# Tunneling Hall Effect 

P. S. Alekseev ${ }^{1)}$<br>Ioffe Physico-Technical Institute, 194021 St. Petersburg, Russia<br>Submitted 4 October 2010<br>Resubmitted 27 October 2010


#### Abstract

Electron tunneling in a semiconductor heterostructure with a barrier in a weak magnetic field applied parallel to the barrier interfaces is analyzed theoretically. A novel mechanism of the Hall effect in this structure is suggested. It is shown that the Hall current in the vicinity of the wide enough barrier is determined by the orbital effect of the magnetic field on the electron motion under the barrier, rather than by the electron $\overrightarrow{\mathcal{E}} \times \overrightarrow{\mathcal{H}}$-drift and scattering in the conductive regions lying to the left and to the right of the barrier.


1. Application of a magnetic field frequently reveals important features of numerous effects in semiconductors and metals and makes it possible to determine parameters of a material. The electron tunneling through a semiconductor barrier in a magnetic field has been extensively studied in this regard. This paper is concerned with the case in which the magnetic field is directed along the barrier interfaces (see figure). It is necessary to distinguish the cases of quantizing and non-quantizing


Energy diagram of the heterostructure. The magnetic field modifies the barrier as shown by use of thick dashed lines. In the inset: heterostructure with a barrier, an in-plane magnetic field, and the directions of the tunneling current $\left(\mathbf{j}_{z}\right)$ and the tunneling Hall current ( $\mathbf{j}_{\|}$)
magnetic fields. As usual, in a quantizing magnetic field the dependence of the current on the magnetic field and the applied bias has an oscillatory character [1]. The effect of a non-quantizing magnetic field on the tunneling current through a single barrier was experimentally and theoretically studied for the first time in [2]. The magnetic field leads to a slight modification of the tunneling current. The value of this modification was explained

[^0]quantitatively in [2] in terms of a semiclassical pattern of the electron motion under the barrier. Such an analysis conforms to the concept of the "traversal time for tunneling" [3].

At the same time, the structure in which 2D electrons in a quantum well (QW) are subjected to an inhomogeneous magnetic field has been fabricated quite recently. The inhomogeneous magnetic field was generated by Abrikosov vortexes in a 3D superconductor placed above the QW [4] or by narrow superconductor strips deposited above the QW [5]. In principle, it is possible to fabricate a 1D magnetic barrier for 2D electrons by using structures of a similar design. The 2D electron tunneling through 1D magnetic barriers was theoretically studied in [6]. It was shown that the barrier transparency coefficient depends not only on the wave vector component in the tunneling direction, but also on that along the 1D barrier interface.

In this paper, I consider the tunneling of 3D electrons through a single 2 D semiconductor barrier in a weak non-quantizing magnetic field directed along the barrier interfaces. It is shown that the tunneling probability depends on the wave vector in the plane of the interface (as in the case of 2D electrons in structures with a 1D magnetic barrier). This dependence gives rise to a surface current along the interface near the barrier. I demonstrate that the density of this current for realistic values of the heterostructure parameters may exceed the 3D Hall current density described by the Drude formulas. Such a generation of the in-plane electric current due to the cyclotron effect of the magnetic field under the barrier can be named the tunneling Hall effect.
2. Let me choose the coordinate axes and the magnetic field direction as shown in figure $\left(\overrightarrow{\mathcal{H}}=\mathcal{H} \mathbf{e}_{x}\right.$, $\mathcal{H}>0)$. Regions to the right and to the left of the barrier are equally strongly doped with donors and the temperature is low enough. If a bias $U_{0} / e$ is applied to the structure ( $e>0$ is the elementary charge), its main part falls
on the dielectric barrier (see figure). The magnetic field is classical: $\hbar \omega_{\mathrm{c}} \ll E_{\mathrm{F}}$. Hence, an electron moves along the classical cycloidal trajectory in both the right- and left-hand regions and this motion is interrupted by scattering events on the chaotic potential of donors and/or on acoustic phonons. The tunneling current flows along the $z$ axis, being largely controlled by the barrier transparency, rather than by the electron scattering (the case of a clean enough sample and high conductivities of the right- and left-hand regions is considered). Far from the barrier, the Hall current is controlled by the $\overrightarrow{\mathcal{E}} \times \overrightarrow{\mathcal{H}}$-drift and scattering. The Hall current density $j_{y}$ of electrons is given by the Drude formulas:

$$
\begin{align*}
& j_{z}=\frac{n e^{2}}{m} \frac{\tau}{1+\left(\omega_{\mathrm{c}} \tau\right)^{2}} \mathcal{E}_{3 \mathrm{D}} \\
& j_{y}=\frac{n e^{2}}{m} \frac{-\omega_{c} \tau^{2}}{1+\left(\omega_{\mathrm{c}} \tau\right)^{2}} \mathcal{E}_{3 \mathrm{D}} \tag{1}
\end{align*}
$$

where $n$ is the concentration of electrons; $m$, effective mass; $\tau$, scattering time considered to be equal within a numerical constant to the momentum relaxation time; $\omega_{\mathrm{c}}=e \mathcal{H} / m c$, cyclotron frequency; $j_{z}$, tunneling current density determined by $U_{0}$ and the barrier configuration; and $\mathcal{E}_{3 \mathrm{D}}$, electric field in the left- and right-hand conducting regions. Note that $\left|\mathcal{E}_{3 \mathrm{D}}\right|$ is far smaller than $\mathcal{E}=U_{0} / e a$, which is the absolute value of $\mathcal{E}_{z}$, the electric field in the barrier.

As the magnetic field is non-quantizing, it is true that $l_{\mathrm{m}} k_{\mathrm{F}} \gg 1$ and $r_{\mathrm{c}} \gg k_{\mathrm{F}}^{-1}\left(l_{\mathrm{m}}=\sqrt{c \hbar / e \mathcal{H}} ; r_{\mathrm{c}}=l_{\mathrm{m}}^{2} k_{\mathrm{F}}\right.$ and $k_{\mathrm{F}}^{-1}$ are the cyclotron radius and the wavelength of a characteristic electron). Thus, electrons in the rightand left-hand regions can be described as classical wave packets with the width $\Delta x, r_{\mathrm{c}} \gg \Delta x \gg k_{\mathrm{F}}^{-1}$, and the center $\overline{\mathbf{r}}(t)$, moving as a classical particle in the magnetic field. When the wave packet center $\overline{\mathbf{r}}(t)$ reaches the barrier (moment $t_{1}$ ), the wave packet is partly transmitted through, and partly reflected from the barrier as a quantum electron with the wave vector $m \dot{\mathbf{r}}\left(t_{1}\right) / \hbar$. For this reason, the effect of the magnetic field on the electron motion outside the barrier should be neglected when we study the tunneling process. Thus, in further calculations I assume that the magnetic field exists only within the barrier. Such an approach was used in papers [7, 2].

The general concept of the tunneling Hall effect is following. Similarly to the usual 3D situation, the electron motion under the barrier is accompanied by a cyclotron effect of the magnetic field in the $(y, z)$-plane. Quantitatively, this is reflected by the fact that the incident electrons with $k_{y}$ and $-k_{y}$ have different semiclassical tunneling times introduced in [3] and different tunneling and reflection amplitudes. In other words, the
magnetic field leads to the filtering of electrons with one preferred direction of the wave vector $k_{y}$ during tunneling through the barrier. Thus, it is reasonable to believe that, for some heterostructure parameters, the Hall current within a distance of about the scattering length from the barrier may be determined by a tunneling process, rather than by the 3D mechanism.

Electrons in the right- and left-hand regions are considered to be in quasi-equilibrium states with the Fermi distributions $f_{\mathrm{L}}$ and $f_{\mathrm{R}}$. Because it is expected that both the 3D and the tunneling Hall currents correspond to a weak modification of the Fermi distributions, but the tunneling Hall current is due to all conduction electrons, the distribution function of incident electrons is supposed to be symmetrical and Fermi-like.
3. With the assumption of the absence of the magnetic field outside the barrier, the classification of electron states is the same as that in the absence of a magnetic field. The electron states in the right- and left-hand regions are plane waves partly transmitted through, and partly reflected from the barrier. It is convenient to take a vector potential in the Landau gauge: $\mathbf{A}(\mathbf{r})=0$ in the left-hand region, $\mathbf{A}(\mathbf{r})=-\mathcal{H} z \mathbf{e}_{y}$ within the barrier, and $\mathbf{A}(\mathbf{r})=-\mathcal{H} a \mathbf{e}_{y}$ in the right-hand region. The Zeeman coupling and the nonparabolicity are neglected. The electron Hamiltonian has the form:

$$
\begin{align*}
\hat{H} & =\frac{1}{2 m}\left(\hat{\mathbf{p}}+\frac{e}{c} \mathbf{A}\right)^{2}+V(z), \\
V(z) & =\left[\begin{array}{ll}
0, & z<0 \\
V_{0}-e \mathcal{E} z, & 0<z<a \\
-e \mathcal{E} a, & z>a
\end{array}\right. \tag{2}
\end{align*}
$$

As the vector potential contains only the $z$ space coordinate, the electron wave functions are the plane waves in the $(x, y)$-plane: $\psi_{\mathbf{k}}(\mathbf{r})=e^{i\left(k_{x} x+k_{y} y\right)} u_{k_{y}, k_{z}}(z)$. The Hamiltonian for the wave function $u_{k_{y}, k_{z}}(z) \equiv u(z)$ takes a form of a magnetic-field-induced correction to the potential energy, $\delta V_{k_{y}}(z): \hat{H}_{z} u=E_{z} u, E=$ $E_{z}+\hbar^{2}\left(k_{x}^{2}+k_{y}^{2}\right) / 2 m$,

$$
\begin{gather*}
\hat{H}_{z}=-\frac{\hbar^{2}}{2 m} \frac{d^{2}}{d z^{2}}+V(z)+\delta V_{k_{y}}(z), \\
\delta V_{k_{y}}(z)=\left[\begin{array}{l}
0, z<0 \\
\frac{m \omega_{\mathrm{c}}^{2}}{2}\left(z-z_{0}\right)^{2}-\frac{\hbar^{2} k_{y}^{2}}{2 m}, 0<z<a \\
\frac{m \omega_{\mathrm{c}}^{2}}{2}\left(a-z_{0}\right)^{2}-\frac{\hbar^{2} k_{y}^{2}}{2 m}, z>a
\end{array}\right. \tag{3}
\end{gather*}
$$

where $z_{0}=k_{y} l_{\mathrm{m}}^{2}$.
Let the barrier width $a$ be substantially smaller than the characteristic electron cyclotron radius $r_{\mathrm{c}}: r_{\mathrm{c}} \gg a$. It is essential to assume that the values of $E_{\mathrm{F}}$ and $U_{0}$ are on the same order of magnitude and far smaller than the barrier height $V_{0}$. The barrier is considered to be wide, $k_{0} a \gg 1\left(k_{0}=\sqrt{2 m V_{0}} / \hbar\right)$.

While $r_{\mathrm{c}}=k_{\mathrm{F}} l_{\mathrm{m}}^{2}, r_{\mathrm{c}} \gg a$ and $m \omega_{\mathrm{c}}^{2} z_{0}^{2} \equiv \hbar^{2} k_{y}^{2} / m$, the inequality $\left|\delta V_{k_{y}}(z)\right| \ll E_{\mathrm{F}}$ is true [see (3)]. Therefore, the correction $\delta V_{k_{y}}$ to the potential energy within the barrier and the electric field energy $-e \mathcal{E} z, e \mathcal{E} z<U_{0}$, are far smaller than $V_{0}$. This circumstance makes it possible to use the WKB asymptotic for the wave function within the barrier ${ }^{2}$ :

$$
\begin{equation*}
u(z)=\frac{A \operatorname{sh}\left[k_{0} z+\delta S / \hbar\right]+B \operatorname{ch}\left[k_{0} z+\delta S / \hbar\right]}{\sqrt{\hbar k_{0}+\delta S^{\prime}}} \tag{4}
\end{equation*}
$$

where the action correction $\delta S$ has the form:

$$
\delta S(z)=\sqrt{\frac{m}{2 V_{0}}}\left[\int_{0}^{z} \delta V_{k_{y}}\left(z^{\prime}\right) d z^{\prime}+\frac{-e \mathcal{E} z^{2}}{2}\right]
$$

