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Renormalization group improved black hole space-time in large extra dimensions

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By taking into account a running of the gravitational coupling constant with an ultra violet fixed point, an improvement of classical black hole space-times in extra dimensions is studied. It is found that the thermodynamic properties in this framework allow for an effective description of the black hole evaporation process. Phenomenological consequences of this approach are discussed and the LHC discovery potential is estimated.

1. Introduction. Models with extra spatial dimensions offer an elegant solution to the hierarchy problem [1, 2]. In the case of [1] this is achieved by banning all standard model particles and forces onto a 4-dimensional subspace, while gravity can propagate also into d additional spatial dimensions. In order to keep the model consistent with todays gravity experiments the additional dimensions are assumed to be compactified in a small volume V_d. By this construction the measured gravitational coupling (or equivalently the Planck mass M_{Pl}) can be explained by a fundamental mass M_f which might be as low as a few TeV. Since this would be much closer to the electro-weak scale, such models give a possible solution of the hierarchy problem. The relation

$$M_{Pl}^2 = V_d M_f^{d+2} \tag{1}$$

connects these two couplings via the volume V_d which is spanned by the extra dimensions. In a world with extra dimensions and a gravitational coupling in a TeV range colliders like the LHC could create tiny black holes (BH) [3 – 8]. The line element of a higher dimensional spherically symmetric black hole is given by [9]

$$ds^2 = f(r)dt^2 - f^{-1}(r)dr^2 - r^2 d\Omega_{d+2}, \tag{2}$$

with

$$f(r) = 1 - R_H^{d+1}/r^{d+1}. \tag{3}$$

The event horizon R_H depends on the black hole mass M and the universal gravitational coupling G

$$R_H^{d+1} = \frac{16\pi G M}{(d+2)A_{d+2}} \ ,$$

where

$$A_{d+2} = \frac{2\pi^{\frac{d+3}{2}}}{\Gamma\left(\frac{d+3}{2}\right)}$$

marks the surface of a $d+3$ unit sphere. Please note, that there are different definitions of the higher dimensional coupling constant G. We use the definition of [6]. For more discussion and other definitions see [5]. By redefining the coupling in terms of the fundamental mass $M_f^{d+2} \equiv 1/G$, the radius of the event horizon is given by

$$R_H^{d+1} = \frac{16\pi}{(d+2)A_{d+2}} \frac{M}{M_f^{d+2}} \ . \tag{4}$$

This form of the black hole horizon holds as long as $R_H \ll R$ which is for TeV masses true since R_H exceeds R typically by fifteen orders of magnitude. A black hole emits thermal radiation [10]. The temperature of this radiation is given by the radial derivative of the metric coefficient $f(r)$ at the horizon. For the case of d extra dimensions this temperature is given by

$$T_H = \frac{d+1}{4\pi R_H} \ . \tag{5}$$

However, this prediction is limited to large black hole masses $M \gg M_f$. For masses close to the fundamental mass one expects modifications of the Hawking temperature and it was conjectured that the thermal radiation could be suppressed, leading to the formation of a stable final state [11 – 13].

Although, some extra dimensional models like in eq. (1) can solve the hierarchy problem, they are not the desired unified description of all forces yet. The reason for this is that gravity (with or without extra dimensions) can not be generalized in the usual loop expansion to a renormalizable quantum field theory. It was

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conjectured that this problem results from expanding the theory in the gravitational coupling instead of solving the complete theory that might even contain higher powers in the curvature R. In [14–17] it was shown that by using special truncation methods an exact renormalization group (RG) equation for the gravitational coupling can be derived. In a first order truncation those studies have been generalized to extra dimensions [18, 19] leading to a fundamental mass that depends on the energy scale k: $M_f^{d+2} \to \tilde{M}_f^{d+2}(k)$. It was shown that the running gravitational coupling has the form

$$\tilde{M}_f^{d+2}(k) = M_f^{d+2}\left[1+\left(\frac{k}{tM_f}\right)^{d+2}\right] \tag{6}$$

which also depends on a parameter, t. Going beyond one loop, this transition behavior between the infrared and ultraviolet regime was found to be even more pronounced with increasing number of extra dimensions [20]. For the case of $d = 0$ the effect of this running coupling on the structure of the Schwarzschild metric was derived in [21]. The aim of this paper is to repeat the construction for $d \neq 0$ and to study its phenomenological implications.

