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Out-of-Equilibrium Admittance of Single Electron Box Under Strong Coulomb Blockade

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We study admittance and energy dissipation in an out-of-equlibrium single electron box. The system consists of a small metallic island coupled to a massive reservoir via single tunneling junction. The potential of electrons in the island is controlled by an additional gate electrode. The energy dissipation is caused by an AC gate voltage. The case of a strong Coulomb blockade is considered. We focus on the regime when electron coherence can be neglected but quantum fluctuations of charge are strong due to Coulomb interaction. We obtain the admittance under the specified conditions. It turns out that the energy dissipation rate can be expressed via charge relaxation resistance and renormalized gate capacitance even out of equilibrium. We suggest the admittance as a tool for a measurement of the bosonic distribution corresponding collective excitations in the system.

It is well-known that the phenomenon of Coulomb blockade is an excellent tool for observation of interaction effects in single electron devices [1 – 5]. Recently, due to the progress in the field of thermoelectricity the Coulomb blockade under out-of-equilibrium conditions has come into the focus of the theoretical [6 – 11] and experimental research [12 – 14]. The simplest mesoscopic system displaying Coulomb blockade is a single electron box (SEB). The properties of such a system are essentially affected by electron coherence and interaction.

The set-up is as follows (see Fig.1). Metallic island is coupled to an equilibrium electron reservoir (temperature T_r) via tunneling junction. The island is also coupled capacitively to the gate electrode. The potential of the island is controlled by the voltage U_g of the gate electrode. The distribution function of electrons in the reservoir is assumed to be equilibrium (Fermi distribution) while the one inside the island is arbitrary.

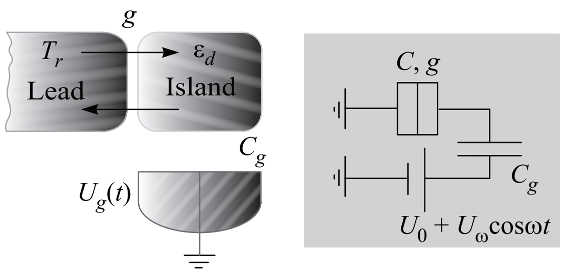

Fig.1. The set-up. A SEB is subjected to a constant gate voltage U_0. The current through the tunneling junction is caused by a weak AC voltage $U_g(t)$

The physics of the system is governed by the Thouless energy of an island E_{Th}, its charging energy E_c, and the mean single-particle level spacing δ. Throughout the paper the Thouless energy is considered to be the largest scale in the problem. This allows us to treat the metallic island as a zero dimensional object with vanishing internal resistance. The characteristic energy (ε_d) of electrons inside the island obeys the condition $\delta \ll \varepsilon_d \ll E_c, E_{\mathrm{Th}}$. This implies that characteristic energy is high enough to render the system incoherent and low enough to keep it strongly correlated due to Coulomb interaction [15]. The dimensionless conductance of a tunneling junction is small $g \ll 1$.

A single electron box does not allow for conductance measurements since there is no DC-transport. This way an essential dynamic characteristic becomes the set-up admittance, which is a current response to an AC-gate voltage $U_g(t) = U_0 + U_\Omega \cos \Omega t$.

Paper [16] sparked both theoretical and experimental attention to the admittance of such a set-up [17 – 22]. As it is well-known, the real part of admittance determines energy dissipation in an electric circuit. Classically, the average energy dissipation rate of a single electron box is given as follows

$$\mathcal{W}_\Omega = \Omega^2 C_g^2 R \frac{|U_\Omega|^2}{2}, \quad R = \frac{h}{e^2 g}, \quad \hbar\Omega \ll g E_c, \qquad (1)$$

where C_g denotes the gate capacitance, e – the electron charge, and $h = 2\pi\hbar$ – the Planck constant. Expression (1) presents us with a natural way of extracting the resistance of a system from its dissipation rate.

