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Statefinder – a new geometrical diagnostic of dark energy

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We introduce a new cosmological diagnostic pair $\{r, s\}$ called Statefinder. The Statefinder is a geometrical diagnostic and allows us to characterize the properties of dark energy in a model independent manner. The Statefinder is dimensionless and is constructed from the scale factor of the Universe and its time derivatives only. The parameter r forms the next step in the hierarchy of geometrical cosmological parameters after the Hubble parameter H and the deceleration parameter q, while s is a linear combination of q and r chosen in such a way that it does not depend upon the dark energy density. The Statefinder pair $\{r, s\}$ is algebraically related to the equation of state of dark energy and its first time derivative. The Statefinder pair is calculated for a number of existing models of dark energy having both constant and variable w. For the case of a cosmological constant the Statefinder acquires a particularly simple form. We demonstrate that the Statefinder diagnostic can effectively differentiate between different forms of dark energy. We also show that the mean Statefinder pair can be determined to very high accuracy from a SNAP-type experiment.

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Recent observations of type Ia supernovae indicate that the expansion of the Universe is accelerating rather than slowing down [1]. These results, when combined with cosmic microwave background (CMB) observations of a peak in the angular power spectrum on degree scales [2, 3], strongly suggest that the Universe is spatially flat with $\sim 1/3$ of the critical energy density being in non-relativistic matter and $\sim 2/3$ in a smooth component with large negative pressure ("dark energy"). Indirect support for dark energy (known long ago) comes from the examination of gravitational clustering within the framework of the standard gravitational instability scenario (see the reviews [4, 5]). Finally, with recent data on the galaxy power spectrum from 2dF Galaxy Survey combined with CMB data, the existence of dark energy can be proved without using the supernovae data at all [6]. A large body of recent work has focussed on understanding the nature of dark energy and its possible relation to a fundamental theory of matter such as M-theory, supergravity etc. Despite the considerable effort in this direction, both the nature of dark energy as well as its cosmological origin remain enigmatic at present.

The simplest model for dark energy is a cosmological constant Λ, whose energy density remains constant with time $\varepsilon = \Lambda/8\pi G$ and whose effective equation of state remains fixed, $w \equiv P/\varepsilon = -1$ (P is the pressure) as the Universe evolves. The cold dark matter (CDM) model with the cosmological constant having the corresponding mass density

$$\rho_\Lambda = \frac{\varepsilon_\Lambda}{c^2} = 6.44 \cdot 10^{-30} \left(\frac{\Omega_\Lambda}{0.7}\right) \left(\frac{h}{0.7}\right)^2 \mathrm{g} \cdot \mathrm{cm}^{-3} , \quad (1)$$

where h is the Hubble constant H_0 in terms of $100\,\mathrm{km{\cdot}s^{-1}{\cdot}Mpc^{-1}}$ and $\Omega_\Lambda = 0.7 \pm 0.1$, $h = 0.7 \pm 0.1$, provides an excellent explanation for the acceleration of the Universe and other existing observational data. However, it remains quite possible that the dark energy density may depend sufficiently weakly upon time. This follows from many proposed models. The possibility that dark energy could be dynamical is also suggested by the remarkable *qualitative* analogy between the observed properties of dark energy and properties of a different type of "dark energy" – namely the inflaton field – postulated in the inflationary scenario of the early Universe.

Once we allow the dark energy density to be time-dependent, then the next simplest class of models are those with a m constant, non-positive w. We shall call this class "quiessence" (Q) ($w < -1/3$ is a necessary condition to make the universe accelerate). Examples include a tangled and "frustrated" network of cosmic strings $w = -1/3$ and domain walls $w = -2/3$. More generally, in a Friedmann-Robertson-Walker (FRW) background with the presence of CDM, an arbitrary but constant w for dark energy from the range $(-1, 0)$ can be achieved by using a scalar field with a hyperbolic sine potential (see Eq. (9) below)[4]. It may be noted that in principle the value of w may be even less than -1; the present observational data do not exclude this possibility but limit the constant w in the range of about $(-1.6, -0.8)$[7].