2. RG improved black holes in extra dimensions and black hole remnants. In flat space-time the de Broglie relation connects energies k and distances d by $k = 1/d$. In curved space-time more care is needed since distances are determined locally by the metric. For the case of a spherically symmetric Schwarzschild space-time, modifications of the de Broglie relation can only depend on the radial coordinate r. This leads to the ansatz

$$k(r) = \frac{\xi}{d(r)}\ , \tag{7}$$

where ξ is a parameter of order one. Before calculating the distance function $d(r)$ it is essential to remember its behavior for large distances $r \to \infty$. In this limit the metric should approach the flat Minkowski metric

$$\lim_{r\to\infty}\frac{d(r)}{r} = 1\ . \tag{8}$$

In that case the scale approaches asymptotically $k(r \to \to \infty) \approx \xi/r$. The distance function is calculated via the definition of distance in general relativity by integrating the line element

$$d(P) = \int_{\mathcal{C}}\sqrt{|ds^2_{class}|} \tag{9}$$

along a curve $\mathcal{C}$. The subscript *class* indicates that the line element is calculated with a fixed coupling M_f. We parameterize $\mathcal{C}$ in Schwarzschild space-time and calculate the distance along the curve

$$ds^2_{class} = \frac{1}{f_{class}(r')}dr'^2,$$
$$d(r) = \int_0^{r(P)}\frac{1}{\sqrt{|f_{class}(r')|}}dr'\ .$$

The parameterization along the radial coordinate r' in the range $r' \in [0, r(P)]$ is chosen like in [21]. In eq. (9) it is necessary to take the absolute value of ds in order to have always a positive distance. Due to the absolute value the distance function differs inside and outside of the event horizon. Together with definition of the event horizon (4) the distance function is expressed in the two regions by

$$d_{r<R_H}(r) = \int dr'\sqrt{\frac{r'^{d+1}}{R_H^{d+1}-r'^{d+1}}}, \tag{10}$$

$$d_{r>R_H}(r) = \int dr'\sqrt{\frac{r'^{d+1}}{r'^{d+1}-R_H^{d+1}}}\ . \tag{11}$$

It is not possible to find a general analytic solution for those two distance functions. Instead, the functions are interpolated between the two limits $r \to 0$ and $r \to \infty$. For small r the denominator of (10) simplifies and the small r limit can be integrated

$$\begin{aligned} d(r) &= \int_0^r dr'\sqrt{\frac{r'^{d+1}}{R_H^{d+1}-r'^{d+1}}} = \\ &\overset{r'\to 0}{=} \int_0^r dr'\sqrt{\frac{r'^{d+1}}{R_H^{d+1}}} = \\ &= \frac{1}{R_H^{\frac{d+1}{2}}}\frac{2}{d+3}r^{\frac{d+3}{2}}\ . \end{aligned} \tag{12}$$

For large r one has to integrate in two steps, first form $r' = 0$ to $r' = R_H$ which just gives a constant summand $\tilde{B}$, and afterwards from $r' = R_H$ to $r' = r$. In the second integration the fraction simplifies to a constant and distance behaves like r, again with a summand $\tilde{A}$

$$\begin{aligned} d(r) &= \int_{R_H}^r dr'\sqrt{\frac{r'^{d+1}}{r'^{d+1}-R_H^{d+1}}} = \\ &\overset{r'\to\infty}{=} \int_{R_H}^r dr'\sqrt{\frac{r'^{d+1}}{r'^{d+1}}} = \\ &= r - \tilde{A}\ . \end{aligned} \tag{13}$$

