Pis'ma v ZhETF, vol. 93, iss. 4, pp. 238 – 241 © 2011 February 25

Influence of disorder on electron-hole pairing in graphene bilayer

D. K. Efimkin[+], V. A. Kulbachinskii[*], Yu. E. Lozovik[+,∇1)]

[+] Institute of Spectroscopy RAS, 142190 Troitsk, Moscow region, Russia

[*] Moscow State University, Department of Physics, 119991 Moscow, Russia

[∇] Moscow Institute of Physics and Technology, 141700 Moscow, Russia

Submitted 20 January 2011

We consider disorder effect on electron-hole pairing in the system of two graphene monolayers separated by dielectric barrier. The influence of charged impurities on temperature of phase transition is studied. In spite of large values of mobility of charge carriers in graphene disorder can considerably reduce temperature of electron-hole condensation in weak-coupling regime. The quantum hydrodynamics of the system is considered and phase stiffness of electron-hole condensate and temperature of Berezinskii-Kosterlitz-Thouless transition to the superfluid state are calculated.

The possibility of superfluidity of spatially separated electrons (e) and holes (h) was predicted [1] rather long ago. A set of unusual effects were suggested to occur in this system: persistent currents flowing in opposite direction in different layers [1, 2], Josephson effect [3], increased drag effect [4]. Interesting coherent effects can also exist in the corresponding quasi-equilibrium system of excitons with spatially separated e and h (see [5] and references therein) and there is great progress in the observation of coherent effects in the system now [6 – 8]. There is also direct experimental evidence of dipole exciton superfluidity and Josephson effect in electron-electron bilayer in quantum Hall regime (see [9] and references therein).

Recently new two-dimensional, one atom thick, material – graphene was fabricated [10]. Mobility of charge carriers in graphene achieves values $\mu \sim 10^5\,\mathrm{sm^2/V\,s}$ so graphene has good perspectives in nanoelectronics. Graphene has unusual electronic structure – its quasiparticles are chiral fermions with linear dispersion law [11]. Many interesting phenomena were discovered experimentally in it: universal minimum of conductivity, positive magnetoresistance, anomalous quantum Hall effect, etc.

We consider the system consisting of two separated by dielectric medium and independently gated graphene layers with equal density of electrons and holes in 1st and 2nd layer accordingly [12 – 14]. Fermi circles of e and h coincide due to perfect symmetry between charge carriers in graphene and e-h pairing can occur due to Coulomb interaction. Screening of electron-hole interaction and influence of graphene peculiarities on e-h pairing were investigated theoretically in [12]. In the present work we consider influence of disorder on e-h pairing and analyze the effect of graphene features on macroscopic quantum hydrodynamics of the system. There are two kind of disorder in graphene: 1) defects of lattice that have short-range potential of interaction; 2) charged impurities from substrate that have long-range potential. From analysis [15] of dependence of conductivity on concentration of charge carriers in graphene in single graphene layer it was obtained that main contribution to damping of carriers results from charged impurities. So we consider here the second type of disorder.

Hamiltonian of ideal system in weak coupling regime is:

$$H = \sum_k \xi_k a_k^+ a_k + \sum_k \xi_k b_k^+ b_k -$$

$$- \sum_{k,k',q} e^2 U_{\mathrm{out}}(\mathbf{q}) \cos\frac{\phi_1}{2} \cos\frac{\phi_2}{2} a_{k+q}^+ b_{k'-q}^+ b_{k'} a_k, \qquad (1)$$

where a_k and b_k are annihilation operators of electron and hole in e-layer and h-layer, correspondingly; $\xi_k = v_F k - E_F$ is dispersion law of quasiparticles; $\phi_1 = \phi_{k+q} - \phi_k$ and $\phi_2 = \phi_{k'-q} - \phi_{k'}$ are scattering angles of electron and hole; additional factors $\cos(\phi_1/2)\cos(\phi_2/2)$ originate from spinor form of envelope wave function of quasiparticles in graphene; $U_{\mathrm{out}}(q)$ is potential of screened electron-hole interaction. Here we suppose that electrons and holes from different valleys and with different projection of spin are pairing independently.

