Pis'ma v ZhETF, vol. 93, iss. 8, pp. 499 – 504 © 2011 April 25

Single-particle relaxation time of the two-dimensional electron gas in Si/SiGe: many-body effects

A. Gold

Centre d'Elaboration de Materiaux et d'Etudes Structurales (CEMES-CNRS), 31055 Toulouse, France

UFR-PCA, Université Paul Sabatier, 31062 Toulouse, France

Submitted 15 February 2011

Resubmitted 15 March 2011

The single-particle relaxation time of the two-dimensional electron gas in SiGe/Si/SiGe quantum wells is calculated. Many-body effects beyond the random-phase approximation become important at low electron density. For charged impurity scattering (remote doped) we analyze the importance of these many-body effects as function of the electron density and the spacer width. Induced by many-body effects a strong reduction of the single-particle relaxation time at low electron density is predicted. We describe the relation with the transport scattering time, we comment on multiple-scattering effects and we discuss the determination of many-body effects in existing samples.

The single-particle relaxation time τ_s is an important scattering time and can be used to characterize the disorder in the two-dimensional electron gas (2DEG). It can be determined by measuring Shubnikov-de Haas-oscillations as function of the temperature and τ_s is different from the transport scattering time τ_t [1]. In lowest order of the disorder one finds for charged-impurity scattering (CIS) in the low density limit $\tau_t^{(0)}/\tau_s^{(0)} \approx 1$ while for high density one finds for remote doping $\tau_t^{(0)} \gg \tau_s^{(0)}$ [2]. In a more detailed analytical calculation it was shown that $\tau_t^{(0)}/\tau_s^{(0)} = (2k_F\alpha)^2$ for $k_F\alpha \gg 1$, where α is the remote doping distance and k_F the Fermi wave number [3]. These results have been obtained within the random-phase approximation (RPA) where many-body effects are neglected. In the present paper we show that many-body effects modify the results mentioned above, especially in the low electron density range. We find that the single-particle relaxation time is strongly reduced, due to the reduced screening properties of the 2DEG, when many-body effects are taken into account.

Shubnikov-de Haas oscillations have been studied in many experiments in order to get information about the effective carrier mass and about scattering mechanisms present in the 2DEG. For instance, it was found that $\tau_t/\tau_s \gg 1$ in the high mobility 2DEG as realized in GaAs/AlGaAs-heterostructures (HS) [4, 5], in Si/SiGe-HS [6–8], in SiGe/Si/SiGe-quantum wells (QW) [9], and in AlGaN/GaN-HS [10]. However, in all these studies the electron density was relatively high and many-body effects were small. Theoretical calculations of τ_s for Si/SiGe-HS have been made within the RPA [11]. Recently, the importance of many-body effects has been deduced from mobility measurements [12] of the 2DEG in SiGe/Si/SiGe-QW and this was verified in calculations for the transport scattering time where many-body effects were been taken into account [13]. For the single-particle relaxation time the importance of many-body effects has not yet been explored, not in experiment and not in theory. In general one can say that the electron density $N > 3 \cdot 10^{11}\,\mathrm{cm}^{-2}$ in measured 2DEG's [4–10] was too large in order to see such many-body effects in experiments in a clear manner. However, recently high mobility Si/SiGe-HS have been produced with electron densities as low as $N > 2 \cdot 10^{10}\,\mathrm{cm}^{-2}$ [14]. In the present paper we show that many-body effects can be measured at such low electron densities.

We consider an interacting 2DEG in the xy-plane with a parabolic dispersion described by an effective masse $m^* = 0.19m_e$, m_e is the free electron mass. For the valley degeneracy in SiGe/Si/SiGe-QW we use $g_v = 2$ [15]. The 2DEG is embedded into an isolating background with dielectric constant $\varepsilon_L = 12.5$. The extension effects of the 2DEG perpendicular to the interface are described by a QW with infinite confinement for $z < 0$ and $z > L$ and with an envelope wave function $\Psi(0 < z < L) \propto \sin(\pi z/L)$. The long-range Coulomb interaction potential $V(q) = (2\pi e^2/\varepsilon_L q)F_C(q, L)$ of the charged carriers is taken into account by using the RPA [16] together with a finite local-field correction (LFC) $G(q)$ [17]. The form factor $F_C(q, L)$ for the finite width was calculated long ago [18].