This is the dependency as required in (8). Now the distance function is interpolated between eq. (12) and (13) by

$$d'(r) = \left(\frac{r^{d+3}}{r^{d+1} + \gamma_d R_H^{d+1}} \right)^{\frac{1}{2}},$$
$$\gamma_d = \frac{(d+3)^2}{4}, \tag{14}$$

which has the correct asymptotic behavior in the limits $r \to \infty$: $d'(r) \to r$ and $r \to 0$: $d'(r) \to r^{(d+3)/2}$. The parameter γ_d is evaluated by the small r limit. Identifying the energy scale k with inverse distance one finds

$$k(r) = \frac{\xi}{d'(r)} = \xi \left(\frac{r^{d+1} + \gamma_d R_H^{d+1}}{r^{d+3}} \right)^{\frac{1}{2}}. \tag{15}$$

This relation between the energy scale k and the radius r in higher dimensional Schwarzschild space-time allows to express the scale dependent fundamental mass $\tilde{M}_f$ (6) in terms of the radial coordinate r.

Modifying the horizon radius R_H by the radius dependent fundamental mass $\tilde{M}_f(r)$ and defining

$$\tilde{t} = (\xi/t)^{d+2}, \tag{16}$$

gives

$$\tilde{R}_H^{d+1}(r) = \frac{16\pi}{(d+2)A_{d+2}} \frac{1}{\tilde{M}_f^{d+1}} \frac{M}{\tilde{M}_f} =$$
$$= R_H^{d+1} \left[1 + \frac{\tilde{t}}{M_f^{d+2}} \left(\frac{r^{d+1} + \gamma_d R_H^{d+1}}{r^{d+3}} \right)^{\frac{d+2}{2}} \right]^{-1}, \tag{17}$$

where $\tilde{t}$ parameterizes the strength of the RG corrections to the classical result. Since this $\tilde{R}_H$ has an explicit r-dependence it can not be interpreted as event horizon. Like usually the event horizon of a spherical symmetric black hole solution is the zero of the radial metric coefficient $f(r) = 1 - \tilde{R}_H^{d+1}(r)/r^{d+1}$. As shown in fig.1 the metric function does not cross $f(r) = 0$ for small values of M and so there is no singularity in the line element ds^2. However, for a larger black hole mass one finds the critical case where the line element gets a zero at one radius. For $M > M_c$ this zero splits up into two zeros, where the outer zero corresponds to the apparent event horizon. As also shown in fig.1 this outer horizon grows for large BH masses ($r \to \infty$) and approaches the classical event horizon of eq. (4). This kind of behavior of the metric function is independent of the number of extra dimensions. As it will be explained in the following

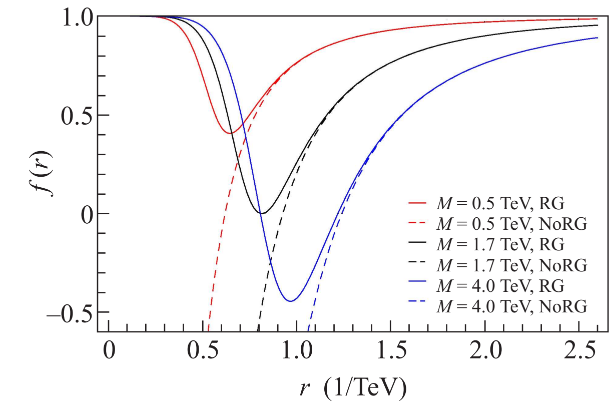

Fig.1. Metric coefficients $f(r)$ for different BH masses M at $d = 2$, $\tilde{t} = 0.002$, and $M_f = 1$ TeV. For those parameters a critical mass of $M_c = 1.67$ TeV is found

sections, it is natural to identify the critical mass with the mass of a black hole remnant

$$M_R = M_c.$$

The critical mass can be calculated in dependence of the parameters $\tilde{t}$ and M_f. As it can be seen in fig.2 for $d = 2, \ldots, 7$, M_c depends strongly on the parame-