The interaction of charge carriers with Coulomb impurities corresponds to additional terms in Hamiltonian of the system:

$$H_{\mathrm{int}}^{e(h)} = \sum_{i,\mathbf{k},\mathbf{k}'} e^{i(\mathbf{k}-\mathbf{k}')\mathbf{r}_i} Z e^2 \cos\frac{\phi_{k,k'}}{2} \times$$

$$\times (U_{\mathrm{in(out)}}(\mathbf{k},\mathbf{k}') a_k^+ a_{k'} - U_{\mathrm{out(in)}}(\mathbf{k},\mathbf{k}') b_k^+ b_{k'}), \qquad (2)$$

1) e-mail: lozovik@isan.troitsk.ru

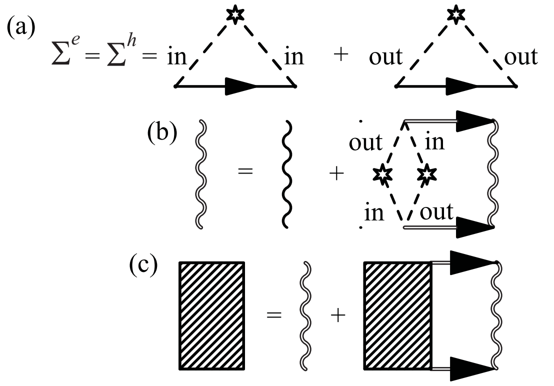

Fig.1. (a) Feynman diagrams for the self-energy of electrons and holes. (b) Feynman diagram for effective e-h interaction. (c) Diagrammatic representation of Bethe-Salpeter equation

where Z, r_i are charge and position of i-th impurity atom. U_{in} and U_{out} are interaction potential between two probe charges in the same layer and in opposite layers, correspondingly. Additional factors $\cos(\phi_{k,k'}/2)$ are also graphene feature. Screened interaction potential U_{in}, U_{out} has the form:

$$U_{\text{in}}(q) = \frac{V + \chi V^2(1 - e^{-2qD})}{1 + 2\chi V + \chi^2 V^2(1 - e^{-2qD})}, \tag{3}$$

$$U_{\text{out}}(q) = \frac{V e^{-qD}}{1 + 2\chi V + \chi^2 V^2(1 - e^{-2qD})}, \tag{4}$$

where $V(q) = 2\pi/\varepsilon q$ is the bare Coulomb interaction in the plane; $\chi = 4e^2\nu_F$ is the static limit of density response function; ν_F is the density of states on Fermi surface. ϵ is dielectric constant of medium that surrounds graphene.

Temperature of the phase transition to coherent state corresponds to appearance of Cooper instability of vertex function. Vertex function satisfy Bethe-Salpeter equation. For determination of critical temperature we use following approximations:

1) BCS-approximation or weak coupling approximation $k_F D \gg 1$.

2) $k_F \ll q_o \ll a_o^{-1}$, thus interaction potential effectively scatter electrons (or holes) to all states on Fermi surface in the same valley but hopping processes between different valleys can be neglected. Here a_o is distance between carbon atoms in graphene lattice.

3) Impurities have the same charge ($Z = 1$) and their concentrations n in both layers coincide.

4) $\gamma \ll E_F$, thus calculating self-energy and vertex function we can neglect diagrams with cross-section of impurity lines [16]. Here $\gamma = \gamma_1 + \gamma_2$ is damping of electron (or hole) on Fermi surface due to scattering on impurities, where γ_1 corresponds to the probability of scattering of charge carrier on impurity of the same layer and γ_2 corresponds to the probability of scattering on impurity of the other layer:

$$\gamma_{1(2)} = \pi n Z^2 e^4 \sum_{k'} |U_{\text{in(out)}}(\mathbf{k}, \mathbf{k}')|^2 \cos^2 \frac{\phi_{k,k'}}{2} \delta E_{kk'} \tag{5}$$

Feynman diagrams for effective e-h interaction and for Bethe-Salpeter equation under described above approximations represented on Fig.1b, c. Self-energy for electrons and holes in first Born approximation is depicted on Fig.2.

