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Nambu-Goldstone explosion under brane perforation

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We show that perforation of the three-brane by mass impinging upon it from the five-dimensional bulk excites Nambu-Goldstone spherical wave propagating outwards with the velocity of light. It is speculated that such an effect can give rise to "unmotivated" energy release events in the brane-world cosmological models.

I. Introduction. Brane-world cosmological scenarios [1, 2] leave open the possibility of presence of matter in the bulk, at least in the form of black holes [3]. Interaction of bulk black holes with the three-brane is important both for the fate of the brane itself [4], as well as for the fate of black holes presumably created at colliders in TeV-scale gravity [5]. This problem was extensively studied recently using the Dirac-Nambu-Goto (DNG) equation on different black hole backgrounds [6] leading to variety of interesting predictions. Here we would like to investigate collision between the mass and the three-brane as genuine two-body problem, treating both of them on equal footing. Clearly, relativistic two-body problem with field mediated interaction has no exact solutions already in the case of two point particles in flat space. The standard approach is perturbation theory in terms of the coupling constant. We therefore adopt the same strategy. In order to maximally simplify the problem, we will treat the mass as point-like, gravity – within the Fierz-Pauli theory, and we restrict by an iterative solution in terms of the coupling constant $\varkappa$, related to the five-dimensional gravitational constant G_5 as $\varkappa^2 = 16\pi G_5$.

Gravitational constant in five dimensions has dimension of length^3. Combining it with the particle mass m (dimension of length^{-1}) and the brane tension μ (dimension of length^{-4}) we have two length parameters: $l = \varkappa^{-2}\mu^{-1}$, $r_g = \varkappa\sqrt{m}$, the first corresponding to the curvature radius of the bulk in the RS II setup, and the second — to gravitational radius of the mass m. To keep contact with the RS II model we have to consider distances small with respect to l, while to justify the linearization of the metric for the mass m we have to consider distances large with respect to r_g. So to apply the linearized theory to both objects we have to assume $l \gg r_g$, or $m\mu^2\varkappa^6 \ll 1$. Clearly, in this approach we will not be able to capture such essentially non-linear phenomena as formation of a hole in the three brane bounded by two-brane, and an associated Chamblin-Eardley instability [?]. Treating the mass as point-like, we exclude formation of a hole, but instead we will be able to describe another interesting manifestation of the brane perforation: an excitation of the quasi-free Nambu-Goldstone (NG) field.

Our model of collision is therefore quite simple: we treat bulk gravity at the linearized level on Minkowski background, with both the brane and the particle being described by geometrical actions. Gravitational interaction between them is repulsive in the co-dimension one, and the force does not depend on distance. When the mass pierces the brane reappearing on the other side of it, the sign of this force changes, and the brane gets shaken. We show that this shake gives rise to spherical NG wave which then expands within the brane with the velocity of light independently on further particle motion. Though we do not discuss here secondary effects due to interaction of the NG wave with matter fields on the brane [8, 9], it is clear that this interaction will transfer energy to the brane matter modes. Thus, for an observer living on the brane the act of brane perforation by a massive body from the bulk will look like a sudden explosive event not motivated by any visible reasons.

II. Set up. Consider the three-brane propagating in the five-dimensional space-time $(\mathcal{M}_5\,,\, g_{MN})$, whose world-volume $\mathcal{V}_4$ is given by the embedding equations $x^M = X^M(\sigma_\mu)$, $M = 0,1,2,3,4$, parameterized by arbitrary coordinates σ_μ, $\mu = 0,1,2,3$ on $\mathcal{V}_4$. The DNG equation reads

$$\partial_\mu \left(X^N_\nu g_{MN} \gamma^{\mu\nu} \sqrt{-\gamma} \right) = \frac{1}{2} g_{NP,M} X^N_\mu X^P_\nu \gamma^{\mu\nu} \sqrt{-\gamma}\,, \tag{1}$$

where $X^M_\mu = \partial X^M / \partial\sigma^\mu$ are the tangent vectors, $\gamma^{\mu\nu}$ is the inverse induced metric on $\mathcal{V}_4$, $\gamma^{\mu\nu}\gamma_{\nu\lambda} = \delta^\mu_\lambda$,

$$\gamma_{\mu\nu} = X^M_\mu X^N_\nu g_{MN}\big|_{x=X}\,, \tag{2}$$

and $\gamma = \det\gamma_{\mu\nu}$.

