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Acceleration of particles by nonrotating charged black holes

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Recently, in the series of works a new effect of acceleration of particles by black holes was found. Under certain conditions, the energy in the centre of mass frame can become infinitely large. The essential ingredient of such effect is the rotation of a black hole. In the present Letter, we argue that the similar effect exists for a nonrotating but charged black hole even for the simplest case of radial motion of particles in the Reissner-Nordström background. All main features of the effect under discussion due to rotating black holes have their counterpart for the nonrotating charged ones.

I. Introduction. Recently, it was made an interesting observation that black holes can accelerate particles up to unlimited energies E_{cm} in the centre of mass frame [1]. This stimulated further works in which details of this process were investigated [2 – 6] and, in particular was found that the effect is present not only for extremal black holes but also for nonextremal ones [2]. These results have been obtained for the Kerr metric (they were also extended to the extremal Kerr-Newman one [3] and the stringy black hole [4]). In the work [7] generalization of these observations was performed and it was demonstrated that the effect in question exists in a generic black hole background (so a black hole can be surrounded by matter) provided a black hole is rotating. Thus, rotation seemed to be an essential part of the effect. It is also necessary that one of colliding particles have the angular momentum $L_1 = E_1/\omega_H$ [7], where E is the energy, ω_H is the angular velocity of a generic rotating black hole. If $\omega_H \to 0$, $L_1 \to \infty$, so for any particles with finite L the effect becomes impossible. Say, in the Schwarzschild space-time, the ratio E_{cm}/m (m is the mass of particles) is finite and cannot exceed $2\sqrt{5}$ for particles coming from infinity [8].

Meanwhile, sometimes the role played by the angular momentum and rotation, is effectively modeled by the electric charge and potential in the spherically-symmetric space-times. So, one may ask the question: can we achieve the infinite acceleration without rotation, simply due to the presence of the electric charge? Apart from interest on its own., the positive answer would be also important in that spherically-symmetric space-times are usually much simpler and admit much more detailed investigation, mimicking relevant features of rotating space-times. As we will see below, the answers is indeed "yes"! Moreover, in [1 – 7] rotation manifested itself in both properties of the background metric and in the nonzero value of angular momentums of colliding particles. However, below we show that both manifestations of rotation can be absent but nonetheless the effect under discussion reveals itself. This is demonstrated for the radial motion of particles in the Reissner-Nordström black hole, so not only $\omega_H = 0$ but also $L_1 = L_2 = 0$ for both colliding particles. It is surprising that the effect reveals itself even in so simple situation (which is discussed even in textbooks).

Formally, the results for the accleration of charged particle by the Reissner-Nordström black hole can be obtained from the corresponding formulas for the Kerr-Newman metric. Although the Kerr-Newman metric was discussed in [3], only motion of uncharged particles with angular momenta was analyzed, the metric being extremal, so there was no crucial difference from the acceleration in the Kerr metric. However, now we are dealing with the situation when a particular case is in a sense more interesting than a general one since it reveals a qualitatively different underlying reason of acceleration to infinite energies. We also discuss both the extremal and nonextremal metrics.

II. Basic formulas. Consider the metric of the Reissner-Nordström black hole

$$ds^2 = -dt^2 f + dr^2/f + r^2 d\omega^2. \tag{1}$$

Here $d\omega^2 = \sin^2\theta d\phi^2 + d\theta^2$, $f = 1 - 2M/r + Q^2/r^2$ where M is the black hole mass, Q is its charge. The event horizon lies at $r = r_H = M + \sqrt{M^2 - Q^2}$. Consider a radial motion of the particle having the charge q and rest mass m. Then, its equations of motion read

$$mu^0 = m\dot{t} = \frac{1}{f}\left(E - \frac{qQ}{r}\right), \tag{2}$$

$$m^2\dot{r}^2 = \left(E - \frac{qQ}{r}\right)^2 - m^2 f. \tag{3}$$

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Here, E is the conserved energy, dot denotes differentiation with respect to the proper time τ, u^μ is the four-velocity. In what follows, we assume that the difference $E - qQ/r_H \geq 0$, so it is positive for all $r > r_H$ (motion "forward in time") .

