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Generation of two-photon correlation in a four-level system

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We study the quantum correlation of a coherently foul-level atomic system coupled by two coherent driven fields. Used the wave function technique, an explicit expression of the photon-photon correlation function is derived. The influence of the driven field as well as the detuning on the second correlation function are investigated. We find that the second correlation function can be controlled by the driven field or the detuning of two coupling lasers. So we can control the magnitude of the joint detection probability by the driving fields. More importantly, we show that this correlated photon states can serve as atomic memory.

I. Introduction. Nonclassical light source is produced artificially and does not exist in nature. It is found that correlated photon pairs can provide optical resolution beyond the diffraction limit through coincident photon detection [1] and is proposed for quantum imaging [2]. As a current interesting subject, the generation of high intensity correlation photon source is studied by many authors [3, 4]. Correlated pairs are very useful because it can be used in quantum information such as quantum memory [5], quantum communication [6], and in controllable few-photon pulse generation [7]. Such pairs can also be used for enhancement of resolution in quantum microscopy and lithography [8 – 10]. The stokes-anti-stokes Raman emission(RED) as a scheme to produce high intensity correlation is discussed in the early works of Agarwal et al. [11] and Scully et al. [12]. A series of experiments is also presented by several groups [5 – 7, 13, 14]. The quantum correlation in a three or four level atomic system shows strong anti-bunching feature and have a controllable correlation [6, 14, 15]. Scully [16] indicates that using strong correlated photon in Raman emission the magnitude of the joint detection probability can be controlled by the classical field.

On the other hand, quantum information requires the use of photons as quantum information carriers and matter as quantum memory [17]. For example, it is shown that intermediate memory nodes are necessary for quantum communication and quantum cryptography over long photonic channels [18]. It is demonstrated that emission of two quantum-mechanically correlated light pulses with a time delay that is coherently controlled via temporal storage of photonic states in an ensemble of atoms.

In the present work, we investigate the intensity correlations that are generated in a coherently foul-level atomic system coupled by two coherent driven fields. A detailed calculation for the Raman two photon quantum state as well as the two photon correlation function is presented. The influence of the driven field on the second correlation function are investigated. The results show that the second correlation function can be controlled by the driven field. That is, we can control the magnitude of the joint detection probability by tuning the driving field. Moreover, we show that this correlated photon states can serve as atomic memory.

II. The model and calculation. This section presents the calculation of two photon correlation. A four-level atom driven by two classical fields is considered. The atomic level configuration is depicted in Fig.1. Off-resonant transitions $|c\rangle \leftrightarrow |d\rangle$ and $|d\rangle \leftrightarrow |a\rangle$ are established by two driving fields with Rabi frequencies Ω_1 and Ω_2. The corresponding detunings are Δ_1 and Δ_2.

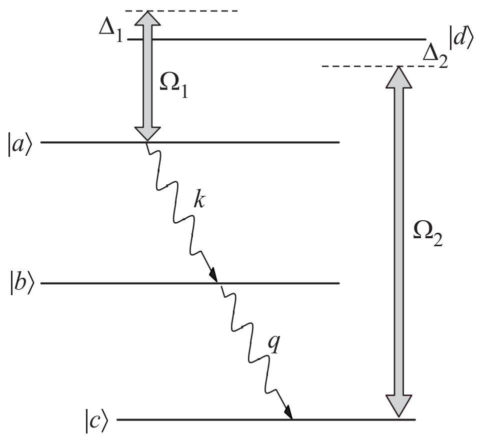

Fig.1. The level configuration of four-level atomic configuration: the transitions from $|a\rangle$ to $|b\rangle$ and $|b\rangle$ to $|c\rangle$ can produce the spontaneous emission spectra of k and q photons, respectively. Off-resonant transitions $|a\rangle \longleftrightarrow |d\rangle$ and $|c\rangle \longleftrightarrow |d\rangle$ are established by using external driving fields with Rabi frequencies Ω_1 and Ω_2. The corresponding detunings are Δ_1 and Δ_2