A more generic alternative to Λ and Q is presented by "kinessence" (K) which refers to dark energy with a *time dependent* w. Examples of kinessence include "quintessence" – a scalar field ϕ with a self-interaction potential $V(\phi)$ minimally coupled to gravity (see [4] for numerous references), as well as the "Chaplygin gas" model [8] and braneworld models of dark energy [9, 10]. These three alternatives are summarized in the Table I (where $z \equiv a(t_0)/a(t) - 1$ is the redshift, $a(t)$ is a FRW scale factor and the subscript 0 denotes the present moment).

The effective equation of state is clearly an important property of dark energy. This has led to numerous attempts to reconstruct the former from observations of high redshift supernovae in a model independent manner [11 – 13]. However, for field-theoretical models of dark energy, the equation of state is not a *fundamental* property. Strictly speaking it has reference only to an exactly isotropic FRW background. For small perturbations superimposed on a FRW background, the pressure tensor is generically non-diagonal (non-barotropic), and the velocity of signal propagation need not be given by the standard hydrodynamic expression $\sqrt{dP/d\varepsilon}$. Moreover, the very notions of ε and P for dark energy presuppose the *Einstein interpretation* of gravitational filed equations (not to be confused with the notion of the Einstein frame which is used in scalar-tensor and string theories of gravity!). Namely, even if the real equations for a given model are not the 4D Einstein equations at all (examples include dark energy models in scalar-tensor [14] and brane [9, 10] gravity), one can still write them formally in the Einstein form, by placing the Einstein tensor $R_{ij} - \frac{1}{2}g_{ij}R$ into the left-hand side, and by grouping all other terms in the right-hand side and calling them (after dividing by $8\pi G$) "the effective energy-momentum tensor of matter". After that, the energy-momentum tensor of dust-like matter (describing CDM and baryons) is subtracted from the latter, and the remaining part is used to define ε and P for "dark energy". All this reveals how ambiguous the notion of "equation of state" can be for a non-Einsteinian model of dark energy.

Fundamental variables (at least, at the field-theoretical level of consideration) are either geometrical (astronomical) – if they are constructed from a space-time metric directly, or physical – those which depend upon properties of physical fields carrying dark energy. Physical variables are, of course, model-dependent, while geometrical variables are more universal. Additionally, the latter do not depend upon uncertainly measured physical quantities such as the present density of dust-like matter Ω_m. That is why we emphasise the use of geometrical variables when describing the present expansion of the Universe and properties of dark energy.

The oldest and most well-known geometric variables are the Hubble constant H_0 and the current value of the deceleration parameter q_0. At present, accurate measurements of the expansion law of the Universe during the past are also possible (e.g., using the luminosity distance to distant supernovae), therefore these variables should be generalized to the Hubble parameter $H(t) \equiv \dot{a}/a$ and the deceleration parameter $q(t) \equiv \equiv -a\ddot{a}/\dot{a}^2 = -\ddot{a}/aH^2$ ($H_0 = H(t_0)$ and $q_0 = q(t_0)$). However, both the necessity of consideration of more general models of dark energy than a cosmological constant, and the remarkable increase in the accuracy of cosmological observational data during the last few years, compel us to advance beyond these two important quantities. For this reason, in this letter we propose a new geometrical diagnostic pair for dark energy. This diagnostic is constructed from the $a(t)$ and its derivatives up to the third order. Namely, we introduce the Statefinder pair $\{r, s\}$:

$$r = \frac{\dddot{a}}{aH^3}, \quad s = \frac{r-1}{3\,(q-1/2)}, \tag{2}$$

$r(z)$ is a natural next step beyond $H(z)$ and $q(z)$. We will soon see that it has a remarkable property for the basic flat ΛCDM FRW cosmological model; $s(z)$ is a linear combination of $r(z)$ and $q(z)$. In a companion paper we shall show that a particular combination of two variables from the above three e.g., q and s, can provide an excellent diagnostic for describing the properties of dark energy [15].