Electron coherence and interaction change the classical result for the dissipation. The low-temperature ($T \ll \delta$) coherent regime was pioneered in Ref. [16]. It

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was shown that the energy dissipation rate $\mathcal{W}_\Omega$ can be factorized in accordance with its classical appearance (1) but the definition of physical quantities comprising it becomes different. Geometrical capacitance C_g should be substituted by a new observable: mesoscopic capacitance C_μ. This leads to the establishment of another observable: *charge relaxation* resistance $\mathcal{R}_q$ such that $R \to \mathcal{R}_q$ in Eq. (1). Charge relaxation resistance of a coherent system differs drastically from its classical counterpart. In particular, the charge relaxation resistance of a single channel junction was predicted to be independent of its transmission and equal to $h/(2e^2)$ at zero temperature [16]. However Coulomb interaction in Ref. [16] and subsequent works [18] was accounted for on the level of classical equations of motion only. Recently the result for quantization of the charge relaxation resistance in SEB at $T \ll \delta$ has been rigorously derived [23]. The admittance in this low temperature regime was investigated experimentally by Gabelli et al. [19].

The knowledge of the charge relaxation resistance has been extended to a SEB at the transient temperatures when thermal fluctuations smear out electron coherence but electron-electron interaction is strong. The expression for the energy dissipation at this transient temperatures keeps its classical appearance if one substitutes the renormalized gate capacitance $\mathcal{C}_g$ and the charge relaxation resistance $\mathcal{R}_q$ for C_g and R respectively [24]. Unlike the latter, $\mathcal{C}_g$ and $\mathcal{R}_q$ have strong temperature and gate voltage U_g dependance.

The recent experiment by Persson et al [22] explored the energy dissipation rate at these transient temperatures. The admittance of SEB was measured at fixed frequency as a function of pumping amplitude U_Ω and the DC part of gate voltage U_0 in a wide range. The theoretical analysis of the data in Ref. [22] was carried out under assumption of linear response to the AC gate voltage: the electrons inside the island were assumed to be in the equilibrium with the reservoir. However it has not been verified experimentally. It is natural to expect that this assumption is violated for the set of data with high values of the amplitude U_Ω.

Motivated by the experiment [22] we study the admittance of a single electron box under the out-of-equilibrium conditions. We consider the linear response of a SEB with *arbitrary* electron distribution function in the island to the AC gate voltage.

A single electron box is described by the Hamiltonian

$$H = H_0 + H_c + H_t, \tag{2}$$

where

$$H_0 = \sum_{k,i} \varepsilon_k a_k^\dagger a_k + \sum_\alpha \varepsilon_\alpha^{(d)} d_\alpha^\dagger d_\alpha \tag{3}$$

– describes free electrons in the lead and the island, H_c describes Coulomb interaction of carriers in the island, and H_t describes the tunneling. Here operators $a_k^\dagger$ ($d_\alpha^\dagger$) create a carrier in the lead (island). Then the tunneling Hamiltonian is

$$H_t = X + X^\dagger, \quad X = \sum_{k,\alpha} t_{k\alpha} a_k^\dagger d_\alpha. \tag{4}$$

The charging Hamiltonian of electrons in the box is taken in the capacitive form:

$$H_c = E_c\big(\hat{n}_d - q\big)^2. \tag{5}$$

Here $E_c = e^2/2C$ denotes the charging energy, $q = C_g U_g/e$ the gate charge, and $\hat{n}_d$ is an operator of a particle number in the island $\hat{n}_d = \sum_\alpha d_\alpha^\dagger d_\alpha$. To characterize the tunneling it is convenient to introduce the Hermitean matrix:

$$\check{g}_{\alpha\alpha'} = (2\pi)^2 \left[\delta(\varepsilon_\alpha^{(d)})\delta(\varepsilon_{\alpha'}^{(d)})\right]^{1/2} \sum_k t_{\alpha k}^\dagger \delta(\varepsilon_k) t_{k\alpha'}. \tag{6}$$