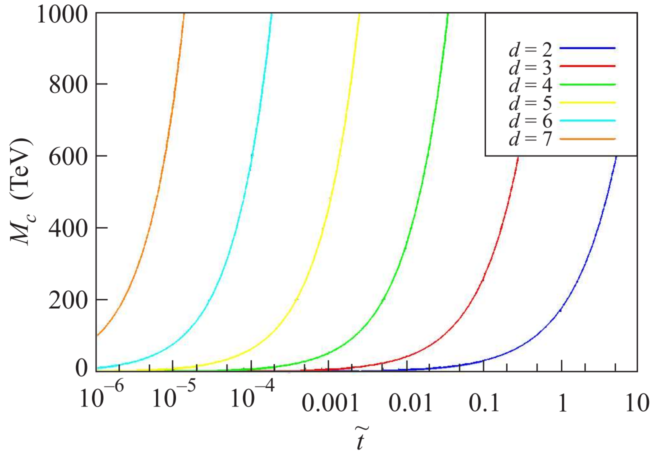

Fig.2. Remnant masses depending on the quantum gravity parameter $\tilde{t}$ for different numbers of extra dimensions d. $M_f = 1$ TeV

ter $\tilde{t}$. While M_c tends to zero for small values of $\tilde{t}$, it grows rapidly beyond the experimentally testable TeV range for larger values of $\tilde{t}$. Since the strength of the quantum gravity corrections is parameterized by $\tilde{t}$, it is interesting to note that by construction the continuous limit $\tilde{t} \to 0$ leads to $M_c \to 0$, whereas in a priory classical calculation ($\tilde{t} = 0$) no critical mass M_c and therefore no remnant exists.

3. Black hole thermodynamics. The predicted thermal decay of black holes due to Hawking radiation can be considered a key testing stone for any suggested theory of quantum gravity. Thus, black hole decay will now be studied in the presented theory with large extra dimensions and RG improved black hole space-times. The derivative of the radial function at the event horizon will still be interpreted as the black hole temperature

$$T_H = \frac{1}{4\pi}(\partial_r f(r))\Big|_{r=Horizon}. \tag{18}$$

In a purely classical calculation, the temperature rises for smaller BH masses, even up to the unphysical case when the typical energy of single quantum emitted by the black hole exceeds the total energy (mass) of the black hole. As it can be seen in fig.3, the temperature

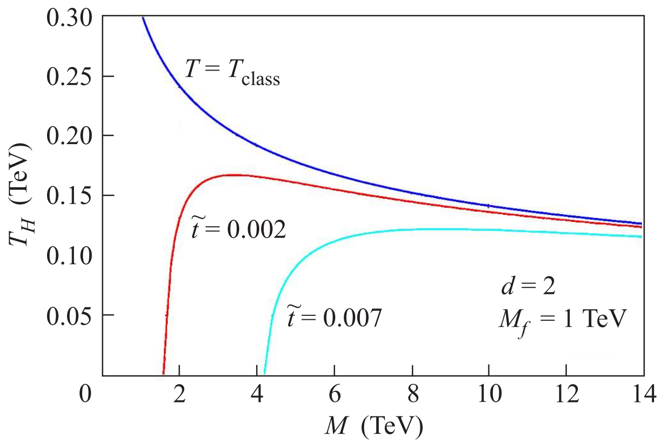

Fig.3. Comparison of the standard and two RG ($\tilde{t} = 0.002$ and $\tilde{t} = 0.007$, $M_f = 1$ TeV) temperature evolution of a black hole with mass M

for the RG behaves like the standard Hawking temperature for large masses $M \gg M_c$. But as soon as the BH mass approaches the critical mass the RG corrected temperature is suppressed until at $M = M_c$ the temperature is zero. This object with non zero mass M_c but zero temperature is not emitting any radiation and can thus be identified with a stable final BH state. Although the existence of such BH remnant states can be motivated from different grounds [11 – 13] it is a natural outcome of the RG improved BH space-time. This behavior solves the information loss problem and the problem of unphysical (the total energy exceeding) radiation for asymptotically slowly evolving black holes.