Let two particles (labeled by $i = 1, 2$) fall into the black hole, so $\dot{r}_1 < 0$, $\dot{r}_2 < 0$. The relevant quantity which we are interested in is the energy in the centre of mass frame [1–7] which is equal to

$$E_{cm} = m\sqrt{2}(1 - u_{1\mu}u^{2\mu}). \tag{4}$$

It follows from (2)–(4) that

$$\frac{E_{cm}^2}{2m^2} = 1 + \frac{X_1X_2 - Z_1Z_2}{fm^2}, \tag{5}$$

where

$$X_i = E_i - \frac{q_iQ}{r},\ Z_i = \sqrt{X_i^2 - m^2 f}. \tag{6}$$

III. Limiting transitions for energy. Now, we are going to examine what happens in different limiting transitions which involve the near-horizon region where $f \to 0$.

1) Let $f \to 0$. Then, we obtain from the (5), (6) that

$$\frac{E_{cm(H)}^2}{2m^2} = 1 + \frac{1}{2}\left[\frac{q_{2(H)} - q_2}{q_{1(H)} - q_1} + \frac{q_{1(H)} - q_{11}}{q_{2(H)} - q_{22}}\right], \tag{7}$$

where $q_{i(H)} \equiv E_i r_H/Q$. It is worth noting that, as $r_H \geq Q$ (for the definiteness, we take $Q > 0$), the critical charge $q_{(H)} > E$. If a particle falls from infinity, $E > m$ whence $q_{(H)} > m$, so a particle with the charge $q_{(H)}$ is overcharged in this sense.

2) If, say, $q_1 = q_{1(H)}(1 - \delta)$ with $\delta \ll 1$ and $q_2 \neq q_{2(H)}$, the energy $E_{cm(H)} \sim 1/\sqrt{\delta}$ can be made as large as one likes. Thus, we have that

$$\lim_{q_1 \to q_{1(H)}} \lim_{r \to r_H} E_{cm} = \infty. \tag{8}$$

3) Let now $q_1 = q_{1(H)}$ from the very beginning, $q_2 \neq q_{2(H)}$. Then, $X_1 = E_1(1 - r_H/r)$. For the nonextremal horizon, in the vicinity of the horizon the expression inside the square root is dominated by the term $-m^2 f$ and becomes negative. This means that the horizon is unreachable, so this case is irrelevant for our purposes. Instead, let us consider the extremal horizon, $M = Q = r_H$, $f = (1 - r_H/r)^2$. After simple manipulations, we find that near the horizon,

$$\frac{E_{cm}^2}{2m^2} = 1 + \frac{X_{2(H)}}{m^2(1 - \frac{r_H}{r})} \times$$
$$\times \left[E_1 - \sqrt{(E_1^2 - m^2)}\right] + O\left(\left(1 - \frac{r_H}{r}\right)\right). \tag{9}$$

Thus, E_{cm} diverges in the horizon limit:

$$\lim_{r \to r_H} \lim_{q_1 \to q_{1(H)}} E_{cm} = \infty. \tag{10}$$

4) For completeness, we should also consider the case $q_1 = q_{1(H)}$, $q_2 = q_{2(H)}$ for the extremal horizon. Then, $X_i = E_i(1 - r_H/r)$, $Z_i = (1 - r_H/r)\sqrt{E_i^2 - m^2}$ and we obtain that

$$\frac{E_{cm}^2}{2m^2} = 1 + \frac{E_1E_2 - \sqrt{E_1^2 - m^2}\sqrt{E_2^2 - m^2}}{m^2}, \tag{11}$$

so the energy remains finite and this case is of no interest in the present context.

IV. Limiting transitions for time and conditions of collision. In the above consideration, we showed that the energy E_{cm} can be made as large as one likes provided $q_1 \to q_{1(H)}$ and collision occurs near the horizon. Meanwhile, it is also essential to be sure that collision itself can be realized. Preliminarily, it can be understood that this is indeed the case, by analogy with the Kerr case where this was issue traced in detail [1–6].

Consider what happens in more detail. Let at the moment of the coordinate time $t = 0$ particle 1 starts to move towards the horizon at the point r_i, at the later moment $t = t_0 > 0$ particle 2 does the same (the precedent history of particles is unimportant). Then, the condition of collision at the point $r = r_f$ reads

$$\begin{gathered} t_0 = t_1 - t_2 > 0, \\ t_1 = \int_{r_f}^{r_i} \frac{dr X_1}{f\sqrt{X_1^2 - m^2 f}}, \\ t_2 = \int_{r_f}^{r_i} \frac{dr X_2}{f\sqrt{X_2^2 - m^2 f}}. \end{gathered} \tag{12}$$

To this end, it is sufficient (say, for $Q > 0$) to take $q_2 \leq q_1$, $E_2 > E_1$. Then, $X_2 > X_1$ for any r, and it is obvious that indeed the time $t_0 > 0$.