The energies $\varepsilon_k, \varepsilon_\alpha^{(d)}$ are accounted from the Fermi level, and the delta-functions should be smoothed on the scale δE, such that $\delta \ll \delta E \ll T_r, \varepsilon_d$. The classical dimensionless conductance (in units e^2/h) of the junction between a reservoir and the island can be expressed as follows $g = \sum_\alpha \check{g}_{\alpha\alpha}$. Therefore, each non-zero eigenvalue of $\check{g}$ corresponds to the transmittance of some 'transport' channel between a reservoir and the island [25]. The effective dimensionless conductance ($g_{\rm ch}$) of a 'transport' channel and their effective number ($N_{\rm ch}$) are given by

$$g_{\rm ch} = \frac{\sum\limits_{\alpha\alpha'} \check{g}_{\alpha\alpha'}\check{g}_{\alpha'\alpha}}{\sum\limits_\alpha \check{g}_{\alpha\alpha}}, \quad N_{\rm ch} = \frac{\left(\sum\limits_\alpha \check{g}_{\alpha\alpha}\right)^2}{\sum\limits_{\alpha\alpha'} \check{g}_{\alpha\alpha'}\check{g}_{\alpha'\alpha}}. \tag{7}$$

The dimensionless conductance becomes $g = g_{\rm ch}N_{\rm ch}$. In what follows we will always assume

$$g_{\rm ch} \ll 1, \qquad N_{\rm ch} \gg 1, \quad g \ll 1. \tag{8}$$

Throughout the paper we keep the units such that $\hbar = e = k_B = 1$ except for the final results.

In the presence of time dependent gate voltage the gate charge q in Eq. (5) is changed as $q \to C_g U_g(t)/e$. The gate voltage is coupled to the operator of particle number inside the island only. Therefore the admittance of the system (the response of the charge in the island to AC part of the gate voltage) is determined by autocorrelation function of fluctuating particle number: $i\theta(t)\langle[\hat{n}_d(t),\hat{n}_d(0)]\rangle$, where $\theta(t)$ is Heaviside step-function. Due to the presence of strong Coulomb interaction the behavior of the autocorrelation function is non-trivial. It corresponds to collective bosonic modes similar to the case of Fermi liquid where the density-density correlator is governed by the electron-hole excitations [26–28]. The latter determines the behavior of the autocorrelation function in the absence of the Coulomb interaction. In an out-of-equilibrium regime we generally expect the collective mode distribution to be different from the distribution of the electron-hole excitations. As shown in Ref. [11], the collective mode distribution coincides with the one for the electron-hole excitations even out of equilibrium:

$$B_\omega(\tau) = \frac{\int \left[1 - F^d_\varepsilon(\tau) F^r_{\varepsilon-\omega}(\tau)\right] d\varepsilon}{\int \left[F^d_\varepsilon(\tau) - F^r_{\varepsilon-\omega}(\tau)\right] d\varepsilon}. \qquad (9)$$

Here function $F^{d,r}_\varepsilon(\tau)$ is given in terms of the Wigner transform $f^{d,r}_\varepsilon(\tau)$ of the electron distribution function $f^{d,r}(t,t')$ inside the island/reservoir: $F^{d,r}_\varepsilon(\tau) = 1 - 2f^{d,r}_\varepsilon(\tau)$, where a slow time $\tau = (t+t')/2$. In the equilibrium $F^{d,r}_\varepsilon = \tanh(\varepsilon/2T_r)$ and $B_\omega = \coth(\omega/2T_r)$.

Results. We focus on the most interesting case of Coulomb peak: the vicinity of a degeneracy point $q = k + 1/2$ where k is an integer. In this parametric regime the transport is dominated by the two closest charging states [29] (see Fig.2) which in the case of $g = 0$

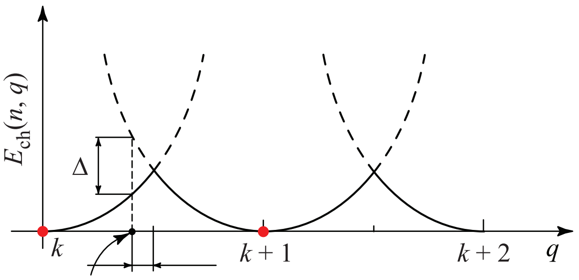

Fig.2. Charging energy $E_{\rm ch} = E_c(n-q)^2$ as a function of gate charge q

are separated by the Coulomb gap $\Delta = 2E_c(k+1/2-q)$. Due to the presence of the tunneling (finite g) all the observables, e.g., Δ, undergo strong renormalization near the Coulomb peak.