Below we will assume that the Universe is spatially flat, $k = 0$. This assumption naturally follows from the simplest versions of the inflationary scenario and is convincingly confirmed by recent CMB experiments [3]. At late times ($z \lesssim 10^4$) the Universe is well described by a two component fluid consisting of non-relativistic matter (CDM+baryons) Ω_m and dark energy $\Omega_X = 1 - \Omega_m$. In this case the Statefinder pair acquires the form

$$r = 1 + \frac{9}{2}\Omega_X w(1+w) - \frac{3}{2}\Omega_X \frac{\dot{w}}{H}, \tag{3}$$

$$s = 1 + w - \frac{1}{3}\frac{\dot{w}}{wH}, \tag{4}$$

where $w = P_X/\varepsilon_X$. Thus, if the role of dark energy is played by a cosmological constant ($w = -1$), then the value of r stays pegged at $r = 1$ throughout the *entire matter dominated epoch and at all future times*; i.e., $r \equiv 1$ for $z \lesssim 10^4$ irrespective of the current value of

Table 1

Dark Energy	State Parameter	Energy Density Parameter
Cosmological constant	$w(z) = \text{const} = -1$	$\rho(z) = \Lambda/8\pi G = \text{const}$
Quiessence	$w(z) = \text{const} < -1/3$	$\varepsilon(z) = \varepsilon_0(1+z)^{3(1+w)}$
Kinessence	$w(z) \neq \text{const}$	$\varepsilon(z) = \varepsilon_0 \exp\left[3\int_0^z dz' \frac{1+w(z')}{1+z'}\right]$

Ω_m. The extreme simplicity of the parameter $r(z)$ for the basic cosmological model (ΛCDM) which also provides the best fit to existing observational data may, in fact, prove not to be a mere coincidence[1)]! Very different behaviour is predicted for quiessence and kinessence for which r is a function of time. In particular, if dark energy is attributed to a minimally coupled scalar field ϕ (quintessence),

$$r = 1 + \frac{12\pi G\dot{\phi}^2}{H^2} + \frac{8\pi G\dot{V}}{H^3}. \tag{5}$$

The properties of the second Statefinder s complement those of the first. For the basic ΛCDM model with any non-zero Λ, $s \equiv 0$. Moreover, s does not depend neither on time, nor on Ω_m, for quiessence models for which $s = 1 + w$. In marked contrast, s generically depends on time for kinessence. E.g., for quintessence:

$$s = \frac{2\left(\dot{\phi}^2 + 2\dot{V}/3H\right)}{\dot{\phi}^2 - 2V}. \tag{6}$$

Thus, properties of the Statefinder pair $\{r, s\}$ enable it to differentiate between the three canonical forms of dark energy described in Table 1.

It is straightforward to invert Eqs. (3), (4) and express w and $\dot{w}$ in terms of the Statefinder pair. However, w is more directly related to the deceleration parameter:

$$w(t) = \frac{2q(t) - 1}{3\Omega_X}. \tag{7}$$

Thus, w a composite quantity since it is constructed out of physical (Ω_X) as well as geometrical (q) parameters. Note that for quintessence, $w > -1$ but $\dot{w}$ may have any sign (for models with $\dot{w} > 0$ and $w < 0$ the epoch of dark energy domination is usually a *transient*). The relationship between geometrical and physical parameters is summarized in Table 2.

Let us now study the Statefinder pair for different models of dark energy in greater detail. As was mentioned already, its value is equal to $\{1, 0\}$ for any ΛCDM model with a non-zero Λ. Quiessence models (QCDM) have a constant w, as a result

Table 2

Relationship between geometrical and physical parameters characterizing the observable Universe