We use the Fermi liquid theory for the 2DEG. The LFC describes many-body effects (exchange and correlation) beyond the RPA and these effects become very important for low electron density where the Wigner-Seitz parameter $r_s \equiv 1/\sqrt{\pi a^{*2} N}$ becomes large. $a^* \equiv$

$\equiv 0.53\,\text{Å} \times \varepsilon_L m_e/m^* = 34.8\,\text{Å}$ represents the effective Bohr radius. For high electron density r_s is small and many body effects are small, too. Therefore, the LFC is less important for $r_s < 1$ and can be neglected if $r_s \ll 1$ [16, 17]. For $r_s > 1$ the electron gas is strongly correlated. In the SiGe/Si/SiGe-QW for $N = 1 \cdot 10^{11}\,\text{cm}^{-2}$ the Wigner-Seitz parameter is $r_s \cong 5.1$.

In the present paper we apply an analytical expressions for $G(q)$ according to the numerical results obtained before in Ref. 19. We discuss the RPA where $G(q) = 0$, the Hubbard (H) approximation $G_H(q) = \frac{q}{2g_v\sqrt{q^2+k_F^2}}$ where exchange effects are taken into account, and the full LFC written as $G(q) = 1.402 r_s^{4/3} q/\sqrt{2.644 C_1^2 q_s^2 + C_2^2 r_s^{4/3} q^2 - C_3 r_s^{2/3} q_s q}$. $q_s = 2g_v/a^*$ represents the screening wave number. $G(q)$ is parameterized by three coefficients $C_i = C_i(r_s)$ $(i = 1, 2, 3)$ calculated before in Ref. [19] and takes exchange and correlation effects into account. It is well known that the LFC in two dimensions has the limiting behavior $G(q \to 0) \propto q$ and $G(q \to \infty) = \text{const}$ [17]. This is also seen in the Hubbard form. The form chosen for $G(q)$ takes these limiting behaviors into account. We have tested the accuracy of the LFC in comparison with Monte-Carlo calculations and we found good agreement for $r_s \le 5$ [19]. For $5 < r_s < 15$ the agreement should be considered as qualitative and we expect that our calculation underestimates many-body effects. But in any case, our approach using the LFC is much better that using the RPA without LFC. Note that our results are for a Coulomb-like disorder and for a long-range Coulomb interaction. The small parameter for this problem is r_s, however, r_s it is not small for low electron density. This is well known and the reason for introducing the LFC [16, 17].

We study the transport properties of the 2DEG in the presence of CIS. We assume a xy-plane of charged impurities at z_i with an impurity density N_i. For $0 < z_i < L$ the impurities are located in the QW (in the Si) and for $z_i < 0$ (or $z_i > L$) they are located outside the QW (in the SiGe). For $z_i = 0$ (or $z_i = L$) the impurities are located at the Si/SiGe interface. The random potential for CIS is described by $\langle |U(q)|^2 \rangle = N_i (2\pi e^2/\varepsilon_L q)^2 F_i(q, L)$ with the form factor $F_i(q, L)$ for the finite width of the QW [18].

The single-particle relaxation time $\tau_s^{(0)}$ in lowest order of the random potential is given by [3]

$$\frac{\hbar}{\tau_s^{(0)}} = \frac{1}{\pi \varepsilon_F} \int_0^{2k_F} dq \frac{\langle |U(q)|^2 \rangle}{\varepsilon(q)^2} \frac{k_F^2}{\sqrt{4k_F^2 - q^2}}, \qquad (1)$$

$\hbar = h/2\pi$ is Planck's constant. ε_F represents the Fermi energy, k_F is the Fermi wave number and $\varepsilon(q \le 2k_F) = 1 + q_s F_C(q, L)[1 - G(q)]/q$ is the dielectric function of the 2DEG including many-body effects, $F_C(q, L)$ represents the form factor for the finite width of the QW [18]. The LFC reduces the screening properties of the 2DEG, which leads to a reduction of $\tau_s^{(0)}$. The single-particle scattering time can be determined measuring Shubnikov-de Haas oscillations in a small perpendicular magnetic field at different temperatures and is related to the Dingle temperature by $k_B T_D = \hbar/2\pi\tau_s$ [1], k_B is Boltzmann's constant. We mention that backscattering with $\Delta q \approx 2k_F$ is most important for the transport scattering time while for the single-particle relaxation time all scattering events, backscattering and forwardscattering, are of equal importance.