For not very high frequencies $\Omega \ll \max\{|\bar{\Delta}|, T_r, \varepsilon_d\}$ we obtained the following expression for admittance of the SEB

$$\mathcal{G}_\Omega = \frac{C_g}{C}\frac{Z^4\bar{g}}{4\pi}\frac{\bar{\Delta}\partial_{\bar{\Delta}} B_{-\bar{\Delta}}}{B_{-\bar{\Delta}}}\frac{i\Omega}{-i\Omega - \bar{g}\bar{\Delta}B_{-\bar{\Delta}}/2\pi}. \qquad (10)$$

Here the scaling parameter Z is defined as

$$Z(\lambda) = \left(1 + \frac{g}{2\pi^2}\lambda\right)^{-1/2}, \quad \lambda = \int \frac{B_\omega}{2\omega} d\omega, \qquad (11)$$

and $\bar{g}, \bar{\Delta}$ are renormalized tunneling conductance and Coulomb gap respectively:

$$\bar{g} = gZ^2(\lambda), \qquad \bar{\Delta} = \Delta Z^2(\lambda). \qquad (12)$$

The integral in Eq. (11) runs over frequencies $E_c \gg |\omega| \gg \omega_0 = \max\{T_r, \varepsilon_d, |\bar{\Delta}|\}$. The energy scale ω_0 determines the natural scale at which the RG procedure has to be stopped [11]. Within logarithmic accuracy we find $\lambda = \ln E_c/\omega_0$.

We stress that our result (10) is valid for an arbitrary electron distribution. To make predictions more concrete we consider the case of quasi-equilibrium $F^d_\varepsilon = \tanh \varepsilon/2T_d$, $T_d > T_r$ as an example. This regime is typical for a SEB with the metallic island. It is achieved when the energy relaxation rate due to electron-electron interaction in the island $1/\tau_{ee} \gg g\delta$ (see e.g., [11]).

The real part of admittance (10) at fixed Ω as a function of q is shown in Fig.3 for the out-of-equilibrium

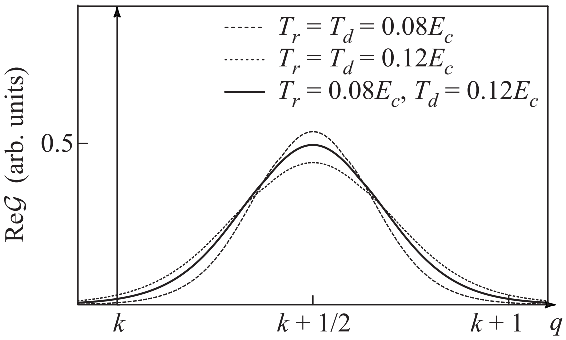

Fig.3. The real part of admittance of the SEB at fixed Ω as a function of q. We use $g = 0.5$, $\Omega = 0.02E_c$ and $C_g/C = 0.24$. See text

regime with $T_d > T_r$. At fixed C_g, C and g the height of the maximum is controled by the effective temperature of electron-hole excitations $T_{\rm eh} = \lim_{\bar{\Delta}\to 0}(\bar{\Delta}/2)B_{\bar{\Delta}}$ [30]. As it was shown, $T_r \leqslant T_{\rm eh} \leqslant T_d$ and $T_{\rm eh} \approx T_d \ln 2$ for $T_d \gg T_r$ [11]. Therefore, out-of-equilibrium admittance is confined within the boundaries $\mathrm{Re}\,\mathcal{G}_{\Omega,T_d} < \mathrm{Re}\,\mathcal{G}_\Omega < \mathrm{Re}\,\mathcal{G}_{\Omega,T_r}$, where $\mathcal{G}_{\Omega,T_d}(\mathcal{G}_{\Omega,T_r})$ are equilibrium admittances at temperatures $T_d(T_r)$.