However, by looking at the thermal spectrum corresponding to a given temperature one sees that one problems remains. For thermal emission onto the four dimensional brane the spectrum I is given by

$$I(\omega, T_H) = N\frac{\omega^3}{\exp(\omega/T_H) + s}, \tag{19}$$

where N is a normalization factor and s is the factor corresponding to the spin of the emitted particle (Fermi-Dirac: $s = 1$, Boltzmann: $s = 0$, Bose-Einstein: $s = -1$). The energy (mass) of the black hole after a single radiation process is then given by $M_{fin} = M - \omega$. Calculating the spectrum (19) for a given black hole mass $M > M_c$ and taking the temperature as the temperature of the black hole before emission $T = T(M)$ one finds that part of the previous problem persists: The spectrum is non zero even for very large values of ω (see fig.4), leading to the problem that the BH mass after

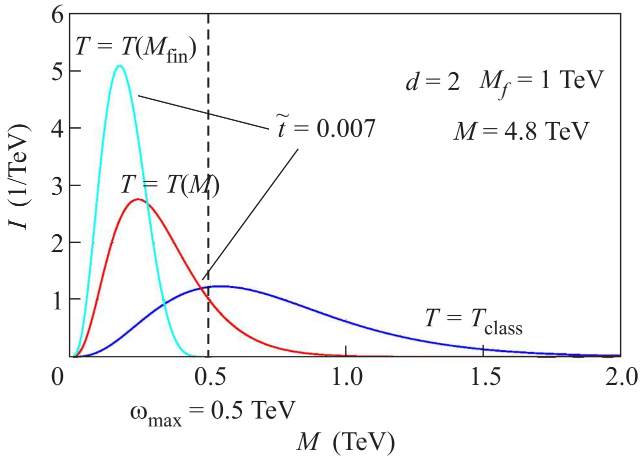

Fig.4. Radiation spectrum for RG ($\tilde{t} = 0.007$, $M_f = 1$ TeV) temperature and a standard temperature (5) for a black hole of the mass $M = 4.8$ TeV. The corresponding critical mass is $M_c = 4.3$ TeV. Thus the maximally allowed ω for $m_\omega = 0$ should be just below 0.5 TeV, which does not hold for the standard blue and the RG red curve, where $T_H = T_H(M)$ is assumed, but which is true for the cyan curve where $T_H = T_H(M_{fin})$ is assumed. Here Boltzmann statistics ($s = 0$) is used

emission $M_{fin} = M - \omega$ still could be below the critical mass or even negative. The nature of this problem is similar to the anterior problem of temperatures exceeding the total energy.

As simple mathematical solution of this problem one can use the conservation of energy and momentum. For a black hole with mass M that sits initially at rest and emits a quantum with mass m_ω and energy ω the final mass is

$$M_{fin} = \sqrt{M^2 + m_\omega^2 - 2E_\omega M}. \tag{20}$$

We propose to take the RG-improved temperature as a function of this final mass

$$T_H = T_H(M_{fin}), \tag{21}$$

as opposed to taking the dependence on the initial mass only $T_H = T_H(M)$. Now, for any unphysical radiation like $\omega \geq M - M_c$, the temperature in the exponential T_H is zero and the spectral density (19) vanishes. This cut off behavior is shown in fig.4. The physical interpretation of this simple modification is: First, one notices that for $\omega \ll (M - M_c)$ the modified and the original spectrum agree. Second, for $\omega \lesssim M - M_c$ the modification becomes important. Further, the form $T_H = T_H(M_{fin})$ means that the black hole must know about the frequency ω of the emitted quantum, already at the moment of emission, a behavior which smells like violation of causality. Nevertheless, in a thermodynamical approach this seems to be simplest solution to the problem of overradiation.