In Fig. 1 we show the single-particle relaxation time $\tau^{(0)}$ versus electron density in different approximations

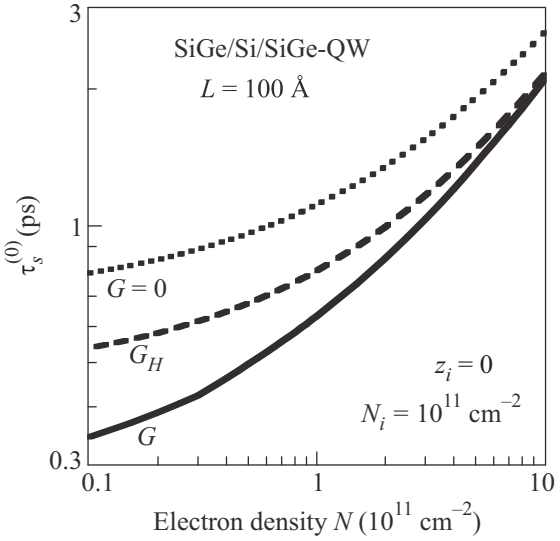

Fig. 1. Single-particle relaxation time $\tau_s^{(0)}$ versus electron density N for the 2DEG in the SiGe/Si/SiGe-QW for charged-impurity scattering ($z_i = 0\,\text{Å}$ and $N_i = 1 \cdot 10^{11}\,\text{cm}^{-2}$). For the dotted line many-body effects are neglected ($G = 0$). For the dashed line exchange effects (G_H) are taken into account. For the solid line exchange-correlation effects (G) are taken into account

for the LFC. The impurities are assumed to be located at the Si/SiGe-interface $z_i = 0$. The smaller screening, when the LFC is taken into account, leads to smaller scattering time compared to the RPA. While at larger density the difference is small the difference at the electron density $N = 1 \cdot 10^{10}\,\text{cm}^{-2}$ is quite high: $\tau_s^{(0,\text{RPA})} \approx 2.3\tau_s^{(0,G)}$. But even at $N = 1 \cdot 10^{11}\,\text{cm}^{-}$ the difference is still large: $\tau_s^{(0,\text{RPA})} \approx 1.7\tau_s^{(0,G)}$. This shows

that many-body effects are quite important for existing samples [9, 12] when $N < 3 \cdot 10^{11}\,\mathrm{cm}^{-2}$.

In Fig. 2 we show $\tau_s^{(0)}$ versus the distance z_i of the impurity layer to the Si/SiGe-interface for the low elec-

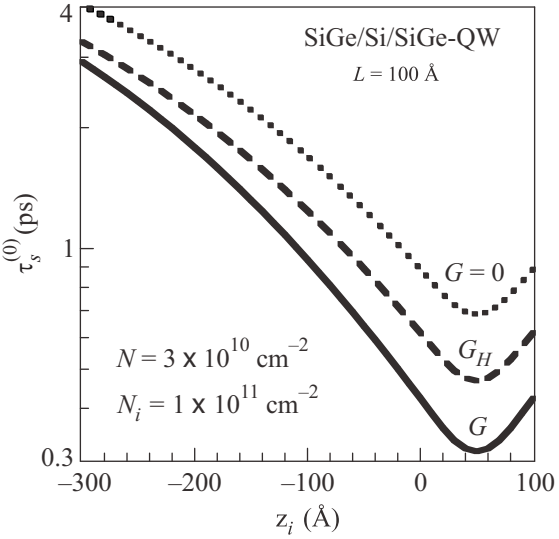

Fig. 2. Single-particle relaxation time $\tau_s^{(0)}$ versus distance z_i of the impurity layer from the SiGe/Si-interface for the 2DEG in the Si/SiGe-QW for charged-impurity scattering ($N = 3 \cdot 10^{10}\,\mathrm{cm}^{-2}$ and $N_i = 1 \cdot 10^{11}\,\mathrm{cm}^{-2}$). For the dotted line many-body effects are neglected ($G = 0$). For the dashed line exchange effects (G_H) are taken into account. For the solid line exchange-correlation effects (G) are fully taken into account