In the interaction picture, the full Hamiltonian for the scheme shown in Fig.1 is given by

$$\hat{H} = \Delta_1|a\rangle\langle a| - \Delta_2|c\rangle\langle c| + \Omega_1|a\rangle\langle d| + \Omega_2|c\rangle\langle d| + \\ + \sum_k g_k^*(r_0)a_k|a\rangle\langle b|e^{-i\delta_1 t} + \\ + \sum_q g_q^*(r_0)a_q|b\rangle\langle c|e^{-i\delta_2 t} + \text{h.c.}, \qquad (1)$$

where $\Omega_{1(2)}$ is Rabi frequencies associated with the classical field, $\delta_1 = \delta_k - \Delta_1$ and $\delta_2 = \delta_q - \Delta_2$ with $\delta_k = \omega_{ab} - \omega_k$ and $\delta_q = \omega_{bc} - \omega_q$. $|i\rangle\langle j|(i, j = a, b, c)$ are the atomic operator. $g_k(r_0)$ and $g_q(r_0)$ are the coupling frequencies for photon emissions between level $|a\rangle \leftrightarrow |b\rangle$ and $|b\rangle \leftrightarrow |c\rangle$, and $a_k(a_k^+)$ and $a_q(a_q^+)$ are the annihilation(creation) operators of photon k and photon q, respectively.

The wave function method is used in treating RED(raman emission doublet) [16] as well as CED (cascade emission doublet) [19]. Here, it is also used to calculate the correlation of spontaneous emission. Assume that initial state of the atom is in the state $|b\rangle$ and the field modes are in the vacuum state $|00\rangle$, the corresponding atom-field state of the system can be written as

$$|\Psi(t)\rangle = B_1(t)|b,0,0\rangle + \sum_q C_q(t)|c,0,1_q\rangle + \\ + \sum_q D_q(t)|d,0,1_q\rangle + \sum_q A_q(t)|a,0,1_q\rangle + \\ + \sum_{kq} B_{kq}(t)|b,1_k,1_q\rangle. \qquad (2)$$

With the help of schrödinger equation, we obtain the equations of motion for the probability amplitudes c_0, c_{qa}, c_{qc} and c_{kq} as

$$\begin{gathered} i\dot{B}_1(t) = \sum_q g_q C_q(t)e^{-i\delta_2 t}, \\ i\dot{C}_q(t) = -\Delta_2 C_q(t) + \Omega_2 D_q(t) + g_q B_1(t)e^{i\delta_2 t}, \\ i\dot{D}_q(t) = \Omega_1 A_q(t) + \Omega_2 C_q(t), \\ i\dot{A}_q(t) = \Delta_1 A_q(t) + \Omega_1 D_q(t) + \sum_k g_k B_{kq}(t)e^{-i\delta_1 t}, \\ i\dot{B}_{kq}(t) = g_k A_q(t)e^{i\delta_1 t}. \end{gathered} \qquad (3)$$

By solving above equations with the Weisskopf-Wigner approximation [15], one can determine the state of atom and the state of the radiation field, and gets the relations $\sum_k |g_k(r_0)|^2 = \Gamma_a, \sum_q |g_q(r_0)| = \Gamma_b$, where Γ_a and Γ_b mean the atomic decay rates from state $|a\rangle$ to $|b\rangle$ and $|b\rangle$ to $|c\rangle$, respectively. Eq. (3) can be rewritten as

$$\begin{gathered} \dot{B}_1(t) = -\Gamma_2 B_1(t), \\ \dot{C}_q(t) = i\Delta_2 C_q(t) - i\Omega_2 D_q(t) - ig_q B_1(t)e^{i\delta_2 t}, \\ \dot{D}_q(t) = -i[\Omega_1 A_q(t) + \Omega_2 C_q(t)], \\ \dot{A}_q(t) = -i[\Delta_1 A_q(t) + \Omega_1 D_q(t)] - \Gamma_1 A_q(t), \\ \dot{B}_{kq}(t) = -ig_k A_q(t)e^{i\delta_1 t}, \end{gathered} \qquad (4)$$