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The energy-momentum tensor of the brane is

$$T_b^{MN} = \mu \int X_\mu^M X_\nu^N \gamma^{\mu\nu} \, \frac{\delta^5 \left(x - X(\sigma) \right)}{\sqrt{-g}} \, \sqrt{-\gamma} \, d^4\sigma \,, \tag{3}$$

where μ is the brane tension,

Bulk gravitational field will be described within the linearized theory expanding the metric as

$$g_{MN} = \eta_{MN} + \varkappa h_{MN} \,, \tag{4}$$

with $\eta_{MN} = \mathrm{diag}(1, -1, -1, -1, -1)$, and the metric deviation h_{MN} being considered as the Minkowski tensor. The linearized Einstein equation in the harmonic gauge $\partial_M h^{MN} = \partial^N h/2, \; h = h_P^P$ reads:

$$\Box_5 h_{MN} = \varkappa \left(T_{MN} - \frac{1}{3} \eta_{MN} T \right) \,, \tag{5}$$

where $\Box_5 = -\partial_M \partial^M$ is the bulk D'Alembertian, and the metric potentials satisfy the superposition principle being the sum of the brane potentials and the mass potentials: $h_{MN} = h_{MN}^b + h_{MN}^m$. For consistency, the respective energy-momentum tensors at the right hand side must be flat space divergenceless $\partial_N T_{MN}^{b,m} = 0$, so, building them, we should not take into account gravitational.

In zero order in $\varkappa$ the brane is assumed to be unexcited $X_0^M = \Sigma_\mu^M \sigma^\mu$, where Σ_μ^M are four constant bulk Minkowski vectors which will be normalized as $\Sigma_\mu^M \Sigma_\nu^N \eta_{MN} = \eta_{\mu\nu}$. Obviously, this is a solution to the Eq. (1) for $\varkappa = 0$, and the corresponding induced metric is four-dimensional Minkowski

$$T_b^{MN} = \mu \int \Sigma_\mu^M \Sigma_\nu^N \eta^{\mu\nu} \delta^5 \left(x - X_0(\sigma) \right) d^4\sigma \,, \quad T = T_P^P \,. \tag{6}$$

One can further indentify the coordinates σ^μ on the unperturbed brane with the bilk coordinates $\sigma^\mu = x^\mu$, in which case $\Sigma_\mu^M = \delta_\mu^M$. Denoting the fifth coordinate as $z = x^4$ we obtain the solution of (5):

$$\begin{aligned} h_{MN}^b &= \frac{\varkappa\mu}{2} \left(\Sigma_M^\mu \Sigma_{N\mu} - \frac{4}{3} \eta_{MN} \right) |z| = \\ &= \frac{\varkappa\mu}{6} \mathrm{diag}(-1, 1, 1, 1, 4) |z|, \end{aligned} \tag{7}$$

this is a potential linearly growing on both sides of the brane. Comparing this with the RS II metric

$$ds_{RS}^2 = e^{-2k|z_{RS}|} \eta_{\mu\nu} dx^\mu dx^\nu - dz_{RS}^2 \,, \quad k = \frac{\varkappa^2 \mu}{12} \,, \tag{8}$$

at the distance z from the brane small compared with the curvature radius of the AdS bulk, $kz \ll 1$, so that $e^{-2k|z_{RS}|} \simeq 1 - 2k|z_{RS}|$ we see that they differ by coordinate transformation. Indeed, the gauge for the RS solution is non-harmonic. To pass to the harmonic gauge one has to transfrom the fifth coordinate as

$$z_{RS} = z - 2kz^2 \mathrm{sign}(z) \,. \tag{9}$$

This reproduces our solution (7). Note that this transformation is non-singular on the brane: $\partial z_{RS}/\partial z = 1$ at $z = 0$.

Consider now motion of the point mass in the bulk along the world line $x^M(\tau) = (t(\tau), 0, 0, 0, z(\tau))$:

$$\frac{d}{d\tau} \left(\dot{x}^N g_{MN} \right) = \frac{1}{2} g_{PQ,M} \dot{x}^P \dot{x}^Q \,. \tag{10}$$

We assume the mass moves in the positive z direction and hits the brane at $t = 0$ having the velocity v, and correspondingly, $\dot{t} = \gamma, \; \dot{z} = \gamma v, \; \gamma = 1/\sqrt{1 - v^2}$. Then we find from (10) that just before and after collision

$$\ddot{t} = \frac{\varkappa^2 \mu}{6} \gamma^2 v \; \mathrm{sign}(z) \,, \quad \ddot{z} = \frac{\varkappa^2 \mu}{12} \gamma^2 (1 + 4v^2) \; \mathrm{sign}(z) \,. \tag{11}$$

Particle energy and momentum $m\dot{t}$, $m\dot{z}$ have no discontinuity at the location of the brane $z = 0$, but their derivative have. It can therefore perforate the brane without loosing the energy-momentum, but with sudden change of acceleration. The sign in (11) corresponds to gravitational repulsion, as could be expected in the co-dimension one case [10].