The dissipative part of the admittance in a SEB has been addressed experimentally via radio-frequency reflectometry measurements. The device was exposed to a continuous rf-signal [22]. In the experiment the tunneling conductance was estimated to be equal $g = 0.5$ such that the SEB was in the strong Coulomb blockade regime. We plot the real part of the admittance (10) at fixed Ω as a function of q in Fig.4. There, for a sake of

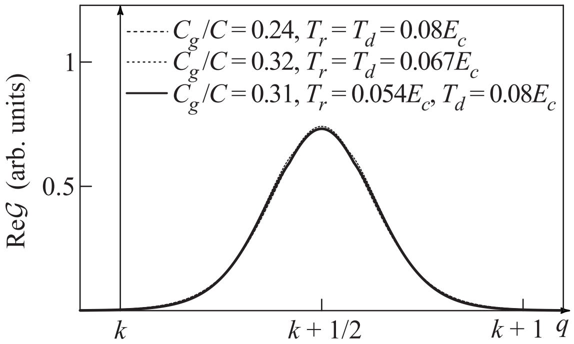

Fig.4. The dissipative part of admittance of the SEB at fixed Ω as a function of q. Three curves corresponding to three different formulae are presented. Dashed line corresponds to Eq. (10) with $Z = 1$, $\bar{\Delta} = \Delta$, $\bar{g} = g$ and $B_{\bar{\Delta}} = \coth \Delta/2T_r$. Dotted line is plotted according to Eq. (10) with $B_{\bar{\Delta}} = \coth \bar{\Delta}/2T_r$. Solid line corresponds to Eq. (10) with non-equilibrium $B_{\bar{\Delta}}$ given by Eq. (9). We use $g = 0.5$ and $\Omega = 0.02E_c$. See text

comparison, we present $\mathrm{Re}\,\mathcal{G}_\Omega$ computed i) in the equilibrium without taking into account the renormalization effects, i.e., with $Z = 1$ and $B_{\bar{\Delta}} = \coth \Delta/2T_r$ (dashed line); ii) in the equilibrium and with the renormalization effects, i.e., with $B_{\bar{\Delta}} = \coth \bar{\Delta}/2T_r$ (dotted line); iii) out of equilibrium with $T_d > T_r$ and $B_{\bar{\Delta}}$ determined by Eq. (9) (solid line). In all three cases, we use the same values of g, E_c and Ω corresponding to the experiment [22]. As one can see from Fig.4, a slight variation of ratios C_g/C and $T_{r,d}/E_c$ allows us to make curves for cases i), ii) and iii) indistinguishable. In Ref. [22] it is assumed that the electrons inside the island are in the equilibrium with the reservoir and the renormalization effects are not important (case i) above). Values of C_g/C and T_r/E_c are used as fitting parameters. The curves presented in Fig.4 however demonstrate a more subtle picture. As one can see, the successful fitting of the experimental data by a 'theoretical' curve gives yet no confidence that these assumptions are satisfied. Therefore, more careful analysis of the experimental data of Ref. [22] is needed.

The electron-hole distribution B_ω enters admittance in a twofold way. The analytical structure of admittance as a function of external frequency Ω is entirely determined by B_ω at $\omega = -\bar{\Delta}$. The scaling parameter Z arising from the renormalization however contains information on B_ω in wide domain $\omega_0 < |\omega| < E_c$. Admittance (10) can serve as a tool for direct experimental measurement of B_ω. As such can be the measurement of a real part of admittance $\mathrm{Re}\,\mathcal{G}_\Omega$ at two different driving frequencies. Other possibility would be the simultaneous measurement of the real and imaginary parts of $\mathcal{G}_\Omega$ at a given frequency [19]. Then one can read out $\bar{\Delta}B_{-\bar{\Delta}}$ in the entire span of $\bar{\Delta}$ by tuning the DC gate voltage U_0. Measurements of frequency dependence of $\mathrm{Re}\,\mathcal{G}_\Omega$ at the Coulomb peak ($\bar{\Delta} = 0$) provide an access to the effective bosonic temperature T_{eh}. Thus the admittance of a SEB under AC gate voltage can be used as the thermometer for the electron-hole excitations similar to the Coulomb blockade thermometer based on measurements of the differential DC conductance in a single electron transistor (SET) [12].