The continuity of the zeroth and first derivatives of the wave function, which are plane waves $u(z)=$ $=e^{i k_{z} z}+r e^{-i k_{z} z}, u(z)=t e^{i k_{z}^{r} z}$ in the left- and right-hand regions and the WKB function (4) within the barrier, should be maintained at the points $z=$ $=0$ and $z=a$. Here, the quantity $k_{z}^{\mathrm{r}}\left(k_{z}, k_{y}\right)=$ $=\sqrt{k_{z}^{2}-2 m\left[\delta V_{k_{y}}(a)-e \mathcal{E} a\right] / \hbar^{2}}$ is the $z$ component of the wave vector in the right-hand region modified by the additional potentials of the electric and the magnetic fields [see (3) and figure].

A full solution of the formulated problem of finding $A, B, r$ and $t$ was made. However, in order to simplify calculations, it is reasonable to accept the following restrictions (which are additional to those made above). First, let me consider not too small wave vectors $k_{y}$ only: $\left|k_{y}\right| l_{\mathrm{m}}^{2} \gg a$. With the fact that $r_{\mathrm{c}} \gg a$ kept in mind, the inequality $\left|k_{y}\right| l_{\mathrm{m}}^{2} \gg a$ is valid for most of electrons in the real situation, when the electron distribution function is the symmetrical Fermi function. Second, let me consider that the magnetic field potential $\delta V_{k_{y}}$ affects the barrier transparency only by modifying the classical action in the argument of the functions sh and ch in (4), rather than by modifying the prefactors and the derivatives of the functions sh and ch (all these modifications are in the equations for $r, t, A$ and $B$ ). It was

[^1]confirmed that this situation takes place if the inequalities $k_{0} a \gg 1$ and $k_{\mathrm{F}} a \gg k_{0} / k_{\mathrm{F}}$ are satisfied. Third, in order to make the magnetic field induced action correction small: $\delta S_{\text {mag }}(a) / \hbar \ll 1$, let the magnetic field be weak enough: $\left(k_{\mathrm{F}} a\right)^{2} \ll k_{0} r_{\mathrm{c}}$. If all the conditions discussed above are satisfied, the transmission coefficient $D=\left(k_{z}^{\mathrm{r}} / k_{z}\right)|t|^{2}=1-|r|^{2}$ is:
\[

$$
\begin{equation*}
D\left(k_{z}, k_{y}\right)=\frac{16 k_{z} k_{z}^{\mathrm{r}}}{k_{0}^{2}} e^{-2 \tilde{k}_{0} a}\left[1+\frac{k_{y} a^{2}}{k_{0} l_{\mathrm{m}}^{2}}\right] \tag{5}
\end{equation*}
$$

\]

where an imaginary wave vector under the barrier, modified by the electric field, $\tilde{k}_{0}=\sqrt{2 m\left(V_{0}-U_{0} / 2\right)} / \hbar$, was introduced. Due to the restrictions imposed on the problem parameters, it should be taken that $k_{z}^{\mathrm{r}}\left(k_{y}, k_{z}\right) \equiv$ $\equiv \sqrt{k_{z}^{2}+2 m U_{0} / \hbar^{2}}$ if $U_{0} \sim E_{\mathrm{F}}$.

The magnetic-field-dependent term in the square brackets in (5) has a semiclassical nature because of the fact that it is due to the "under-barrier" action correction $\delta S$. Thus, this term is determined only by the properties of the "imaginary", but classical motion of an electron under the barrier in the $z$-direction with $\tilde{E}_{z}=-E_{z}$, $\tilde{V}(z)=-V(z)$, and $\delta \tilde{V}_{k_{y}}(z)=-\delta V_{k_{y}}(z)$ and can be written in the semiclassical form: $\left(m \omega_{\mathrm{c}} v_{y} a^{2} / v_{0}\right) / \hbar$, where $v_{0, y}=\hbar k_{0, y} / m$. This statement follows from the connection between the solutions of the Schrodinger equation and the classical motion equations (in the case when the WKB method can be used) and is in the spirit of the "imaginary time" method of calculation of the atom ionization probability in an electric field [9].