Using Laplace transformations, above equations can be derived as

$$\begin{aligned} B_1(s) &= \frac{1}{s+\Gamma_2}, \\ D_q(s) &= -\frac{\Omega_2 g_q(s+\Gamma_1+i\Delta_1)}{(s-i\delta_2+\Gamma_2)[s(s-i\Delta_2)(s+i\Delta_1+\Gamma_1)+\Omega_2^2(s+i\Delta_1+\Gamma_1)+\Omega_1^2(s-i\Delta_2)]}, \\ C_q(s) &= -\frac{ig_q[s(s+\Gamma_1+i\Delta_1)+\Omega_2^2]}{(s-i\delta_2+\Gamma_2)[s(s-i\Delta_2)(s+i\Delta_1+\Gamma_1)+\Omega_2^2(s+i\Delta_1+\Gamma_1)+\Omega_1^2(s-i\Delta_2)]}, \\ A_q(s) &= \frac{i\Omega_1\Omega_2 g_q}{(s-i\delta_2+\Gamma_2)[s(s-i\Delta_2)(s+i\Delta_1+\Gamma_1)+\Omega_2^2(s+i\Delta_1+\Gamma_1)+\Omega_1^2(s-i\Delta_2)]}, \\ B_{kq}(s) &= \frac{\Omega_1\Omega_2 g_k g_q}{[(s-i\delta_1)(s-i\delta_1-i\Delta_2)(s-i\delta_1+i\Delta_1+\Gamma_1)+\Omega_2^2(s-i\delta_1+i\Delta_1+\Gamma_1)+\Omega_1^2(s-i\delta_1-i\Delta_2)]}\times \\ &\quad \times\frac{1}{s(s-i\delta_1-i\delta_2+\Gamma_2)}. \end{aligned} \qquad (5)$$

When the time tends to infinite, we can obtain

$$|C_q(t\to\infty)|^2=|B_1(t\to\infty)|^2= \\ =|D_q(t\to\infty)|^2=|A_q(t\to\infty)|^2=0, \\ |B_{kq}(t\to\infty)|^2=|-ig_kA_q(s=i\delta_1)|^2= \\ =\left|\frac{\Omega_1\Omega_2g_kg_q}{(\delta_1+\delta_2+i\Gamma_2)}\left[\frac{\chi_1}{\delta_1+i\lambda_1}+\frac{\chi_2}{\delta_1+i\lambda_2}+\frac{\chi_3}{\delta_1+i\lambda_3}\right]\right|^2, \quad (6)$$

with

$$\chi_1=\frac{\lambda_3-\lambda_2}{\lambda_1^2(\lambda_2-\lambda_3)+\lambda_2^2(\lambda_3-\lambda_1)+\lambda_3^2(\lambda_1-\lambda_2)}, \\ \chi_2=\frac{\lambda_1-\lambda_3}{\lambda_1^2(\lambda_2-\lambda_3)+\lambda_2^2(\lambda_3-\lambda_1)+\lambda_3^2(\lambda_1-\lambda_2)}, \quad (7) \\ \chi_3=\frac{\lambda_2-\lambda_1}{\lambda_1^2(\lambda_2-\lambda_3)+\lambda_2^2(\lambda_3-\lambda_1)+\lambda_3^2(\lambda_1-\lambda_2)},$$

where λ_1, λ_2 and λ_3 are the roots of the following cubic equation

$$x^3-x^2[\Gamma_1+i(\Delta_1-\Delta_2)]+x(\Delta_1\Delta_2+\Omega_1^2+ \\ +\Omega_2^2-i\Delta_2\Gamma_1)+i\Omega_1^2\Delta_2-\Omega_2^2(\Gamma_1+i\Delta_1)=0. \quad (8)$$

The second order correlation function can be expressed as

$$G^{(2)}(1,2)=\langle\Psi|E_1^{(-)}E_2^{(-)}E_2^{(+)}E_1^{(+)}|\Psi\rangle= \\ =\langle\Psi|E_1^{(-)}E_2^{(-)}|0\rangle\langle 0|E_2^{(+)}E_1^{(+)}|\Psi\rangle=|\Psi^{(2)}(1,2)|^2, \quad (9)$$

where $\Psi^{(2)}(1,2)=\langle 0|E_2^{(+)}E_1^{(+)}|\Psi\rangle$, in which $E^{(+)}$ is the positive frequency (annihilation operator) part of the electric field operator. In this system, k photon is detected then q photon successively, we obtain