Now compute the gravitational potentials of the mass constructing the source energy-momentum tensor in zero order in $\varkappa$. With gravitational interaction being neglected, the mass moves freely with the velocity v, so

$$x^M(\tau) = u^M \tau = \gamma(1, 0, 0, 0, v)\tau \,, \quad u^M = \mathrm{const.} \tag{12}$$

Substituting the corresponding energy-momentum tensor

$$T_m^{MN} = m \int u^M u^N \delta(x - x(\tau)) d\tau \tag{13}$$

into the Eq. (5), we obtain

$$h_m^{MN} = -\frac{\varkappa m}{(2\pi)^2} \frac{\left(u_M u_N - \frac{1}{3} \eta_{MN} \right)}{\gamma^2 (z - vt)^2 + r^2} \,, \tag{14}$$

where $r^2 = (x^1)^2 + (x^2)^2 + (x^3)^2$. This is nothing but the Lorentz-contracted five-dimensional Coulomb gravitational field of a moving point particle.

III. Brane deformation. The transverse coordinates of the brane can be viewed as Nambu-Goldstone bosons (branons) which appear as a result of spontaneous breaking of the translational symmetry [8]. These are coupled to gravity and matter on the brane via the induced metric (for a recent discussion see [9]). In our case of co-dimension one there is one such branon, which is a massless field coupled to bulk gravity.

To derive the NG wave equation we consider the deformation of the brane $X^M = X_0^M + \delta X^M$ caused by gravitational field h^m_{MN} due to matter in the bulk, compute the induced metric and linearize the resulting equation with respect to δX^M. Only transverse to the brane perturbation $\Phi(x^\mu) = \delta X^4$ is physical, for which we obtain the wave equation

$$\Box\Phi(x) = J(x), \tag{15}$$

where $\Box_4 = -\partial_\mu\partial^\mu$ is the flat D'Alembertian on $\mathcal{V}_4$, and the source term $J = J^4$, where

$$J^N = \varkappa\left(\frac{1}{2}\partial^N h_{PQ} - \partial_P h_Q^N\right)\Bigg|_{z=0}\Sigma^P_\mu\Sigma^Q_\nu\eta^{\mu\nu}, \tag{16}$$

with $h_{PQ} = h^m_{PQ}$. Substituting here the potentials (14), we obtain explicitly:

$$J(x) = \frac{\lambda v t}{(r^2+\gamma^2v^2t^2)^2}, \qquad \lambda = \frac{m\varkappa^2\gamma^2}{(2\pi)^2}\left(\gamma^2v^2+\frac{1}{3}\right). \tag{17}$$

The retarded solution of the Eq. (15) reads:

$$\Phi = \int G_{\mathrm{ret}}(x-x')J(x')d^4x' = = \frac{1}{(2\pi)^4}\int\frac{e^{-ikx}}{k^2+2i\epsilon\omega}J(k)\,d^4k, \tag{18}$$

where $k^\mu = (\omega, \mathbf{k})$ and $J(k)$ is the Fourier-transform of the source:

$$J(k) = \int e^{ikx}J(x)d^4x = \frac{2\pi^2\lambda}{\gamma}\frac{i\omega}{\omega^2+\gamma^2v^2\mathbf{k}^2}. \tag{19}$$

Evaluating the integral we find the following solution consisting of two terms

$$\Phi = \frac{\lambda}{2\gamma^3}\left(\frac{F_0(r,t)}{r} - \frac{F_1(r,t)}{r}\right), \tag{20}$$

where

$$F_0(r,t) = \frac{\pi}{2}\theta(t)\left[\epsilon(r+t)+\epsilon(r-t)\right], \qquad F_1(r,t) = \arctan\frac{r}{\gamma v t}. \tag{21}$$

The first part F_0, proportional to Heviside function of time $\theta(t)$, is zero until the moment $t = 0$ of perforation. It describes an expanding wave caused by the perforation shake discussed above (the first term in F_0 looks contracting, but actually it is a constant, $\theta(t)\epsilon(r+t) = 1$). This wave, propagating with the velocity of light, does not correlate with further motion of the particle.