It is noteworthy that the magnetic field correction to the tunneling probability (5) can be obtained by use of the perturbation theory in the continuous spectrum. The unperturbated wave functions $u_{k_{z}, 1 / \mathbf{r}}^{(0)}(z)$ correspond to the Hamiltonian (2) with $\mathbf{A}=0$ and are plane waves incident from the left or from the right, partly transmitted through, and partly reflected from the barrier $V(z)$. The perturbation is the potential energy correction $\delta V_{k_{y}}$ inside the barrier. A calculation based on the formulas from [10] leads to the correction to the tunneling amplitude:

$$
\delta t_{\mathrm{l}}^{(1)}=r_{\mathrm{r}}^{(0)} \hbar \omega_{\mathrm{c}} k_{y} \frac{i m}{\hbar^{2} k_{z}^{\mathrm{r}}} \int_{0}^{a}\left[u_{k_{z}, \mathrm{r}}^{(0)}(z)\right]^{*} u_{k_{z}, \mathrm{l}}^{(0)}(z) z d z
$$

where $r_{\mathrm{r}}^{(0)} \approx-1$ is the reflection amplitude of the waves incident from the right. It was proved that $\left(k_{z}^{\mathrm{r}} / k_{z}\right)\left|t_{1}^{(0)}+\delta t_{1}^{(1)}\right|^{2}$ is equal to (5) in the first order by $\mathcal{H}$.
4. The fact that the tunneling probability (5) depends on $k_{y}$ results in that the tunneled electrons, which had a symmetrical distribution function in the left-hand region, will have a nonzero mean $y$-directed momentum
in the right-hand region. During the motion of the tunneled electrons in the $z$ direction in the right-hand region, their scattering and, thus, the relaxation of their mean momentum will occur. So, a surface current will flow near the right-hand barrier edge in a layer with a width of about $l_{\text {sc }}$ (or the Hall voltage will be generated, as in the common Hall effect measurements).

Let me restrict the analysis to calculation of the tunneling Hall current in the right-hand region, which is due to electrons tunneled from the left-hand region only, with the $k_{y}$-dependent reflection of electrons in the righthand region disregarded. This corresponds to the case in which the applied bias is high enough: $U_{0}>E_{\mathrm{F}}$ (but, nevertheless, $U_{0} \sim E_{\mathrm{F}}$; see figure) and $T=0$. Thus, the aim of the present calculation is an order-of-magnitude estimation of the surface current in the situation of its saturation, rather than a detailed calculation of its $U_{0}$ dependence for $0<U_{0}<E_{\mathrm{F}}$. In spirit of the simple Landauer approach of calculation of tunneling currents and following [11], I calculate the flux (in the $z$ direction) of the $y$ component of the velocity of tunneled electrons as a sum of values $v_{z} v_{y}$ multiplied by the transmission coefficient over all the populated states in the left-hand region with $k_{z}>0$. The surface current is a product of this flux by $-e$ and $\tau$ :

$$
j_{| |}=-\frac{2 e \hbar^{2} \tau}{(2 \pi)^{3} m^{2}} \int_{k_{z}>0} f_{\mathrm{L}}\left(\varepsilon_{\mathbf{k}}\right) k_{y} k_{z} D\left(k_{y}, k_{z}\right) d \mathbf{k}
$$

where 2 in the numerator is due to the spin degeneracy. If $T=0$, a simple calculation based on (5) gives ${ }^{3)}$ :

$$
\begin{equation*}
j_{\|}=-0.152 \frac{e \hbar^{2} \tau}{\pi^{2} m^{2}} \frac{e^{-2 \tilde{k}_{0} a}\left\langle k_{z}^{\mathrm{r}}\right\rangle_{1} k_{\mathrm{F}}^{7}}{k_{0}^{2}} \frac{a^{2}}{l_{\mathrm{m}}^{2} k_{0}} \tag{6}
\end{equation*}
$$

where $\left\langle k_{z}^{\mathrm{r}}\right\rangle_{1}=\left\langle k_{z}^{\mathrm{r}}\right\rangle_{1}\left(U_{0}\right)$ is a quantity on the same order of magnitude as $k_{\mathrm{F}}$. A similar calculation for the tunneling current