Geometrical parameters	Related physical parameters
$H = \dot{a}/a$	$\Omega_{\text{total}}, \Omega_{\text{curv}}$
$q = -\ddot{a}/aH^2$	Ω_i, w_i
$r = \dddot{a}/aH^3$	$\Omega_i, w_i, \dot{w}_i$
$s = (r-1)/3(q-1/2)$	$w_i, \dot{w}_i$

$$r = 1 + \frac{9}{2}\Omega_Q w(1+w), \quad s = 1 + w\ . \tag{8}$$

Two values of the equation of state are singled out for special attention: $w = -1/3$ (cosmic strings) and $w = -2/3$ (domain walls). In both cases the first Statefinder has the simple form $r(t) = 1 - \Omega_Q(t) = \Omega_m(t)$. As a result, $r(t) \to 1$ for $t \ll t_0$, $r(t) \to 0$ for $t \gg t_0$ and $r_0 \simeq 0.3$ at the present time when $\Omega_Q(t_0) \simeq 0.7$. This leads to a degeneracy in r_0 for the dual value $w = -1/3, -2/3$. Though generic, this degeneracy is easily broken when one adds information from the second Statefinder s. Note that the case of an arbitrary $-1 < w < 0$ in the presence of a non-zero Ω_m can be achieved using quintessence with the potential [4] (see also [17]) [2)]

$$V(\phi) = \frac{3H_0^2(1-w)(1-\Omega_{m0})^{1/|w|}}{16\pi G\Omega_{m0}^{(1+w)/|w|}} \sinh^{-2(1+w)/|w|} \times$$
$$\times \left(|w|\sqrt{\frac{6\pi G}{1+w}}(\phi - \phi_0 + \phi_1)\right), \tag{9}$$
$$\Omega_{m0} = \Omega_m(t_0), \quad \phi_0 = \phi(t_0),$$
$$\phi_1 = \sqrt{\frac{1+w}{6\pi G}}\frac{1}{|w|}\ln\frac{1+\sqrt{1-\Omega_{m0}}}{\sqrt{\Omega_{m0}}}.$$

In this case, $r < 1,\ 0 < s < 1$.

Let us now turn to the quintessence case where r and s are given by Eqs. (5) and (6) correspondingly. To this category belong scalar fields with "tracker" potentials, for which the scalar field ϕ approaches a common evo-

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1)Note that the quantity $r(z)$ was also considered in the paper [16] for a non-flat case when it is time-dependent. However, its remarkable property for the flat ΛCDM model was not emphasized. For completeness, let us mention that $r = 2q = \Omega_m(z)$, $s \equiv 2/3$ for a matter dominated non-flat CDM model with negligible amounts of dark energy and radiation.

2)There are some misprints in numerical coefficients in Eqs.(119)–(121) of [4] which are corrected here.

lutionary path from a wide range of initial conditions [18]. Tracker potentials satisfy $V''V/(V')^2 \geq 1$. We consider the simplest case of an inverse power-law potential $V(\phi) = V_0/\phi^\alpha$, $\alpha > 0$ first studied in [19]. For this potential, the region of initial conditions for ϕ for which the tracker regime has been reached before the end of the matter-dominated stage is $\phi_{in} \ll M_P \equiv \sqrt{G}$, and the present value of quintessence is $\phi(t_0) \sim M_P$. The evolving values of the Statefinder pair for this potential with $\alpha = 2$ and $\alpha = 4$ are shown in Figs. 1 and 2. Also shown are results for the cosmological constant and quiessence. During tracking $\varepsilon_\phi/\varepsilon_m \propto t^{4/(2+\alpha)}$ as a result quintessence always becomes dominant at late times. The equation of state of quintessence and the corresponding value of the Statefinder pair is given by

$$w = -\frac{w_B + 2}{\alpha + 2}, \quad r \approx 1, \quad s \approx 1 + w \qquad (10)$$

($w_B = 1/3, 0$ during the radiation- and matter-dominated epochs respectively).