The result of this section is that RG (for a positive parameter $\tilde{t}$) allows for a consistent description of the thermodynamic evolution of black holes. It further predicts the formation of a final stable black hole state with temperature $T_H = 0$ and mass M_c.

4. Phenomenology. Several models with extra dimensions allow for an effective Planck mass at the order of a few TeV. The most exciting prediction of such models is the production of mini black holes due to particle collisions at the TeV scale. Already a simple implementation of a running of the gravitational coupling has a significant phenomenological impact on models with large extra dimensions [19]. Therefore, it is straight forward to study how the predictions about mini black holes due to high energy particle collisions change for the RG-improved black holes. The necessary condition for doing phenomenology with such black hole is that they are produced at all and that they are produced at sufficient rates. Therefore, we will leave the analysis of the specific thermodynamical properties or of the direct detection stable remnants to future studies and focus on the RG effects on black hole production.

The semi-classical cross section for the production a black holes due to a particle collisions with invariant energy $\sqrt{s}$ is given by

$$\sigma(\sqrt{s}) = \pi R_H^2 \theta(\sqrt{s} - M_{cut}), \tag{22}$$

where R_H is the Schwarzschild radius corresponding to the energy $\sqrt{s}$. The production threshold is usually associated with the higher dimensional Planck scale $M_{cut} = M_f$. At this point it should be mentioned that the possible production of mini black holes at particle colliders does not imply any risk [22 – 25]. This approximation of the cross section turned out to also be valid in different approaches (for a discussion see [26 – 31]). A first generalization is achieved by replacing the classical Schwarzschild radius R_H with the RG improved Schwarzschild radius $\tilde{R}_H$ (17) (evaluated at the outer horizon). The second generalization comes when replacing the heuristic threshold mass M_{cut} by the physical mass scale M_c. This leads to a physically intuitive threshold, since only for $M > M_c$ an event horizon (and therefore a black hole) exists. Thus, taking renormalization group into account the black hole cross section reads

$$\tilde{\sigma}(\sqrt{s}) = \pi \tilde{R}_H^2 \theta(\sqrt{s} - M_c). \tag{23}$$

As one can see in fig.5, for the production of very massive black holes $M \gg M_c$ the improved cross-section agrees with the semi-classical estimate. Only when M is slightly higher than M_c, the numerical values start to differ significantly. The most drastic difference to the semi-classical estimate appears when $M = M_c$, since this defines the new threshold for black hole production. One sees that this threshold M_c can differ largely from the ad hoc threshold M_f. Since the remnant mass depends strongly on the RG parameter $\tilde{t}$, the black hole threshold also depends strongly on $\tilde{t}$. An analogous result was obtained in [32]. Please note that if one just replaces M_f by $\tilde{M}_f(k)$ without taking the modification of $f(r)$ into account one finds that the cross section for large masses $M \gg M_f$ is damped to zero [33]. As it can be seen in fig.5 the RG-improved result does approach the semi-classical estimate for $M \gg M_c$ and therefore the damping behavior was just an artefact of not taking the modification of $f(r)$ into account.

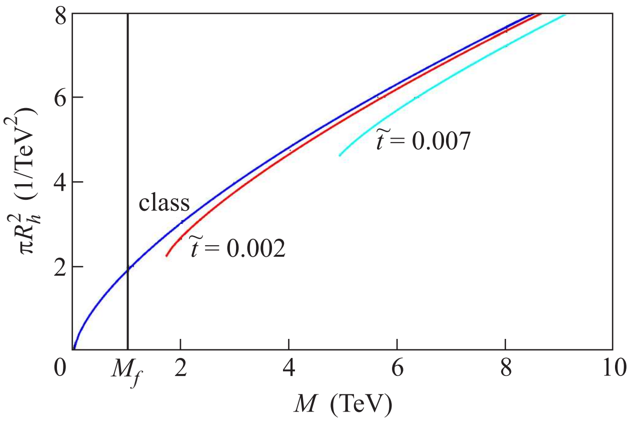

Fig.5. Black hole cross sections for $d = 2$ and $M_f = 1$ TeV, varying $\tilde{t}$