tron density $N = 3 \cdot 10^{10}\,\mathrm{cm}^{-2}$ in different approximations for the LFC. It is clearly seen that the many-body corrections are most important for impurities located in or near the QW and they decrease when the impurities are located far away from the QW. This can be understood by the long-range nature of the random potential when impurities are far away. In the q-integral for $\hbar/\tau_s^{(0)}$ large q-values are cut-off by a factor $\exp(-2k_F|z_i|)$, coming from $F_i(q, L)$, and only small q values determine the single-particle relaxation time when $|z_i|$ is large. However, for small q-values the LFC $G(q) \propto q$ is small and the many-body effects are small, too. In order to determine many-body effects one should use samples with small spacer width or with impurities located in the QW. We mention that $\tau_s^{(0)} \propto 1/N_i$. Experimental results together with Fig. 1 and/or Fig. 2 can be used to determine N_i.

Within the RPA we note that for an ideal (no extension in z-direction) 2DEG and remote doping at a distance α one gets, using results obtained before [3, 20], the analytical expression

$$\tau_s^{(0)} = 4\frac{m^* g_v^2}{\hbar N_i}(k_F\alpha) \propto \frac{N^{1/2}\alpha}{N_i} \qquad (2)$$

and $\tau_t^{(0)} \propto \alpha^3 N^{3/2}/N_i$ for $k_F\alpha \gg 1$. In a 2DEG with finite QW width one should replace α by $\alpha' \approx L/2 - z_i$. The behavior $\tau_s^{(0)} \propto (L/2 - z_i)$ is clearly seen in Fig. 2. For $\alpha = 300$ Å and parameters as used in Fig. 2 we find numerically within the RPA $\tau_s^{(0)} = 4.05$ ps. Eq. (2) results in $\tau_s^{(0)} = 3.85$ ps. Using $\alpha' = 350$ Å we get with Eq. (2) $\tau_s^{(0)} = 4.49$ ps.

Many-body effects not only are important for the single-particle relaxation time but also for the transport scattering time $\tau_t^{(0)}$ [21]. They enter the two scattering times $\tau_s^{(0)}$ and $\tau_t^{(0)}$ via the screening function and are even more important for the transport scattering time. Including many-body effects results in $\tau_r^{(0)}/\tau_s^{(0)} < 1$ if $k_F|z_i| \ll 1$ while the RPA result is $\tau_t^{(0)}/\tau_s^{(0)} \cong 1$. The ratio $\tau_t^{(0)}/\tau_s^{(0)}$ versus electron density is shown in Fig. 3 for impurities at the Si/SiGe-interface

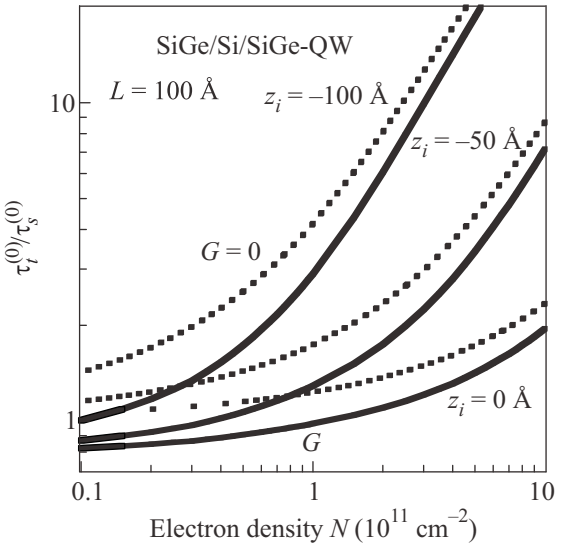

Fig. 3. Ratio $\tau_t^{(0)}/\tau_s^{(0)}$ versus electron density N for the 2DEG in the SiGe/Si/SiGe-QW for charged-impurity scattering for $z_i = 0$ Å, $z_i = -50$ Å and $z_i = -100$ Å. For the solid lines many-body effects (G) are taken into account and for the dotted lines many-body effects ($G = 0$) are neglected

$z_i = 0$ and for impurities with spacer $z_i = -50$ Å and $z_i = -100$ Å. At large density the RPA result is given by $\tau_t^{(0)}/\tau_s^{(0)} \cong (2k_F|z_i|)^2 \propto N/g_v$ and the modification due to many-body effects are small but increase with decreasing electron density. Moreover, for a given z_i the random potential always becomes short-ranged when the electron density is sufficiently reduced. From Fig. 3 we conclude that many-body effects can be measured for densities $N \leq 3 \cdot 10^{11}\,\mathrm{cm}^{-2}$. Of course, this implies that

the position of the impurities is known, which is best determined from the density dependence of the transport scattering time $\tau_t^{(0)} \propto \alpha^3$.