$$
j_{z}=-\frac{2 e \hbar}{(2 \pi)^{3} m} \int_{k_{z}>0} f_{\mathrm{L}}\left(\varepsilon_{\mathbf{k}}\right) k_{z} D\left(k_{y}, k_{z}\right) d \mathbf{k}
$$

yields:

$$
\begin{equation*}
j_{z}=-0.533 \frac{e \hbar}{\pi^{2} m} \frac{e^{-2 \tilde{k}_{0} a}\left\langle k_{z}^{\mathrm{r}}\right\rangle_{2} k_{\mathrm{F}}^{5}}{k_{0}^{2}} \tag{7}
\end{equation*}
$$

where $\left\langle k_{z}^{\mathrm{r}}\right\rangle_{2}=\left\langle k_{z}^{\mathrm{r}}\right\rangle_{2}\left(U_{0}\right)$ is also a quantity on the same order of magnitude as $k_{\mathrm{F}}$. A numerical calculation of

[^2]$\left\langle k_{z}^{\mathrm{r}}\right\rangle_{1}$ and $\left\langle k_{z}^{\mathrm{r}}\right\rangle_{2}$ shows that they lie between $2 k_{\mathrm{F}}$ and $3 k_{\mathrm{F}}$ when $U_{0}$ lies between $U_{0}$ and $3 U_{0}$.

For the considered configuration of the structure (figure), it is true that $\mathcal{E}_{z}=-\mathcal{E}<0$ and $\mathcal{E}_{3 \mathrm{D}}<0$. Thus, on base of (1) and (6), one concludes the 3D Hall current and the tunneling Hall current have different directions. It is noteworthy that the result (6) is linear in $\mathcal{H}$, whereas the magnetic field correction to $j_{z}$ is quadratic in $\mathcal{H}$ [2].

The surface tunneling Hall current can be observed if its density exceeds that of the 3D Hall current. To estimate the tunneling Hall current density (near the barrier), $j_{\|}$should be divided by the scattering length $l_{\text {sc }}=$ $\hbar\left\langle k_{z}^{\mathrm{r}}\right\rangle \tau / m$, where $\left\langle k_{z}^{\mathrm{r}}\right\rangle$ can be taken the same as $\left\langle k_{z}^{\mathrm{r}}\right\rangle_{1}$ in (6). Using (7), we obtain for the ratio of this current density to the tunneling current density (with a numerical coefficient omitted): $j_{y}^{\text {tunn }} / j_{z} \sim k_{\mathrm{F}} a^{2} / k_{0} l_{\mathrm{m}}^{2}$. Compare this formula with the formulas (1) for the 3D Hall current density. Equations (1) leads to $j_{y}^{3 \mathrm{D}}=j_{z} \omega_{c} \tau$. Therefore, the ratio of the tunneling Hall current density to the 3D Hall current density is given by

$$
\frac{j_{y}^{\text {tunn }}}{j_{y}^{3 \mathrm{D}}} \sim \frac{k_{\mathrm{F}} a^{2}}{v_{0} \tau} \sim \frac{\left(k_{\mathrm{F}} a\right)^{2}}{k_{0} l_{\mathrm{sc}}} .
$$

Now, we can formulate a criterion for prevalence of the tunneling Hall current over the 3D Hall current: $k_{\mathrm{F}} a \gg$ $\sqrt{k_{0} l_{\mathrm{sc}}}$ (the barrier must be wide enough). At the same time, it is necessary that $a<l_{\text {sc }}$ for the ballistic tunneling picture to be relevant. It is noteworthy that, according to these estimations, $\mathcal{E}_{3 \mathrm{D}} \sim-\left(a k_{\mathrm{F}}^{2} / l_{\mathrm{sc}} k_{0}^{2}\right) e^{-2 \tilde{k}_{0} a} \mathcal{E}$.