Then $r_f \to r_H$, each of integrals in (12) diverges in accordance with the well known fact that when the horizon is approached, the time measured by clocks of a remote observer is infinite. Let us discuss what happens to t_0 in the limiting situations 1)-4) discussed above

If we take the horizon limit 1) we find that t_0 is finite for $q_1 \neq q_{1(H)}$, $q_2 \neq q_{2(H)}$. Both proper times τ_1, τ_2 are also finite. If, afterwards, we consider $q_1 = q_{1(H)}(1 - \delta)$ with $\delta \ll 1$, the time t is still finite, the allowed region for particle 1 near the horizon shrinks since the positivity the allowed region for particle 1 near the horizon shrinks since the positivity of (3) entails that $r_H < r < r_H + A\delta^2$ where $A = \frac{r_H^2}{2\sqrt{M^2 - Q^2}}\left(\frac{E}{m}\right)^2$. The proper time $\tau_1 \sim \delta, \tau_2 \sim \delta^2$.

In case 3) the horizon is extemal and $q = q_{(H)}$. Then, one can obtain the exact explicit expressions:

$$t_1=\frac{E}{\sqrt{E_1^2-m^2}}\left[r+2r_H\ln(r-r_H)-\frac{r_H^2}{r-r_H}\right]_{r_f}^{r_i}, \quad (13)$$

$$\tau_1=\frac{m}{\sqrt{E_1^2-m^2}}\left(r_i-r_f+\ln\frac{r_i-r_H}{r_f-r_H}\right). \quad (14)$$

If $r_f \to r_H$, $t_1 \sim (r_f - r_H)^{-1} \sim t_2$. The proper time τ_1 diverges logarithmically, τ_2 is finite, so that the situation is very similar to the case of the extremal rotating black holes (cf. [2, 7]). Moreover, calculating the second derivative $\ddot{r}$ from (3), one can see that in the case under discussion both $\dot{r}$ and $\ddot{r}$ asymptotically vanish as the particle approaches the horizon, so particle 1 halts in the sense that r almost does not change (in terms of the proper distance l, the derivative $dl/d\tau$ is finite but l itself diverges for the extremal horizon). Correspondingly, particle 2 will inevitably will come up with a slow falling particle 1 and will collide with it.

Thus, we checked that in all cases of interest particle 2 can indeed overtakes particle 1, so collision will occur. This happens for a finite (or even almost vanishing) interval of the proper time of particle 2 after the start of motion in point r_i.

V. Extraction of energy after collision. Up to now, we discussed the effect of infinitey growing energy in the centre of mass frame. Meanwhile, for observations in laboratory, it is important to know what can be seen by an observer sitting at infinity. This poses a question about the possibility of extraction of the energy after collision. Below, we suggest preliminary analysis for the process near the horizon similar to what has been carried out in Sec.2 of [2] for rotating black holes.

Let two particles with energies E_1, E_2 and charges q_1, q_2 experience collision and turn into two other particles with energies $\varepsilon_1, \varepsilon_2$ and charges e_1, e_2. From the conservation law we have for the energy, the radial momentum (3):and the electric charge:

$$E_1 + E_2 = \varepsilon_1 + \varepsilon_2, \quad (15)$$

$$-\left[\sqrt{\left(E_1-\frac{q_1 Q}{r}\right)^2-m^2 f}\right]+$$
$$+\left[\sqrt{\left(E_2-\frac{q_2 Q}{r}\right)^2-m^2 f}\right]=$$
$$=\left[\sqrt{\left(\varepsilon_1-\frac{e_1 Q}{r}\right)^2-m^2 f}\right]-$$
$$-\left[\sqrt{\left(\varepsilon_2-\frac{e_2 Q}{r}\right)^2-m^2 f}\right], \quad (16)$$

$$q_1 + q_2 = e_1 + e_2. \quad (17)$$

The signs are chosen so, that before the collision both particles move towards a black hole and after it particle 1 goes outside a black hole and particle 2 goes inside. If collision occurs at the horizon $f = 0$, the system simplifies and one finds explicitly

$$\varepsilon_1=\frac{Q}{r_+}e_1,\ \varepsilon_2=E_1+E_2-\varepsilon_1,\ e_2=q_1+q_2-e_1. \quad (18)$$

