle exchange terms in hard-wall and soft-wall models explicitly. Namely, the processes involving two real (or nearly real) external photons, which have unity "bulk-to-boundary" propagators $V=1$.

A. *All four photons are real.* The most simple limit is to put all the external momenta on shell: $k_i^2=0$. Note, in this case the terms in Eqs. (8) and (11), which represent the interaction of particles via the intermediate longitudinal particle vanish. The next simplification comes from the fact that all $V_i(z)=1$. This means that there is only one type of nontrivial z-integral:

$$I_s=-i\int\frac{dz}{z}e^{-\Phi(z)}\frac{dz'}{z'}e^{-\Phi(z')}G_s\left(z,z'\right). \tag{12}$$

This integral can be calculated precisely in both hard-wall and soft-wall models. To do so, let us introduce the function

$$K_s(z)=-i\int\frac{dz'}{z'}e^{-\Phi(z')}G_s\left(z,z'\right), \tag{13}$$

which satisfy the equation

$$\left(ze^{\Phi(z)}\partial_z\frac{e^{-\Phi(z)}}{z}\partial_z+s\right)K_s(z)=1. \tag{14}$$

For the soft-wall after changing the variables $t=\Lambda^2z^2$ and shifting K_s Eq. (14) turns into Kummer's equation with the parameters $a=-s/4\Lambda^2$, $b=0$. For the hard-wall model the solution of Eq. (14) is just Bessel functions. For any model we write

$$K_s(z)=\frac{1}{s}\left\{1-V(s,z)\right\}, \tag{15}$$

where $V(s,z)$ coincide with $V(k^2,z)$ defined in (8) and (11). For the hard-wall and soft-wall models respectively we obtain

$$V^{\rm hw}(s,z) = = \frac{z}{\epsilon}\frac{J_1(\sqrt{s}z)\,Y_0(\sqrt{s}z_{\rm IR}) - J_0(\sqrt{s}z_{\rm IR})\,Y_1(\sqrt{s}z)}{J_1(\sqrt{s}\epsilon)\,Y_0(\sqrt{s}z_{\rm IR}) - J_0(\sqrt{s}z_{\rm IR})\,Y_1(\sqrt{s}\epsilon)},$$

$$V^{\rm sw}(s,z) = \frac{U\left(-\frac{s}{4\Lambda^2}, 0, \Lambda^2 z^2\right)}{U\left(-\frac{s}{4\Lambda^2}, 0, \Lambda^2\epsilon^2\right)}, \tag{16}$$

where $J_n(z)$, $Y_n(z)$ – Bessel functions, $U(a,b,z)$ – the confluent hypergeometric function U.

Performing integration in (12) gives for the hard-wall model

$$I_s^{\rm hw} = \int \frac{dz}{z} e^{-\Phi(z)} K_s(z) = = \frac{1}{2s}\left\{\ln\frac{s z_{\rm IR}^2}{4} + 2\gamma - \frac{\pi Y_0(\sqrt{s}z_{\rm IR})}{J_0(\sqrt{s}z_{\rm IR})}\right\} \tag{17}$$

(γ is Euler's constant). At large s $I_s^{\rm hw}$ behaves as

$$I_s^{\rm hw} \approx \frac{1}{2s}\left\{\ln\frac{s z_{\rm IR}^2}{4} + 2\gamma - \pi + \frac{2\pi}{1+\tan(\sqrt{s}z_{\rm IR})}\right\}. \tag{18}$$

The structure of $I_s^{\rm hw}$ at large s is particularly remarkable. Apart from the logarithm and the constant term in brackets it contains all the "mass" poles $s = m^2 \sim n^2$.