Let me summarize all the inequalities, discussed above, which guarantee that the tunneling Hall current is observable. The structure configuration must satisfy the inequalities: $k_{\mathrm{F}} \ll k_{0}$ and $k_{\mathrm{F}} a \gg k_{0} / k_{\mathrm{F}}$. The scattering length must lie within the interval: $a<l_{\mathrm{sc}} \ll\left(k_{\mathrm{F}} a\right)^{2} / k_{0}$. The magnetic field must have a cyclotron radius $r_{\mathrm{c}}$ far greater than $\left(k_{\mathrm{F}} a\right)^{2} / k_{0}$. Note that it follows from these inequalities that $a \gg k_{\mathrm{F}}^{-1}$ and $r_{\mathrm{c}} \gg a, l_{\mathrm{sc}}$.

In paper [2], a GaAs/AlGaAs heterostructure with the parameters $V_{0}=43 \mathrm{meV}, a=25 \mathrm{~nm}, E_{\mathrm{F}}=12 \mathrm{meV}$ was studied. For this structure, the "geometrical" inequalities are valid with a twofold safety margin, which is sufficient for a qualitative manifestation of the predicted effect. The optimal scattering length for this structure is $l_{\mathrm{sc}} \sim 3 \cdot 10^{-6} \mathrm{~cm}$. Such $l_{\mathrm{sc}}$ is a physically reasonable value and provides a 1.5 -fold safety margin for the inequalities for $l_{\mathrm{sc}}$. The corresponding magnetic fields must be lower than $\sim 1 \mathrm{~T}$ (this value also corresponds to a twofold safety margin for the magnetic field inequality). The surface current calculated using (6) is $j_{\|} \sim 3 \cdot 10^{-7} \mathrm{~A} / \mathrm{cm}$ for this structure at $\mathcal{H}=1 \mathrm{~T}$. It is noteworthy that raising the barrier width by even a factor of 1.5 , with all the other parameters remaining the
same, will make the ranges of the appropriate values of $l_{\text {sc }}$ considerably wider.

It is reasonable to try to measure the suggested effect in a heterostructure with several barriers (i.e., in a superlattice with wide QW regions). If the distance between barriers is about $l_{\mathrm{sc}}$, the Hall effect in the whole superlattice will be mainly due to the sum of the tunneling Hall effects near each barrier.
5. The spin-orbit coupling can influence the probability of tunneling in clean enough heterostructures [11, 12]. For example, the surface current along the interface can be generated as a result of the spin-orbit coupling of the spin-polarized electrons tunneling through a barrier grown of noncentrosymmetrical semiconductors (tunneling spin-galvanic effect [11]). Of interest is the possible effect of the spin-orbit coupling on the tunneling Hall effect under study. Let spin-polarized electrons tunnel through the barrier in a GaAs/AlGaAs heterostructure in a weak in-plain magnetic field. The goal is to calculate the total surface current in the right-hand region. It was proved that, in the case of spin-polarized electrons tunneling in a weak magnetic field, the tunneling Hall effect and the tunneling spin-galvanic effect are independent and give additive contributions to the surface current near the barrier. The estimate of the tunneling Hall current, obtained above for the structure studied in [2], is on the same order of magnitude $\left(10^{-7} \mathrm{~A} / \mathrm{cm}\right)$ as the estimate of the tunneling spin-galvanic current in [11] for a similar GaAs/AlGaAs structure.

In conclusion, I would like to make two comments. First, the nature of the tunneling Hall effect is similar to that of the Hall effect in the hoping conductivity mode [13] in which, as in the present study, the Hall current component arises due to the electron motion with a negative kinetic energy. Second, in atomic physics, the "imaginary time" method of calculation of the tunneling probability of an electron from an atom, similar to the consideration of the present paper, was used to study the effect of a magnetic field on the atom ionization in an electric field [14].

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[^0]:    1) e-mail: pavel.alekseev@mail.ioffe.ru
[^1]:    ${ }^{2)}$ It is impossible to use the WKB asymptotic for the wave function in the whole structure. This is associated with the sharp edges of the barrier and gives rise to a pre-exponential factor in the semiclassical tunneling probability $D \sim e^{-2 k_{0} a}$ [8].

[^2]:    ${ }^{3)}$ The numerical constants in (6) and (7) are exact fractions, but are written in the decimal mode due to their being lengthy and because of the presence of numerically calculated values $\left\langle k_{z}^{\mathrm{r}}\right\rangle_{i}$.