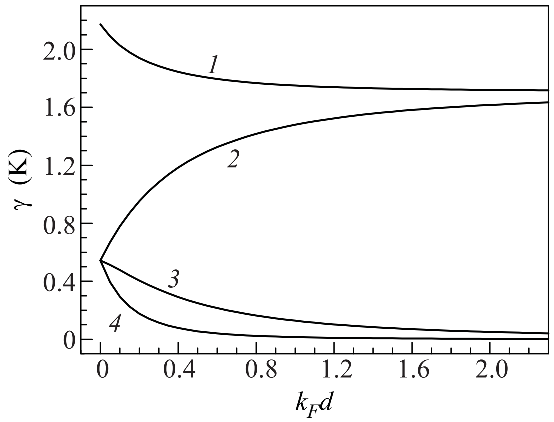

Fig.2. Dependence of values of damping γ_1 (line *2*), γ_2 (line *4*), $\tilde{\gamma}$ (line *3*) and γ^* (lines *1*) on dimensionless distance $k_F D$ between graphene layers. Concentration of Coulomb impurities is $n = 10^{10}\,\text{cm}^{-2}$

The temperature of phase transition to coherent state is defined by the equation:

$$\ln \frac{T}{T_o} = \Psi\left(\frac{1}{2} + \frac{\gamma^*}{2\pi T}\right) - \Psi\left(\frac{1}{2}\right), \tag{6}$$

where Ψ is logarithmic derivative of Гamma-function, T_o is critical temperature in the system without disorder. Damping $\gamma^* = \gamma_1 + \gamma_2 + 2\tilde{\gamma}$ can be interpreted as the probability of decay of Cooper pairs. Value of damping $\tilde{\gamma}$ is:

$$\tilde{\gamma} = \pi n Z^2 e^4 \sum_{k'} U_{\text{out}}(\mathbf{k}, \mathbf{k}') U_{\text{in}}(\mathbf{k}', \mathbf{k}) \cos^2 \frac{\phi_{kk'}}{2} \delta E_{kk'}, \tag{7}$$

and it can be interpreted as the probability of scattering of electron and hole on impurities situated in one of the layers. It corresponds to impurity interaction lines that connect electron and hole propagator lines as depicted on Fig.1b. Values of all damping are proportional to the concentration of impurities so it is convenient to fix concentration and consider the dependence of values γ_2, γ_1 $\tilde{\gamma}$

only on distance between graphene layers D. These dependencies are presented in Fig.2. Despite of the strong dependence of all damping on distance their combination γ^* smoothly decrease with increasing distance D. This compensation is caused by two competition processes. If distance D decrease: 1) interaction $U_{\rm out}$ of electrons (holes) with impurities of h(e)-layer increases; 2) charge carriers from *both* layers play more significant role in screening of interactions $U_{\rm out}, U_{\rm in}$ decrease.

Numerical solution of eq.(6) is presented in Fig.3. In first order of parameter γ^*/T_o the equation has simple analytic solution:

$$T = T_0 - \frac{\pi}{4}\gamma^*. \qquad (8)$$

So e-h pairing would be destroyed at $\gamma^* = 0.88T_o$, and minimal value of critical temperature $T_o^{\rm min} = 1.14\gamma^*$. Minimal value of the disorder concentration in relatively clean samples of graphene now is $n \sim 10^{11}\,{\rm sm}^{-2}$ [15], which corresponds to minimal value of critical temperature equals $T_o^{\rm min} = 19.8\,{\rm K}$.

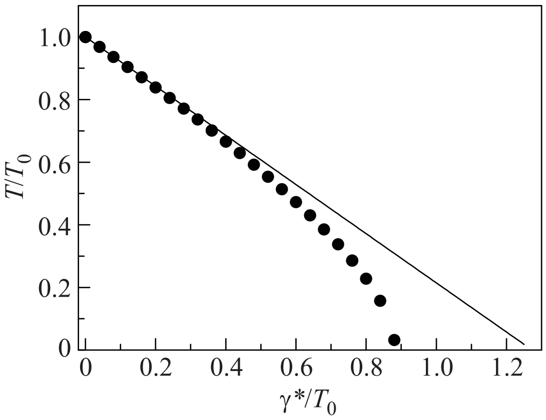

Fig.3. Dependence of dimensionless temperature of phase transition to coherent state on dimensionless value of damping γ^*. Solid line: analytic solution $T = T_o - \frac{\pi}{4}\gamma^*$; dotted line: numerical solution of Eq. (5)

Disorder can destroy coherent state because impurity potential acts differently on components of Cooper pair. There are two causes: 1) e and h have different charge 2) e and h are situated in different layers. It is analogous to the influence of paramagnetic impurities on superconductivity [17].