The second part F_1 is non-zero and smooth both before ($t < 0$) and after ($t > 0$) the perforation. It does correlate with position of the mass describing a continuous deformation of the brane caused by its gravitational field. It is small when the particle is far away from the brane, and grows up to the maximal absolute value equal to $\pi/2$ when it approaches the brane.

It is easy to verify that for all $t \neq 0$, $r \neq 0$ this term satisfies the *inhomogeneous* wave equation:

$$\Box\,\frac{F_1(r,t)}{r} = \frac{1}{r}(\partial_t^2 - \partial_r^2)F_1(r,t) = \frac{2\gamma^3 v t}{(r^2+\gamma^2v^2t^2)^2}, \tag{22}$$

reproducing the right hand side of the Eq. (15). But it has finite and unequal limits as $t \to \pm 0$ for all $r \neq 0$:

$$\lim_{t\to\pm 0}\arctan\frac{r}{\gamma v t} = \frac{\pi}{2}\epsilon(t). \tag{23}$$

As was already explained, change of sign is due to change of direction of the gravitational force between the brane and the particle. The repulsive nature of this force manifests itself in signs: when the mass approach the brane ($t < 0$) the repulsive deformation is directed along z, which corresponds to $\Phi > 0$, when the particle reappears on the other side of the brane it repels the brane in the negative z direction, thus $\Phi < 0$. Since the force does not vanish and instantaneously changes sign at the moment of perforation, the second part of the solution has singular t-derivatives. Indeed, applying the box operator in the sense of distributions we get an additional delta-derivative term at the right hand side of the Eq. (22) which was obtained for $t \neq 0$:

$$\Box\frac{F_1(r,t)}{r} = \frac{2\gamma^3 v t}{(r^2+\gamma^2v^2t^2)^2} + \frac{\pi\delta'(t)}{r}. \tag{24}$$

The delta-derivative term describes the instantaneous shaking force exerted upon the brane. It excites the NG shock wave described by the first term in (21). Indeed, acting by D'Alembert operator on the F_0 part of the solution, we find exactly the same delta-derivative term:

$$\Box\frac{F_0(r,t)}{r} = \frac{\pi\delta'(t)}{r}. \tag{25}$$

Actually, the right-hand side arises as the second time derivative of Heviside function $\theta(t)$ entering F_0, while the remaining factor $[\epsilon(r+t)+\epsilon(r-t)]/r$ describes spherical shock waves satisfying the *homogeneous* wave equation:

$$\Box \frac{\epsilon(r \pm t)}{r} = \frac{1}{r} \, (\partial_t^2 - \partial_r^2) \epsilon(r \pm t) = 0 \, . \qquad (26)$$

The sum of the two terms in (20) therefore has no discontinuity at $t = 0$, while the discontinuity at $t = r$ corresponds to the expanding shock wave, satisfying the homogeneous wave equation. One could expect that brane deformation caused by the gravitational field of the mass could be tight to the mass motion, i.e. be of the F_1 type only. However, as we have shown, non-smoothness of gravitational interaction at the moment of piercing gives rise to NG explosive wave F_0 which then propagates freely along the brane.

Even more surprising is that in the limit of zero mass velocity our solution remains non-zero:

$$\Phi_0 = \lim_{v \to 0} \Phi = \frac{m \varkappa^2}{48 \pi} \, \frac{\epsilon(r - t)}{r} \, . \qquad (27)$$

Moreover, acting on this expression by box operator, one obtains zero, except for the point $r = 0$, at which one has to perform calculations in terms of distributions. By virtue of the identity $\Delta \frac{1}{r} = -4\pi \delta^3(\mathbf{r})$ we then find an extra term:

$$\Box \Phi_0 = Q_B \delta^3(\mathbf{r}) \, , \quad Q_B = \frac{m \varkappa^2}{12} \epsilon(t) \, . \qquad (28)$$

This might seem paradoxical, since the source term (17) in the NG equation (15) looks to be zero for $v = 0$. The paradox is solved if we regard the source $J(x)$ as distribution. It is easy to see, that in the limiting cases $t \to 0$ or $v \to 0$, $J(x)$ exhibits properties of the three-dimensional delta-function. Denoting $\alpha = \gamma v t$, we have:

$$\lim_{\alpha \to \pm 0} \frac{\alpha}{(r^2 + \alpha^2)^2} = \begin{cases} 0, & \text{if } r \neq 0, \\ \pm \infty, & \text{if } r = 0. \end{cases} \qquad (29)$$