The real part of admittance determines energy dissipation rate: $\mathcal{W}_\Omega = (C_g/2C)\mathrm{Re}\,\mathcal{G}_\Omega|U_\Omega|^2$. At quasi-static regime $\Omega \to 0$ we find that even out of equilibrium the energy dissipation rate factorizes into the product of well-defined physical observables in full analogy with classical expression (1):

$$\mathcal{W}_\Omega = \frac{\Omega^2}{2}\mathcal{C}_g^2\mathcal{R}_q|U_\Omega|^2, \quad \mathcal{R}_q = \frac{h}{e^2 g'}, \quad \mathcal{C}_g = \frac{\partial q'}{\partial U_0}. \quad (13)$$

The charge relaxation resistance $\mathcal{R}_q$ and the renormalized gate capacitance $\mathcal{C}_g$ are related to physical observables formally defined as [31]

$$g' = 4\pi \mathrm{Re}\,\frac{\partial K^R(\omega)}{\partial \omega}, \quad q' = Q + \mathrm{Re}\,\frac{\partial K^R(\omega)}{\partial \omega}. \quad (14)$$

Here $Q = \langle \hat{n}_d \rangle$ is the average charge in the island, the correlation function $K^R(t) = i\theta(t)\langle [X(t), X^\dagger(0)]\rangle$ and the limit $\omega \to 0$ is assumed. The physics behind quantities (14) can be understood if we turn from a SEB to a SET. In the absence of source-drain voltage a SET is physically equivalent to the SEB. The quantity g' then coincides with the SET conductance [32, 33]. The quantity q' is specific to Coulomb blockade physics and can be addressed as the quasi-particle charge [31].

With the help of definitions (14) we obtained the following results in the out-of-equilibrium regime (for $g \ll 1$)

$$g' = -\frac{1}{2}\bar{g}\bar{\Delta}\partial_{\bar{\Delta}} \ln B_{-\bar{\Delta}}, \qquad q' = k + \frac{1}{2} + \frac{1}{2}\frac{1}{B_{-\bar{\Delta}}}. \quad (15)$$

Equations (15) generalize the results for g' [32, 33] and q' [31] derived under the equilibrium conditions.

Derivation. Below we describe the main steps of the derivation. Further details will be given in [34]. Following Ref. [29], we write the Hamiltonian (2) in the truncated Hilbert space of electrons on the island accounting for two charging states: with $Q = k$ and $Q = k + 1$ only(see Fig.2). The projected Hamiltonian then takes a form of 2×2 matrix acting in the space of these two charging states:

$$\mathcal{H} = H_0 + H_t + \Delta S_z + \Delta^2/4E_c, \tag{16}$$

where

$$\mathcal{H}_t = \sum_{k,\alpha} t_{k\alpha} a_k^\dagger d_\alpha S^- + \text{H.c.}, \tag{17}$$

and S^z, $S^\pm = S^x \pm i S^y$ are ordinary (iso)spin 1/2 operators. Admittance is proportional to dynamical (iso)spin susceptibility $\Pi_s^R(t) = i\theta(t)\langle[S^z(t), S^z(0)]\rangle$ [24]:

$$\mathcal{G}_\Omega = -i\Omega C_g \Pi_s^R(\Omega)/C. \tag{18}$$

To deal with spin operators out of equilibrium the generalization of Abrikosov's pseudo-fermions (PF) $\psi_\alpha^\dagger$, ψ_α is used [35, 36]. Integrating out electrons in the limit $N_{\text{ch}} \gg 1$, we arrive at the following effective action [11]

$$S = \int dt \bar\psi\Big(i\partial_t - \frac{\sigma_z\Delta}{2} + \eta\Big)\psi + \frac{g}{8}\int \bar\psi(t)\gamma_i\sigma_-\psi(t)\, \times$$
$$\times\, \Pi_{ij}(t,t')\bar\psi(t')\gamma_j\sigma_+\psi(t')\, dt dt'. \tag{19}$$