Now, an estimate of the LHC discovery potential on the parameter space d, M_f, and $\tilde{t}$ will be given. Clearly, an upper limit of the accessable production threshold M_c will be given by the maximal LHC center of mass energy of 14 TeV. While an upper limit on the accessable cross-section area $\pi \tilde{R}_H^2$ will be given by the first mileston of the integrated LHC luminosity $L_{LHC} \sim 100$ fb^{-1}. The same estimate can be made for the Tevatron with $M_c < 1.8$ TeV and $\pi\tilde{R}_H^2 < L_{Tev} = 10\mathrm{fb}^{-1}$). Combining the estimates already gives a rough estimates on lower limits on the LHC discovery potential in this model. For a more quantitative estimate of the discovery potential the luminosities (L_{LHC}, L_{Tev}) are compared to integrated partonic cross section

$$\sigma(\sqrt{s}) = \sum_{i,j}^{partons} \int_0^1 dx \int_0^1 dy\, f_i(x,q) f_j(y,q) \tilde{\sigma}(\sqrt{\hat{s}}), \tag{24}$$

by using the parton distribution functions f_i [34]. Here, the center of mass energy in the partonic reference frame is given by $\sqrt{\hat{s}} = \sqrt{(xy)s}$. For $d = 2$ the result of this scan over the parameter space is given in fig.6, showing

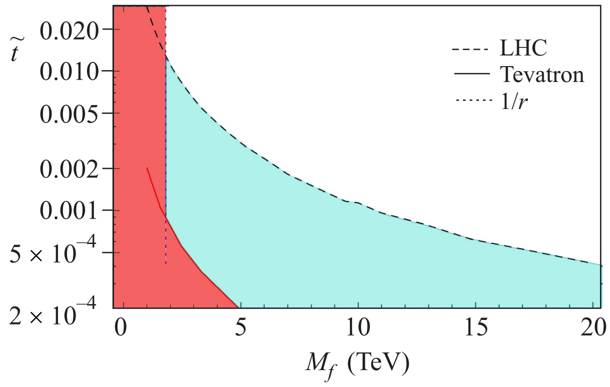

Fig.6. LHC ($\sqrt{s} = 14$ TeV, $L_{LHC} = 100\ \mathrm{fb}^{-1}$) discovery potential (turquoise region) in the parameterspace $\tilde{t}$ and M_f for $d = 2$. The Tevatron line is calculated with equation (24) for $\sqrt{s} = 1.8$ TeV and $L_{Tev} = 10\ \mathrm{fb}^{-1}$. For $d = 2$ the relation (1) in combination with the experimental limit on the compactification radius $R \leq 0.1$ mm [35] gives the vertical line at $M_f \approx 1.8$ TeV. Thus the red region is already excluded and the white region is not accessable by the LHC experiment

that the parameter $\tilde{t}$ has to be tuned to small values in order to be testable at one of the LHC experiments. On the other hand one sees that given a small value of $\tilde{t}$, the LHC might produce mini black holes even if the fixed point mass M_f exceeds the center of mass energy of 14 TeV. For each additional number of extra dimensions, the LHC and Tevatron lines are shifted about one order magnitude towards smaller values of $\tilde{t}$.

5. Summary and conclusion. We studied the impact of an effective running of the gravitational coupling on Schwarzschild black hole space-times for the case of large extra dimensions. The analysis of the black hole evaporation process revealed that this approach allows to solve the problem of overradiation for the whole process even for black hole masses comparable to the fundamental mass scale $M \sim M_f$, which was not possible in the standard description (see fig.3, 4). Further, it was found that this evaporation ends in a stable final state with mass M_c. Finally the impact of this approach on the predicted black hole production at the large hadron collider is calculated and an estimate of the possible discovery potential is given (see fig.6).

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