Let us consider now peculiarities of quantum hydrodynamics of the system. Behavior of the system on spatial scales larger than coherence length $\xi_c = \hbar v_F/\Delta$ can be described by order parameter $\Delta e^{i\varphi}$. Gauge invariant expression for free energy (at negligible interlayer tunneling) has the form:

$$F = \frac{\rho_S}{2}\int \left(\nabla\varphi(\mathbf{r}) - \frac{e}{\hbar c}\left(\mathbf{A}^e(\mathbf{r}) - \mathbf{A}^h(\mathbf{r})\right)\right)^2 d\mathbf{r}. \qquad (9)$$

Here coefficient ρ_S is phase stiffness, that describe system on large scales; $\mathbf{A}^{e(h)}(\mathbf{r})$ is value of vector potential in e(h)-layer. The last term in brackets dramatically changes hydrodynamics of the system. It permits parallel magnetic field between the layers to excite persistent currents flowing through layers in opposite directions:

$$\mathbf{j}^{e(h)} = -c\frac{\delta F}{\delta A^{e(h)}} = \pm\frac{e}{\hbar}\rho_S\left(\nabla\varphi - \frac{e}{\hbar c}\delta\mathbf{A}\right). \qquad (10)$$

For microscopic calculation of phase stiffness it is convenient to redefine reaction functions in the following way: 1) Response function of electric current: $\mathbf{j}^{e(h)} = \mp\frac{e^2}{\hbar^2 c}\chi_j\delta\mathbf{A}^{e(h)}$; 2) Response function of momentum: $\mathbf{P}^{e(h)} = \frac{e^2}{\hbar^2 c}\chi_P\delta\mathbf{A}^{e(h)}$. In this case values of χ_j and ρ_s coincide by the definition.

According to Kubo linear response theory we calculate reaction functions χ_j and χ_p. In the system without disorder at $T = 0K$ we obtain:

$$\chi_j = \frac{E_F}{8\pi} = \frac{v_F}{4}\sqrt{n}, \qquad \chi_P = \frac{n}{2}. \qquad (11)$$

Here n is the concentration of electrons (holes) in a single valley and with fixed projection of spin. The same reaction functions for the system of nonchiral fermions with quadratic dispersion law $\xi = p^2/2m - E_F$ in bilayer formed by two quantum wells or semimetallic layers have the form:

$$\chi_j = \frac{E_F}{4\pi} = \frac{n}{2m}, \quad \chi_P = \frac{n}{2}. \qquad (12)$$

Unusual electronic properties of graphene results in the following: 1) Contrary to the case of massive fermions response functions χ_j and χ_p in graphene bilayer don't not proportional to each other. This unusual result is the consequence of independence of velocity of chiral fermions on momentum of the quasiparticle. 2) Value χ_j (current is observable physical quantity) does not proportional to the concentration of charge carriers in graphene; 3) Value χ_P for massive fermions and for chiral massless fermions coincide.

Full values of reaction functions are four time greater because of independent contribution of electrons and holes from different valleys and different projection of spin.