Here the pseudo-fermion fields ψ, $\bar\psi$ are understood as vectors in the tensor product of isospin and Keldysh spaces. We inserted the factor $\exp(\eta\bar\psi\psi)$ with $\eta\to-\infty$ into the density matrix in order to fulfill the constraint $\bar\psi(t)\psi(t) = 1$. The matrices σ_z, $\sigma_\pm = (\sigma_x \pm i\sigma_y)/2$, and $\gamma_1 \equiv \tau_x$, $\gamma_2 \equiv \tau_0$ are the Pauli matrices in (iso)spin and Keldysh spaces respectively. Π_{ij} stands for the matrix:

$$\Pi = \begin{pmatrix} 0 & \Pi^A \\ \Pi^R & \Pi^K \end{pmatrix}, \tag{20}$$

$$\Pi^{R,A,K}(t,t') = \int \frac{d\omega}{2\pi}\Pi_\omega^{R,A,K}(\tau) e^{-i\omega(t-t')}, \tag{21}$$

$$\Pi_\omega^{R,A}(\tau) = \mp i\int \left[F_\varepsilon^d(\tau) - F_{\varepsilon-\omega}^r(\tau)\right]\frac{d\varepsilon}{2\pi}, \tag{22}$$

$$\Pi_\omega^K(\tau) = 2i\int (1 - F_\varepsilon^d(\tau) F_{\varepsilon-\omega}^r(\tau))\frac{d\varepsilon}{2\pi}. \tag{23}$$

The PF dynamical spin susceptibility is given as [34]:

$$\Pi_{s,pf}^R(\omega) = Z^2\sum_\sigma \int\Bigg\{\mathbf{\Gamma}_\sigma^{RKR}(\varepsilon+\omega,\varepsilon,\omega)\overline{G}_{\sigma,\varepsilon+\omega}^R\overline{G}_{\sigma,\varepsilon}^R +$$
$$+\mathbf{\Gamma}_\sigma^{RAR}(\varepsilon+\omega,\varepsilon,\omega)\Big[\overline{G}_{\sigma,\varepsilon+\omega}^R\overline{G}_{\sigma,\varepsilon}^K + \overline{G}_{\sigma,\varepsilon+\omega}^K\overline{G}_{\sigma,\varepsilon}^A\Big] +$$
$$+\mathbf{\Gamma}_\sigma^{KAR}(\varepsilon+\omega,\varepsilon,\omega)\overline{G}_{\sigma,\varepsilon+\omega}^A\overline{G}_{\sigma,\varepsilon}^A\Bigg\}\frac{d\varepsilon}{16\pi i}, \tag{24}$$

where the renormalized Green's function [24]

$$\overline{G}_{\sigma,\varepsilon}^{R,A} = \frac{Z(\lambda)}{\varepsilon - \bar\xi_\sigma \pm i\bar g\Gamma_\sigma(\varepsilon)}, \qquad \bar\xi_\sigma = -\eta + \sigma\bar\Delta/2,$$
$$\Gamma_\sigma(\varepsilon) = \frac{1}{8\pi}(\varepsilon - \bar\xi_{-\sigma})[\mathcal{F}_{\bar\xi_{-\sigma}}^{-\sigma} + B_{\varepsilon-\bar\xi_{-\sigma}}]. \tag{25}$$

The pseudo-fermion distribution $\mathcal{F}_\varepsilon^\sigma$ is not known a priori. It is to be determined self-consistently from corresponding kinetic equation. It obeys [11]:

$$\mathcal{F}_\varepsilon^\sigma = \frac{B_{-\sigma(\varepsilon+\frac{\Delta\sigma}{2}+\eta)}\mathcal{F}_{\bar\xi_{-\sigma}}^{-\sigma} - \sigma}{B_{-\sigma(\varepsilon+\frac{\Delta\sigma}{2}+\eta)} - \sigma\mathcal{F}_{\bar\xi_{-\sigma}}^{-\sigma}}. \tag{26}$$