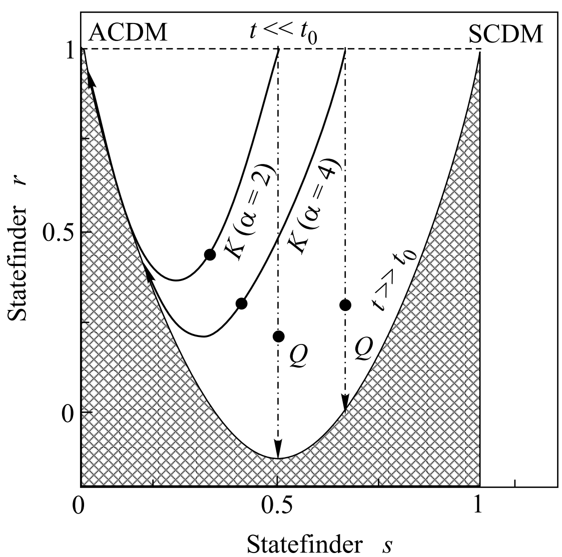

Fig.1. The Statefinder pair (r, s) is shown for different forms of dark energy. In quiessence (Q) models (w =constant $\neq -1$) the value of s remains fixed at $s = 1 + w$ while the value of r asymptotically declines to $r(t \gg t_0) \simeq 1 + \frac{9w}{2}(1 + w)$. Two models of quiessence corresponding to $w_Q = -0.25, -0.5$ are shown. Kinessence (K) models are presented by a scalar field (quintessence) rolling down the potential $V(\phi) \propto \phi^{-\alpha}$ with $\alpha = 2, 4$. These models commence their evolution on a tracker trajectory described by (10) and asymptotically approach ΛCDM at late times. ΛCDM ($r = 1, s = 0$) and SCDM in the absence of Λ ($r = 1, s = 1$) are the fixed points of the system. The hatched region is disallowed in quiessence models and in the kinessence model which we consider. The filled circles show the *current values* of the Statefinder pair (r, s) for the Q and K models ($\Omega_{m0} = 0.3$)

Constraints from structure formation and the CMB suggest that dark energy must be subdominant at $z \gtrsim 1$. Primordial nucleosynthesis arguments impose the stringent constraint: $\Omega_X < 0.05$ at $z \sim 10^9$ [20]. Small values of Ω_X and w substantially decrease the terms $\Omega_X w$ and $\Omega_X \dot{w}/H$ which appear in the right-hand side of (3) and ensure that the Statefinder r remains close to unity at high z. This is exactly what one finds from Fig.2. The extreme sensitivity of r to an evolving equation of state

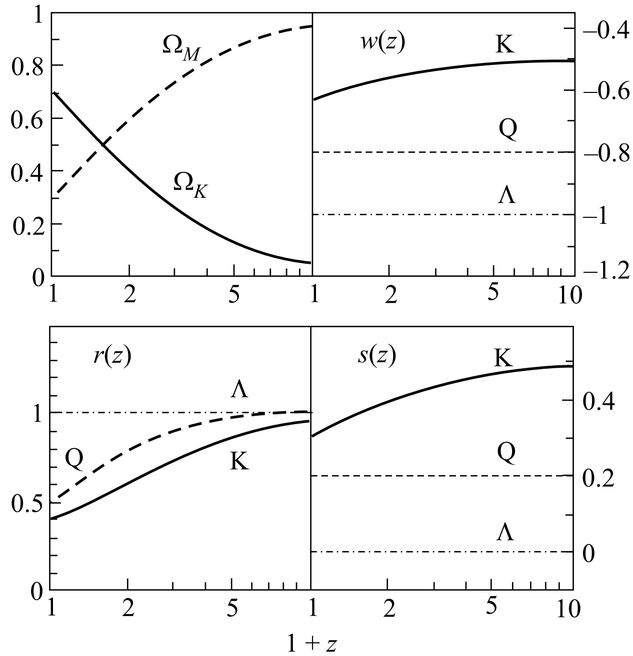

Fig.2. The Statefinder pair $\{r, s\}$ is shown for dark energy consisting of a cosmological constant Λ, quiessence Q with an unevolving equation of state $w = -0.8$ and the inverse power law tracker model $V = V_0/\phi^2$, referred here as kinessence "K". The lower left panel shows $r(z)$ while the lower right panel shows $s(z)$. Kinessence has a time-dependent equation of state which is shown in the top right panel. The fractional density in matter and kinessence is shown in the top left panel

of the tracker field is reflected by the fact that the value of r declines rapidly as the Universe expands, dropping to $\sim 50\%$ from its starting value by $z \sim 1$, even though dark energy remains subdominant at this epoch.