Multiple-scattering effects (MSE), described by the dimensionless parameter A, are important for the transport scattering time $\tau_t = \tau_t^{(0)}(1-A)$ and lead to a metal-insulator transition (MIT) with $\tau_t \to 0$ at a critical electron density $N \equiv N_{MIT}$ were $A = 1$ [21]. MIT's have been observed in the 2DEG in Si/SiGe [9,12], and in other 2DEG's, for a review see Ref. [22]. For $N \gg N_{MIT}$ these MSE are negligible $\tau_t \approx \tau_t^{(0)}$. The importance of MSE on the single-particle relaxation time was studied before and for $N \geq N_{MIT}$ it was found that $\tau_s^{(0)} < \tau_s < 1.1\tau_s^{(0)}$, see Fig. 8 in Ref. [3]. We conclude that MSE don't lead to a dramatic change of τ_s, quite differently to the case of the transport scattering time. Therefore, in the following we neglect MSE and use $\tau_s \approx \tau_s^{(0)}$.

In Fig. 4 we show a representative example of scattering times of a QW where the mobility versus density was measured before [9]. The calculation [13] of the mobility was in good agreement with experiment. A high impurity density $N_i = 1 \cdot 10^{12}\,\mathrm{cm}^{-2}$ was necessary to explain the measured mobility. Due to the remote doping

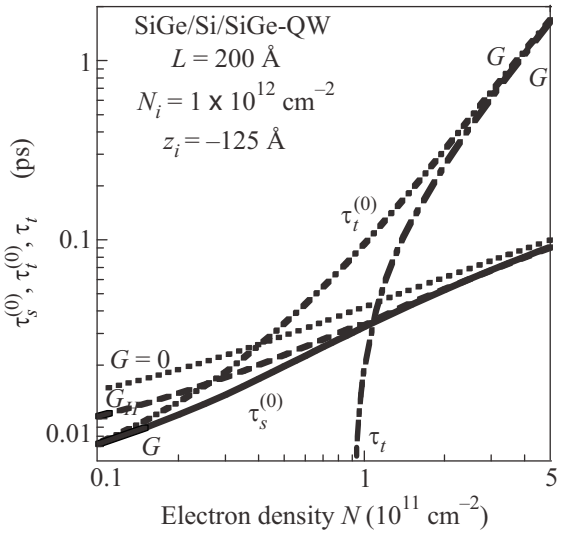

Fig. 4. Scattering times versus electron density N for the 2DEG in the SiGe/Si/SiGe-QW for charged-impurity scattering ($z_i = -125$ Å and $N_i = 1 \cdot 10^{12}\,\mathrm{cm}^{-2}$). The dashed-dotted lines represents the transport scattering time τ_t and $\tau_t^{(0)}$ including many-body effects (G). Multiple-scattering effects are taken into account for τ_t and are neglected for $\tau_t^{(0)}$. The dotted, dashed, and solid lines represent $\tau_s^{(0)}$ without many-body effects ($G = 0$), within the Hubbard approximation (G_H), and with exchange-correlation effects (G), respectively

with $z_i = -125$ Å many-body effects give small modifications of $\tau_s^{(0)}$ in the density range of the experiment. From Fig. 4 it is clearly seen that in this sample the low density range is not accessible because a MIT occurs at the density $N = N_{MIT} \approx 9.1 \cdot 10^{10}\,\mathrm{cm}^{-2}$. Due to the large impurity density in this sample it might be difficult to observe Shubnikov-de Haas oscillations near N_{MIT}. Note the small values of the single-particle relaxation time with $\tau_s^{(0)} \propto N^{1/2}$, characteristic for remote doping, see Eq. (2). For low density the random potential becomes short-range $\tau_t^{(0)}/\tau_s^{(0)} \approx 1$. In Fig. 4 we also see that for the transport scattering time MSE are very important and might make a determination of many-body effects difficult. We stress that the single-particle relaxation is only weakly dependent on MSE and this makes the study of many-body effects via τ_s very interesting: the interpretation of experimental results on τ_s is not contaminated by MSE. But to reach the low density region one needs sample with $N_{MIT} < 5 \cdot 10^{10}\,\mathrm{cm}^{-2}$.