From (18), we obtain the bound $\varepsilon_1 \leq e_1$. If, say, all particles have charges of the same order $q_i \sim e_i \sim q$ (i=1,2) which remain the same after elastic scattering, then $\varepsilon_1 \lesssim q$. As far as the extraction of the energy is concerned, the collision with the critical value of the charge q_H of the falling particle leading to the unbound energy in the centre of mass is not singled out in this process. Thus, from the viewpoint of an observer at infinity, one cannot gain much energy in elastic scattering. Rather, the main new physical effects from the collision with unbound energy in the centre of mass can be connected with new (yet unknown) physics at Planck scale due to creation of new kinds of particles.

At the first glance, the bound for the extracted energy is similar to that claimed in [6] for the rotating case, with m replaced by q. However, this bound was refuted in [2] where no bound was found at all. The difference between the situations considered here and in [6, 2] consists in that the angular momentum of a particle can be taken arbitrarily large after collision while we assume that an electric charge does not change (at least, significantly) after collisions.

VI. Critical electric charge and creation of pairs. Up to now, our consideration was pure classical. Meanwhile, it is known that in the electric field of a Reissner-Nordström black hole, creation of electron-positron pairs leads to diminishing of its electric charge [9]. Does this process influence significantly the effect discussed in our article? The pair production is energetically favorable, if $Q \geq \frac{\mu}{e} r_H c^2$ where μ is the electron mass, e its electric charge (in this section we restore explicitly the speed oflight c, the gravitational constant G and the Planck constant $\hbar$). Meanwhile, in our consideration the charge of one of two colliding particles should be close but slightly less that $q_H = \frac{mc^2}{Q} r_H$ (for simplicity, we assume that the energy $E \approx mc^2$

that corresponds to the particle nonrelativistic at infinity). Therefore, if one tries to apply our formulas to this case directly, it is seen that the system is close to the threshold of pair production but is somewhat below it. However, particle production comes into play in an indirect way. Consider the collision of (quasi)classical heavy particles such that $\frac{\mu}{e} r_H c^2 < Q < \frac{m}{q} r_H c^2$. Then, the creation of light particles (electrons and positrons) will change the value of the black hole charge and, thus, affect the critical value for heavy particles. As is known [9], the charge, independently on the exact initial value, falls off rather rapidly to the value $Q_1 = \pi\mu^2 c^3 r_H^2/e\hbar$. This entails the grow of the electric charge needed for the effect of acceleration under discussion to occur. We assume that our colliding particles represent or consist of stable atoms, so it is natural to assume that $q_H < 137e = \hbar c/e$ to avoid quantum instability of particles themselves. Then, after the substitution of Q_1 into the formula for q_H we obtain the inequality

$$M > \frac{m}{G}\left(\frac{e}{\mu}\right)^2 \approx 10^{42} m. \qquad (19)$$

It is seen that this inequality does not contain the Planck constant. If we take, say, $m \sim 100 m_p$, where m_p is the proton mass, we have that $M > 10^{20} g$ that is not restrictive from the astrophysical viewpoint.

VII. Conclusions. Thus, we reproduced all main features, existing for acceleration of particles in rotating black holes: for the nonextremal horizon the energy in the centre of mass frame is finite but can be made as large as one like, for the extremal case with the critical value of charge $q_{(H)}$ it diverges. In the latter case the proper time is also infinite, so in a sense the situation resembles the remote singularity unreachable in a finite proper time.

In our consideration, we neglect gravitational and electromagnetic radiation and backreaction on the metric. The case of nonrotating black holes is especially useful in a given context in that it seems to facilitate the evaluation of the role of such effects which is a separate subject for further investigation.

In the present work, we restricted ourselves by the Reissner-Nordström metric but it is clear from the method of derivation that effect should persist in more general situation of a black hole surrounded by matter (so-called "dirty" black hole). In this sense, the effect is as universal as its counterpart for rotating black holes [7], provided one of colliding particle has a special value of the electric charge.

In spite of the infinite energy of collision in the centre of mass frame, the energy of charged particles remains bound after elastic scattering. Therefore, in this context one can expect new physical effects detectable in observations (at least, in principle) not from high energy bursts but, rather due to new channel of reactions at Planck scale, entailing new scenarios in particle physics and astrophysics.

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