$$\psi_1^{(2)}(r_1,t_1;r_2,t_2)= \\ =i\Omega_1\Omega_2g_{a,k_0}g_{b,q_0\sigma(k_0)\sigma(q_0)}\chi_1\int_0^\infty q^2dq\int_0^\pi\sin\theta_qd\theta_q\times \\ \times\int_0^{2\pi}d\phi_q\int_0^\infty k^2dk\int_0^\pi\sin\theta_kd\theta_k\int_0^{2\pi}d\phi_k\times \\ \times[e^{-ickt_1+ik\Delta r_1cos\theta_k}e^{-icqt_2+iq\Delta r_2cos\theta_q}+(1\leftrightarrow 2)]\times \\ \times\frac{1}{(\delta_1+\delta_2+i\Gamma_2)}[\frac{\chi_1}{\delta_1+i\lambda_1}+\frac{\chi_2}{\delta_1+i\lambda_2}+\frac{\chi_3}{\delta_1+i\lambda_3}]= \\ =i\Omega_1\Omega_2\frac{(2\pi)^2}{i^2\Delta r_1\Delta r_2}k_0g_{a,k_0}q_0g_{b,q_0\sigma(k_0)\sigma(q_0)}\chi_1\times \\ \times\int_0^\infty\int_0^\infty dkdq\times \\ \frac{e^{-ickt_1}(e^{-ik\Delta r_1}-e^{ik\Delta r_1})e^{-icqt_2}(e^{-iq\Delta r_2}-e^{iq\Delta r_2})}{[i(\omega_{ab}-ck)+i(\omega_{bc}-cq)+\Gamma_2][-i(\omega_{ab}-ck)-\lambda_1)]}. \quad (10)$$

Neglecting the unphysical incoming waves which go as $e^{-ik\Delta r_1}$, we perform integration on the q photon and k photon, subsequently, to find

$$G^{(2)}(r_1,t_1;r_2,t_2)\propto \\ \propto|\Omega_1\Omega_2|^2|e^{[i(\omega_{ac}-\Delta_1-\Delta_2)-\Gamma_2]}\Theta(\tau_2)\times \\ \times(\chi_1e^{-[\lambda_1-i(\omega_{ab}-\Delta_1)](\tau_1-\tau_2)}+ \\ +\chi_2e^{-\{\Re(\lambda_2)-i[\Im(\lambda_2)+\omega_{ab}-\Delta_1]\}(\tau_1-\tau_2)}+ \\ +\chi_3e^{-\{\Re(\lambda_2)-i[-\Im(\lambda_2)+\omega_{ab}-\Delta_1]\}(\tau_1-\tau_2)}|^2+(1\leftrightarrow 2)= \\ =|\Omega_1\Omega_2|^2|e^{-\Gamma_2\tau_2}\Theta(\tau_2)\{\chi_1e^{-\lambda_1(\tau_1-\tau_2)}+ \\ +\chi_2e^{-\lambda_2(\tau_1-\tau_2)}+\chi_3e^{-\lambda_3(\tau_1-\tau_2)}\}|^2+(1\leftrightarrow 2), \quad (11)$$

where $\tau_1=t_1-\frac{\Delta r_1}{c}$ and $\tau_2=t_2-\frac{\Delta r_2}{c}$.

III. Numerical results and discussion. Now we present a few numerical results about the photon-photon correlation function based above Eq. (11). All parameters are scaled with Γ ($\Gamma=\Gamma_1=\Gamma_2$). We plot the second correlation function in Fig.2a with different field strengths. In the calculation, we assume that Rabi frequency $\Omega_1=\Omega_2=\Omega$, and the pump fields to be on resonance with $\Delta_1=\Delta_2=\Delta=0$. Fig.2a shows that curves of correlation function vanish at $\tau_1-\tau_2=0$. Physically, this can be understandable since once the q photon is emitted from the level $|b\rangle$ to the level $|c\rangle$, the atom goes back to the state $|b\rangle$ and needs to be pumped to the state $|a\rangle$ for emission of k photon. From three curves of Fig.2a, one can also see that the photon-photon correlations can be enhanced while their linewidth decreas with the Rabi frequency increasing. The reason is: if Rabi frequency $\Omega_{1(2)}$ is too weak, the photon-photon correlation will also be weak. Therefore, one can conclude that the correlation is strongly affected by the classical fields. On the other hand, although our atomic configuration can also be viewed as cascade system, our system shows anti-bunching, that is different from usually bunching behavior of CED [19]. The different initial condition and the introduction of classical fields are the reason. The most important feature is that this system can be used as quantum memory while usually CED can not. In order to estimate the influence of the detunings $\Delta_{1(2)}$ on the photon-photon correlation, we plot the corresponding carves in Fig.2b. As can be seen, with the increasing of detuning Δ, both the strength and their linewidth are decreased, suppressed, and finally die away. Therefor, near-resonant light is necessary to achieve strongly photon-photon correlation.