As is apparent from Fig.2, the discriminating power of r and s can be significant even at moderate redshifts. Since Ω_Λ and Ω_Q usually decrease faster with redshift than Ω_K, the value of $r(z)$ for both the cosmological constant and quiessence is generally closer to unity at a given large redshift than the corresponding value for a tracker field (kinessence). Thus, whereas the current value of r_0 allows us to differentiate Λ from Q and K, the value of r at moderate redshifts distinguishes K from

Λ and Q. This feature is even more pronounced in the second Statefinder s, whose value does not explicitly depend upon Ω_X and whose capacity to distinguish between Λ and quiessence on the one hand, and from kinessence on the other, actually *increases* with redshift (see Fig.2). The present CMB, SNe and galaxy clustering data strongly suggest that $\alpha \lesssim 1$ for quintessence with the inverse power-law potential [7]. However, even then the Statefinder remains a useful diagnostic as will be shown below.

Let us consider another form of kinessence. Below we determine the value of the Statefinder pair for the simplest of brane cosmological models – the Dvali-Gabadadze-Porrati (DGP) model [9]. It is important to note that in this model "dark energy" is not the energy associated with a new form of matter, rather its origin is geometrical in nature and is entirely due to the fact that general relativity is formulated in 5 dimensional space-time. The model has only one adjustable parameter r_c – the scale beyond which gravity becomes five-dimensional. This scale can be related to the current values of H_0 and Ω_{m0} by the relation $H_0 r_c = 1/(1-\Omega_{m0})$. The FRW equation for this model reads:

$$H = \sqrt{\frac{8\pi G\varepsilon_m}{3} + \frac{1}{4r_c^2}} + \frac{1}{2r_c} \tag{11}$$

(the choice of sign in front of the last term in the right-hand side corresponds to the "Brane2" class of models according to the terminology of [10]).

The solution to (11) can be written in the following parametric form:

$$a = a_1 \sinh^{2/3}\psi, \quad \frac{3t}{2r_c} = \psi + \frac{1-e^{-2\psi}}{2},$$
$$H = \frac{e^{\psi}}{2r_c \sinh\psi}, \quad \varepsilon_m = \frac{3}{32\pi G r_c^2 \sinh^2\psi},$$
$$\Omega_m \equiv \frac{8\pi G\varepsilon_m}{3H^2} = e^{-2\psi}. \tag{12}$$

The values of the deceleration parameter and the Statefinder pair read:

$$q = \frac{2\Omega_m - 1}{1+\Omega_m}, \quad r = 1 - \frac{9\Omega_m^2(1-\Omega_m)}{(1+\Omega_m)^3},$$
$$s = \frac{2\Omega_m^2}{(1+\Omega_m)^2}. \tag{13}$$

In particular, $r = 0.74$, $s = 0.11$ for $\Omega_m = 0.3$. At large redshifts the universe becomes matter dominated and $r \to 1$, $s \to 0.5$.

At the end of the paper, we estimate the accuracy with which the Statefinder pair (averaged over a range of z) can be determined in future SNAP-type satellite missions. The "SuperNovae Acceleration Probe" (SNAP) is expected to observe approximately 2000 type Ia supernovae within a year up to a redshift $z \sim 2$ and to improve luminosity distance statistics by over an order of magnitude [21]. Measurement of the luminosity distance $D_L(z)$ allow us to determine the Hubble parameter, since [11, 4]

$$H(z) = \left[\frac{d}{dz}\left(\frac{D_L(z)}{1+z}\right)\right]^{-1}. \tag{14}$$