As was shown in [24] all terms of G^RG^R and G^AG^A type are controlled by renormalization scheme and can be discarded. Then, Eq. (24) becomes simplified:

$$\Pi_{s,pf}^R(\omega) = \frac{Z^4}{8}\sum_\sigma \partial_{\bar\xi_\sigma}\mathcal{F}_{\bar\xi_\sigma}^\sigma\Big[1 - \frac{\omega\mathbf{\Gamma}_\sigma^{RAR}(\bar\xi_\sigma+\omega,\bar\xi_\sigma,\omega)}{\omega + 2i\bar g\Gamma_\sigma}\Big], \tag{27}$$

where $\Gamma_\sigma = \Gamma_\sigma(\bar\xi_\sigma)$. The vertex function $\mathbf{\Gamma}^{RAR}$ solves the following Dyson equation

$$\mathbf{\Gamma}_\sigma^{RAR}(\varepsilon+\omega,\varepsilon,\omega) = 1 + \frac{ig}{4}\int\frac{dx}{2\pi}\overline{G}_{-\sigma,\varepsilon+\omega+x}^R\overline{G}_{-\sigma,\varepsilon+x}^A\, \times$$
$$\times \operatorname{Im}\Pi_x^R(B_x - \sigma)\mathbf{\Gamma}_{-\sigma}^{RAR}(\varepsilon+\omega+x,\varepsilon+x,\omega). \tag{28}$$

By using Eqs (26)–(27) and the solution of Eq. (28):

$$\frac{\mathbf{\Gamma}_\sigma^{RAR}(\bar\xi_\sigma+\omega,\bar\xi_\sigma,\omega)}{\omega+2i\bar g\Gamma_\sigma} = \frac{1}{\omega}\frac{\omega + 2i\bar g(\Gamma_{-\sigma} - \Gamma_\sigma)}{\omega + 2i\bar g(\Gamma_{-\sigma}+\Gamma_\sigma)}, \tag{29}$$

we obtain expression (10) for the admittance.

The computation of q' and g' is entangled with the computation of K_ω^R (see Eq. (14)). Using the definition of $K^R(t)$ in terms of the operators $X(t)$, one can obtain the following expression [34]:

$$K_\omega^R = -\frac{g}{8\pi}\int\frac{d\omega'}{2\pi}\Big[i\operatorname{Im}\mathcal{D}_{\omega'}(B_{\omega'} - B_{\omega'-\omega}) +$$
$$+\operatorname{Re}\mathcal{D}_{\omega'}B_{\omega'-\omega}\Big]\int(F_{\varepsilon+\omega'-\omega}^d - F_\varepsilon^r)d\varepsilon. \tag{30}$$

Here we introduce the transverse spin susceptibility $\mathcal{D}^R(t) = i\theta(t)\langle[S^-(t), S^+(0)]\rangle$. Following Eq. (14) one straight forwardly establishes:

$$g' = g\int\frac{d\omega}{2\pi}\operatorname{Im}\mathcal{D}_\omega^R\omega\partial_\omega B_\omega, \tag{31}$$

$$q' = Q + \frac{g}{4\pi}\int\frac{d\omega}{2\pi}\operatorname{Re}\mathcal{D}_\omega^R\partial_\omega(\omega B_\omega). \tag{32}$$

The average charge in the island is given in terms of the average isospin as: $Q = k + 1/2 - \langle S_z\rangle$. Using the result for the transverse spin susceptibility [11]:

$$\mathcal{D}_\omega^R = \frac{1}{B_{-\bar\Delta}}\frac{Z^2(\lambda)}{\omega + \bar\Delta + i0^+}, \tag{33}$$

we obtain results (15) for g' and q'.

In summary, the paper addresses the admittance and energy dissipation in an out-of-equlibrium single electron box under strong Coulomb blockade ($g \ll 1$). We deal with the regime when electron coherence can be neglected but quantum fluctuations of charge are strong due to Coulomb interaction. We derived the expression for the admittance at frequencies $\Omega \ll \max\{T_r, \varepsilon_d, |\bar{\Delta}|\}$. We found that the energy dissipation rate retains its universal appearance in the quasi-stationary limit even out of equilibrium. It is achieved in terms of specially chosen physical observables: the charge relaxation resistance and the renormalized gate capacitance. We propose the admittance as a tool for a measurement of the effective bosonic distribution corresponding to electron-hole excitations in the system.

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