IV. Atomic memory for correlated photons. We now investigate a technique in which two correlated photons with a time delay are controlled by the storage of quantum photonic state in an ensemble of Rb atoms. This resonant nonlinear process is necessary to potential applications in quantum communication. An ensemble of atoms is initially prepared in level $|b\rangle$. Atomic spin excitations can be generated via spontaneous Raman scattering from levels $|b\rangle$ to $|c\rangle$ (This Raman transition is

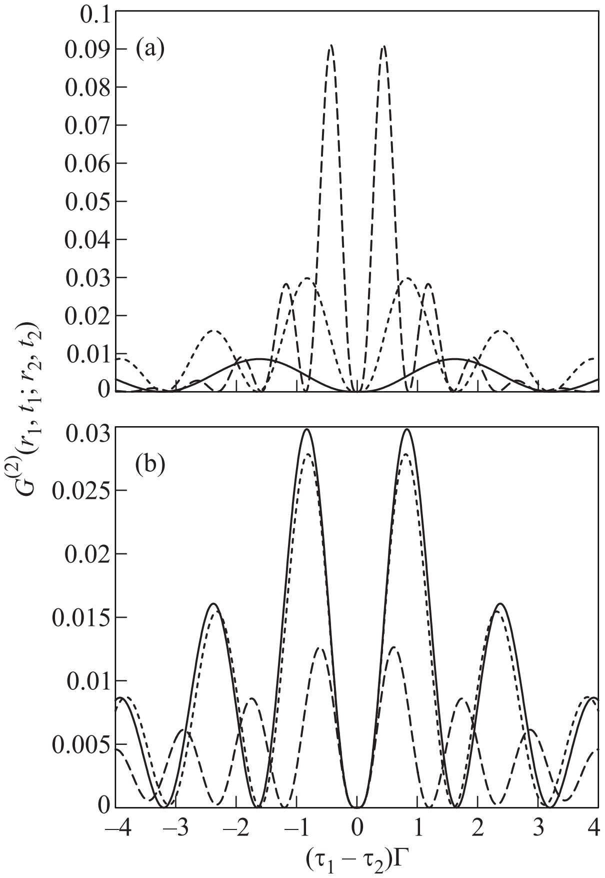

Fig.2. The two photons correlation function $G^{(2)}(r_1,t_1;r_2,t_2)$ is plotted as a function of the time delay $\tau_1 - \tau_2$ with different Rabi frequencies (a) and different detundings (b), in units of Γ that describes the spontaneous decay of the atom. The parameters are (a) $\Delta_1 = \Delta_2 = \Delta = 0$, (1) $\Omega_1 = \Omega_2 = \Gamma$ (solid line); (2) $\Omega_1 = \Omega_2 = 2\Gamma$(dotted line); (3) $\Omega_1 = \Omega_2 = 4\Gamma$ (dashed line). (b) $\Omega_1 = \Omega_2 = 2\Gamma$, (1) $\Delta_1 = \Delta_2 = 0$ (solid line); (2) $\Delta_1 = \Delta_2 = \Gamma$(dotted line); (3) $\Delta_1 = \Delta_2 = 4\Gamma$ (dashed line)

spontaneous), which we call the written beam. In this process, correlated Stokes photons and flipped spins of the atoms are produced (corresponding to Raman transitions to the state $|c\rangle$). To take into account energy and momentum conservation, each Stokes photon corresponds one flipped spin quantum in a spin-wave mode. Therefor, the number of Stoke-photons and the number of spin-wave quanta are strong correlation.

In the second Raman process, we can retrieve the state stored in the spin wave: Anti-Stokes field is produced by employing two control beams (the retrieve beams) with Rabi frequencies Ω_1 and Ω_2. We note that, this Raman process is not spontaneous, and the generation of the anti-Stokes field results from the atomic spin coherence interacting with the retrieve beam. The physical mechanism of this process is identical to the context of light storage experiments [20]. Because of the suppression of resonant absorption related to EIT, the anti-Stokes field (retrieved field) is not reabsorbed by the cloud of atoms. Therefor, this retrieval process allows for an ideal mapping of the quantum state of a spin wave onto a propagating anti-Stokes field, and this process is delayed but identical to the Stokes field according to photon number correlations. The storage time $(\tau_1 - \tau_2)$ and the rate of retrieval are controlled by the timing and the intensity of the retrieve beam (Rabi frequencies Ω_1 and Ω_2).