The analogous expression can be written for the soft-wall model,

$$I_s^{\rm sw} = \int \frac{dz}{z} e^{-\Phi(z)} K_s(z) = \frac{H_{-s/4\Lambda^2}}{2s}, \tag{19}$$

where harmonic number function for integer n is defined as $H_n = \sum_{k=1}^n \frac{1}{k}$. As $s \to \infty$

$$I_s^{\rm sw} \approx \frac{1}{2s}\left(\ln\frac{s}{4\Lambda^2} + \gamma + \pi\cot\frac{\pi s}{4\Lambda^2}\right), \tag{20}$$

and soft-wall model correctly reproduces Regge poles $s = m^2 \sim n$. As we see, the pole structure is different in each model, but large s dependence remains the same. This means that one can calculate large s dependence in "any-wall" model (with any z dependence in IR region) and it will coincide with hard/soft-wall model results, but the pole structure may be different.

It could be also useful to calculate (12) for slightly virtual particles, but computation gives the same $\ln s$ dependence, where s can be any k^2.

B. *Only two photons are real.* In the limit $k_1^2 = x$, $k_2^2 = y$, $k_3^2 = k_4^2 = 0$ it is not possible to compute all the z integrals in (8) and (11) even in the large s limit. But one can take the integral of type

$$I_s(x,y) = -i\int \frac{dz}{z} e^{-\Phi(z)} \frac{dz'}{z'} e^{-\Phi(z')} V(x,z) G_s(z,z') V(y,z'). \tag{21}$$

It can be computed in both models. To do so, let us introduce the function

$$K_{s,y}^{(V)}(z) = -i\int \frac{dz'}{z'} e^{-\Phi(z')} G_s(z,z') V(y,z'), \tag{22}$$

which satisfy the Eq. (14) with $V(y,z)$ at right hand side instead of 1. The solution for the equation

$$K_{s,y}^{(V)}(z) = \frac{s}{s-y} K_s(z) + \frac{y}{y-s} K_y(z) \tag{23}$$

is a linear combination of $K(z)$ defined in (13). Now, the integral (21) can be computed:

$$I_s(x,y) = \int \frac{dz}{z} e^{-\Phi(z)} V(x,z) K_{s,y}^{(V)}(z) = = -i\frac{s}{s-y}\int \frac{dz}{z} e^{-\Phi(z)} \frac{dz'}{z'} e^{-\Phi(z')} V(x,z) G_s(z,z') - -i\frac{y}{y-s}\int \frac{dz}{z} e^{-\Phi(z)} \frac{dz'}{z'} e^{-\Phi(z')} V(x,z) G_y(z,z'). \tag{24}$$

Repeating again the trick in Eqs. (22), (23) we obtain

$$I_s(x,y) = \frac{s^2}{(s-x)(s-y)} I_s + + \frac{x^2}{(x-s)(x-y)} I_x + \frac{y^2}{(y-s)(y-x)} I_y. \tag{25}$$

It is worth mentioning that the last expression is symmetric in s, x and y.

V. Conclusions. In the present paper we made the predictions for the strong coupling limit of four-point vector correlators in QCD from AdS/QCD. The final expressions are represented by formulas (8) and (11). The answer includes the nontrivial external momenta dependent factors, which are represented as integrals over the fifth coordinate. We have also computed some particular on-shell and large-energy limits and found that these integral factors correctly reproduce meson spectrum at small and at large energies.

The nonzero expression (11) suggests naively a nonzero possibility of emitting real longitudinal photons. This is not the case [3, 4]. The 5-dimensional gauge invariance prohibits this. The explanation of the paradox is related to the fact that the limit of longitudinal photon zero virtuality is not smooth. The virtual longitudinal photon processes contribute to the full amplitude and cross-section. Such processes in principle could be detected in the experiment of deep inelastic scattering but the smallest observable photon virtuality is of order E/L with L being the typical apparatus size.

The $\log s$ dependence does not coincide with what was obtained in [10] for planar $\mathcal{N}=2$ super Yang-Mills theory.

It will be interesting to make the similar derivations for the axial four-point correlators as it requires including the contribution of the axial current interaction with the scalar field. Moreover, the Chern-Simons term may also give contribution. See also recent papers [9, 11 – 14] on the subject.

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