Two-dimensional system under consideration became superfluid bellow the critical temperature of Berezinski–Kosterlitz–Thouless (BKT) transition. T_{BKT} is always less than temperature obtained from mean field theory T_o and satisfies the equation [18]:

$$T_{BKT} = \frac{\pi}{2}\rho_s(T_{\rm BKT}). \qquad (13)$$

Temperature of BKT transition depends on phase stiffness of one of four decoupled condensates since each condensate undergoes its own phase transition. In first order on value T_o/E_F solution of the equation takes the form

$$T_{\mathrm{BKT}} = T_o(1 - 4T_o/E_F). \qquad (14)$$

In weak coupling regime $T_o/\rho_s \sim T_o/E_F \ll 1$, and difference between temperatures T_o and T_{BKT} is insignificant. Weak disorder $\gamma^* {\sim} T_o$ cannot change the ratio between them because it reduces phase stiffness only on value $\Delta\rho_S \sim \gamma^*$. So relative difference between T_{BKT} and T_o in system with disorder can be estimated as $(T_{\mathrm{BKT}} - T_o)/T_o \sim T_o/E_F$ and can be neglected.

Conclusion. In this work we calculate temperature of phase transition to coherent state and temperature of Berezinski-Kosterlitz-Thouless transition to superfluid state taking into account Coulomb impurities. The minimal value of temperature of "clean" system in which pairing won't be destroyed by disorder was obtain. Also the peculiarities of quantum hydrodynamics of the system were considered and the exotic dependance of phase stiffness on density of charge carriers was obtained.

The authors thank A.A. Sokolik for fruitful discussions. The work was supported by RFBR and RAS programs.

1. Yu. E. Lozovik and V. I. Yudson, JETP Lett. **22**, 274(1975); Sov. Phys. JETP **44**, 389 (1976); Solid. Sate Commun. **19**, 391 (1976); **21**, 211 (1976).
2. Yu. E. Lozovik and O. V. Berman, JETP Lett. **64**, 573(1996); JETP **84**, 1027 (1997).
3. Yu. E. Lozovik and A. V. Poushnov, Phys. Lett. A **228**, 399 (1997).
4. G. Vignale and A. H. MacDonald, Phys. Rev. Lett. **76**, 2786 (1996).
5. Yu. E. Lozovik, Phys. Usp. **52**, 286(2009); Yu. E. Lozovik and I. V. Ovchinnikov, Phys. Rev. B **66**, 075124 (2002).
6. V. B. Timofeev, Phys.-Usp. **48**, 295 (2005), references therein; A. V. Gorbunov and V. B. Timofeev, JETP Lett. **84**, 390 (2006); A. V. Gorbunov, A. V. Larionov, and V. B. Timofeev, JETP Lett. **86**, 8 (2007).
7. A. T. Hammack, L. V. Butov, J. Wilkes et al., Phys. Rev. B **80**, 155331 (2009); M. M. Fogler, S. Yang, A. T. Hammack et. al., Phys. Rev B **78**, 035441 (2008).
8. D. W. Snoke, Nature **443**, 403 (2006); Advances in condensed matter physics **2011**, 938609 (2010).
9. J. P. Eisenstein and A. H. MacDonald, Nature **432**, 691 (2004).
10. K. S. Novoselov, A. K. Geim, S. V. Morozov et. al, Nature **438**, 197 (2005).
11. A. H. Castro Neto, F. Guinea, and N. M. P. Peres, Rev. Mod. Phys. **81**, 109 (2009); Yu. E. Lozovik, S. P. Merkulova, and A. A. Sokolik, Phys.-Usp. **51**, 727 (2008).
12. Yu. E. Lozovik and A. A. Sokolik, JETP Lett. **87**, 55 (2007); Eur. Phys. **73**(2), 195 (2009).
13. H. Min, R. Bistrizer, J. J. Su et. al., Phys. Rev. B **78**, 121401(R) (2008); R. Bistrize and A. H. MacDonald, Phys. Rev. Lett. **101**, 256406 (2008).
14. M. Yu. Kharitonov and K. B. Efetov, Phys. Rev. B **78**, 241401(R) (2008).
15. K. Nomura and A. H. MacDonald, Phys. Rev. Lett. **98**, 076602 (2008); S. Adam and S. Das Sarma, Sol. St. Comm. **146**, 356 (2008).
16. A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloschinskii, *Quantum field theory in statistical physics*, Pergamon, New York, 1965.
17. A. A. Abrikosov and L. P. Gorkov, JETP, **12**, 1243 (1960).
18. J. M. Kosterlitz and D. J. Thouless, J. Phys. C. **6**, 1181 (1973).