Since the box operator in (15) contains the three-dimensional Laplace operator, $\Box = \Delta - \partial_t^2$, it is reasonable to consider $J(x)$ in the sense of distributions on R^3. Now, the integral of $J(x)$ over the three-dimensional space is α-independent up to the sign and finite,

$$\int J(x) d^3x = \frac{4\pi\lambda}{\gamma} \int\limits_0^\infty \frac{\alpha \, r^2}{(r^2 + \alpha^2)^2} dr = \frac{\pi^2 \lambda}{\gamma} \epsilon(\alpha) \, , \qquad (30)$$

where $\epsilon(\alpha) = \alpha / |\alpha|$ so, taking into account that $\epsilon(\alpha) = \epsilon(t)$, we obtain:

$$\lim_{\alpha \to \pm 0} J(x) = \frac{\pi^2 \lambda}{\gamma} \epsilon(t) \delta^3(\mathbf{r}) \, , \qquad (31)$$

which for $v = 0$ reproduces the right hand side of (28). Remarkably, this limiting value is the same if we consider time in the close vicinity of the perforation moment $t \to 0$ for any velocity v of the mass m, or if we consider the limit of small velocity $v \to 0$. In the latter case of quasi-static perforation this limit holds for any t, and since the coefficient λ remains finite as $v \to 0$, the point-like source in (15) might be attributed to some NG "charge" (shake charge).

This notion allows us to make distinction between two static situations. The first is that of the point mass sitting on the brane all the time. In this case, coming back to the Eq. (16) for brane perturbations, we find that the source term at the right hand side will be zero. This "eternally" siting on the brane mass is not in fact the bulk particle. On the contrary, the bulk particle arriving at the brane with zero velocity interacts gravitationally with the brane in such a way that a non-zero NG charge as a source term in the branon wave equation is produced. Therefore, it does generate the NG wave (28). This NG "charge" is a manifestly non-conserved quantity, changing sign at the moment of perforation. For an observer on the brane the perforation is felt as a sudden shake, and the corresponding NG field will be not a static Coulomb field, but an expanding wave.

IV. Conclusions. We have considered a simple model of collision between an infinitely thin three-brane and a point-like bulk particle interacting gravitationally in five-dimensional space-time within the linearized Einstein theory. In this setup the particle impinging normally on the brane freely passes through it to reappear on the other side. Since the particle has no size, no hole is created in the three-brane and no two-brane appears surrounding the hole. These phenomena which have to arise in the case of an impinging mass of finite radius (a black hole) are beyond the scope of the linearized gravity theory we adopt here. But as we have shown, perforation has another important effect which *is* well captured by the linear gravity approximation: detonation of a shock NG expanding wave at the moment of piercing. This might seem surprising, since no direct non-gravitational force between the mass and the brane exists in our model, and the particle passes through the brane feeling only its gravitational field. Similarly, the brane is not hit by the mass in the mechanical sense, but only reacts on its gravitational field. But the potential energy of the point mass in the field of the brane immersed in space-time with co-dimension one is linearly growing, so the force is repulsive and distance-independent. When the mass pierces the brane, this finite repulsive force instantaneously changes sign, so its time derivative behaves as delta-function of time.

Apart from this shock NG wave expanding with the velocity of light, the retarded solution contains the precursor/tail part which is non zero both before and after the perforation. This component is in direct correlation with position of the particle, so its measurement may serve as a tool to see invisible matter in the bulk.

Our model was inspired by the RS II setup. Indeed, using the linearized Einstein theory we were able to reproduce the RS II solution at distances small with respect the curvature radius of the bulk. But differently from the RS approach, we have considered both the brane and the particle on equal footing as test interacting objects in the Minkowski background. This allowed us to tackle the problem relativistically (in the special relativity sense) and to reveal existence of explosive NG wave triggered at the moment of perforation. One can speculate that perforation of the three-brane in the brane-world models may give rise to "unmotivated" explosive events in the observed Universe. Indeed, the NG field universally interacts with matter on the brane via the induced metric [8, 9], and, consequently, the NG explosion will transform into the matter explosion. In absence of matter, gravitational radiation will be excited, to see this it is enough to pass to the second postlinear order of Einstein theory. NG explosion can be expected to hold in the full non-linear treatment as well. Indeed, excitation of the brane oscillations in the black hole case is likely to have been observed in numerical experiments [4].

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