To determine the Statefinder pair we use the following model independent parameterization of $H(z)$:

$$H^2(x) = H_0^2\,[\tilde{\Omega}_{m0}x^3 + A + Bx + Cx^2], \tag{15}$$

where $x = 1+z$ and $A+B+C = 1-\tilde{\Omega}_{m0}$. This form is simpler than that used in [12] but it is sufficient for our purpose. It becomes exact in the case of the ΛCDM model (i.e., dark energy being a cosmological constant). Note that the fact that we parameterize $H^2(z)/H_0^2$ by a 3-parameter fit means that the real $H(z)$ curve is *smoothed* over some redshift interval $z \sim z_{\max}/3$. In principle, the value of $\tilde{\Omega}_{m0}$ can be somewhat larger than the current density in CDM + baryons if dark energy has a tracker component having equation of state equal to that of matter at high z. However, the difference between $\tilde{\Omega}_{m0}$ and Ω_{m0} (if exists at all) is known to be small: $\tilde{\Omega}_{m0} \lesssim 1.1\Omega_{m0}$. Supernova observations of D_L and relations (14) and (15) can be used to determine A, B, C and the Statefinder pair $\{r, s\}$, since

$$r = 1 - \frac{(B+Cx)x}{\tilde{\Omega}_m x^3 + A + Bx + Cx^2},$$
$$s = \frac{2(B+Cx)x}{3(3A+2Bx+Cx^2)}. \tag{16}$$

In Fig.3 we present the results obtained from 1000 random simulations of a SNAP-type experiment for the "mean Statefinder statistic"

$$\bar{r} = \frac{1}{z_{\max}}\int_0^{z_{\max}} r(z)\,dz, \tag{17}$$

$$\bar{s} = \frac{1}{z_{\max}}\int_0^{z_{\max}} s(z)\,dz, \tag{18}$$

with $z_{\max} = 1.7$. The simulated numbers of SNe Ia events for one year period of observations are taken to be 50, 1800, 50 and 15 for the redshift intervals (0 – 0.2), (0.2 – 1.2), (1.2 – 1.4) and (1.4 – 1.7) respectively. The statistical uncertainty in the magnitude of SNe is assumed to be constant over redshift and is given by $\sigma_{\rm mag} = 0.15$. Details will be presented in a companion paper [15]. Fig. 3 shows that a future SNAP-

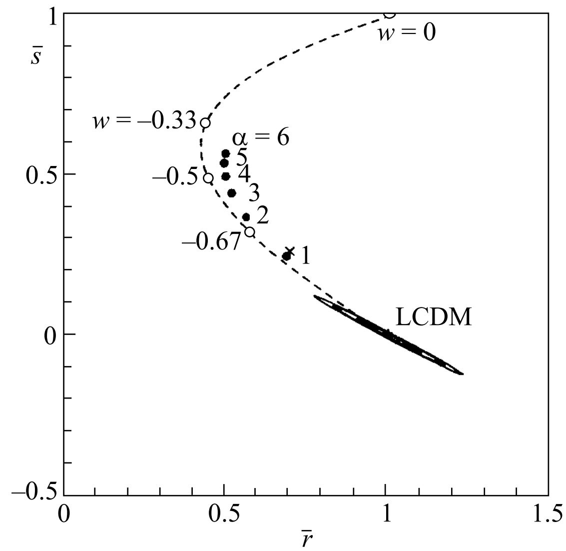

Fig.3. Confidence levels at 1σ, 2σ, 3σ of $\bar{r}$ and $\bar{s}$ computed from 1000 random realizations of a SNAP-type experiment probing a ΛCDM fiducial model with $\Omega_{m0} = 0.3$, $\Omega_{\Lambda 0} = 0.7$. The open circles represent the values of $\bar{r}$ and $\bar{s}$ for the quintessence potential $V(\phi) \propto \phi^{-\alpha}$ with $\alpha = 1, 2, 3, 4, 5, 6$ (bottom to top). The filled triangles represent quiessence with $w = -2/3, -1/2, -1/3, 0$ (bottom to top). The cross shows the mean Statefinder value $\bar{r} = 0.70$, $\bar{s} = 0.27$ for the DGP brane model with $H_0 r_c = 1.43$ ($\Omega_{m0} = 0.3$). Note that all inverse power-law models, as well as the DGP model, lie well outside of the three sigma contour centered around the ΛCDM model

type experiment determining $\{r, s\}$ can easily distinguish a fiducial ΛCDM model from several alternative time-dependent forms of dark energy, including the inverse power-law quintessence potential $V \propto \phi^{-\alpha}$ with $\alpha \sim 1$ and the DGP brane cosmological model.

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