In an experimental scenario, the required atomic level configuration of our proposed scheme can be achieved in ^{87}Rb atom. The levels of our system $|a\rangle$, $|b\rangle$, $|c\rangle$ and $|d\rangle$ are corresponding to the Rb atom with $|5P_{3/2}, F = 2, m_F = 1\rangle$, $|5S_{1/2}, F = 2, m_F = 0\rangle$, $|5S_{1/2}, F = 1, m_F = 0\rangle$ and $|5S_{1/2}, F = 1, m_F = 1\rangle$, respectively. The pump field between $|5P_{3/2}, F = 2, m_F = 1\rangle$ and $|5S_{1/2}, F = 1, m_F = 1\rangle$ with Rabi frequency Ω_1 can be obtained by external cavity diode laser, and the coupling of levels $|5S_{1/2}, F = 1, m_F = 0\rangle$ and $|5S_{1/2}, F = 1, m_F = 1\rangle$ can be chosen by a microwave field with frequency about 120GMz, while the transition $|5P_{3/2}, F = 2, m_F = 1\rangle \to |5S_{1/2}, F = 2, m_F = 0\rangle \to |5S_{1/2}, F = 1, m_F = 0\rangle$ can be coupled with the vacuum modes in the free space. Moreover, To avoid the Doppler broadening effects, atoms must be trapped and cooled with the techniques of magneto-optical trap (MOT)[21].

V. Conclusion. In summary, we propose a scheme of generating the quantum correlation in a foul-level atomic system coupled by two coherent driven fields. Using the wave function technique, we derive an explicit expression of the photon-photon correlation function. The influence of the driven field and the detuning on the second correlation function are investigated. It shows that the second correlation function can be controlled by the driven field. Thus, we can control the magnitude of the joint detection probability by tuning the driving field. More importantly, we find that this correlated photon states can serve as atomic memory.

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1. M. D. Angelo, M. V. Chekhova, and Y. Shih, Phys. Rev. Lett. **87**, 013602 (2001).
2. Y. Shih, J. Mod. Opt. **49**, 2275 (2002).
3. C.H. Raymond Ooi and B. L. Lan, Phys. Rev. A **81**, 063832 (2010); C.H. Raymond Ooi, Phys. Rev. A **75**, 043817 (2007); C.H. Raymond Ooi, Phys. Rev. A **75**, 043818 (2007).

4. D. Sarchi and I. Carusotto, Phys. Rev. B **81**, 075320 (2010).
5. C. H. vander Wal, M. D. Eisaman, André A et al., Science **301**, 196 (2003)
6. A. Kuzmich, W. P. Bowen, A. D. Boozer et al., Nature(London) **423**, 731 (2003).
7. M. D. Eisaman, L. Childress, A. André et al., Phys. Rev. Lett. **93**, 233602 (2004).
8. U. W. Rathe and M. O. Scully, Lett. Math. Phys. **34**, 297 (1995).
9. A. N. Boto, P. Kok, D. S. Abrams et al., Phys. Rev. Lett. **85**, 2733 (2000).
10. M. D. Angelo, M. V. Chekhova, and Y. Shih, Phys. Rev. Lett. **87**, 013602 (2001).
11. G. S. Agarwal and S. S. Jha, Z. Phy. B **35**, 391 (1979).
12. M. O. Scully and K. Drűhl, Phys. Rev. A **25**, 2208 (1982).
13. C. W. Chou, S. V. Polyakov, A. Kuzmich et al., Phys. Rev. Lett. **92**, 213601 (2004).
14. V. Balic, D. A. Braje, P. Kolchin et al., Phys. Rev. Lett. **94**, 183601 (2005).
15. A. K. Patnaik, G. S. Agarwal, C. H. R. Ooi et al., Phys. Rev. A **72**, 043811 (2005).
16. M. O. Scully, C. H. R. Ooi, J. Opt. B: Quantum Semiclass. Opt. **6**, 816 (2004).
17. D. Bouwmeester, A. Ekert, A. Zeilinger et al., *The physics of Quantum information*, Springer, Berlin, 2000.
18. H. J. Brigel, W. Dur, J. I. Cirac et al., Phys. Rev. Lett. **81**, 5932 (1998).
19. M. O. Scully and M. S. Zubairy, *Quantum optics*, Cambridge. University Press, 1997.
20. D. F. Phillips, A. Fleischhauer, A. Mair, et al., Phys. Rev. Lett. **86**, 783 (2001).
21. G. Uhlenberg, J. Dirscherl, and H. Walther, Phys. Rev. Lett. **62**, 063404 (2000).