For the transport properties of the Si/SiGe-QW studied in Ref. [9] with $1.6 \cdot 10^{11}\,\mathrm{cm}^{-2} < N < 5.4 \cdot 10^{11}\,\mathrm{cm}^{-2}$ a ratio $\tau_t/\tau_s \approx 15$ was estimated. The electron density for this ratio was not reported. From Fig. 4 we get $\tau_t/\tau_s \approx 14$ for $N = 4 \cdot 10^{11}\,\mathrm{cm}^{-2}$ and $\tau_t/\tau_s \approx 18$ for $N = 5 \cdot 10^{11}\,\mathrm{cm}^{-2}$, which is reasonable agreement with the experimental result. However, at $N \geq 4 \cdot 10^{11}\,\mathrm{cm}^{-2}$ the modification of τ_s due to the finite LFC is too small (< 10%) to be measurable, compared to other uncertainties, as for instance the doping distance z_i. Nevertheless, we expect that future samples with low N_{MIT} allow to reach the low density range and to find evidence of many-body effects in the single-particle relaxation time.

A first important step has been made with the ultra-high mobility Si/SiGe-HS studied in Ref. [14] with a peak mobility $\mu \approx 1.5 \cdot 10^6\,\mathrm{cm}^2/\mathrm{Vs}$ and $z_i = -1400$ Å. The peak mobility corresponds to $\tau_t \approx 180$ ps and we find numerically $\tau_s^{(0)} = 1.1$ ps at $N = 3 \cdot 10^{11}\,\mathrm{cm}^{-2}$ for CIS. In fact, in this sample the single-particle relaxation time is determined by CIS with $z_i = -1400$ Å. We predict for this sample $\tau_s^{(0)}/\mathrm{ps} = 0.63(N/10^{11}\,\mathrm{cm}^{-2})^{1/2}$ for $2 \cdot 10^{10}\,\mathrm{cm}^{-2} < N < 5 \cdot 10^{11}\,\mathrm{cm}^{-2}$. Unfortunately, many-body effects in $\tau_s^{(0)}$ are small in this sample due to the large spacer width, see Fig. 2. For more details concerning the relevant disorder and the transport properties in this high-mobility sample, see Ref. [23].

In order to observe Shubnikov-de Haas oscillations the condition $\omega_c \tau_s > 1$ with $\omega_c = eB/m^*$ must be fulfilled. For electrons in Si/SiGe a magnetic field of 1T corresponds to $\omega_c \cong 0.92 \cdot 10^{12}/\mathrm{s}$. This means that with increasing disorder the magnetic field must increase. A Dingle temperature of $T_D = 1\,\mathrm{K}$ corresponds

to $\tau_s = 1.2$ ps. This estimation shows that it might be difficult to determine experimentally τ_s with reasonable magnetic fields $B < 10$ T when τ_s is small, as for instance for the Si/SiGe-QW studied in Ref. [12]. For the sample of Ref. [9] the situation is somewhat better due to higher mobility, see Fig. 4. For the ultra-high mobility sample [14] we expect that one can determine τ_s, see for this sample our prediction given above.

In our calculations we supposed a constant effective mass for the 2DEG in Si/SiGe. In the 2DEG as realized in Si/SiGe no anomalies of the effective mass have been reported yet. The Shubnikov-de Haas effect was used to study the density dependence of the effective mass and the Dingle-temperature in the 2DEG at the Si/SiO_2-interface [1, 24–28]. For the 2DEG in $(100)Si/SiO_2$ the Dingle temperature was determined in the high density range [24] $2 \cdot 10^{12}\,\mathrm{cm}^{-2} \leq N \leq 5 \cdot 10^{12}\,\mathrm{cm}^{-2}$ and $\tau_t/\tau_s < 1$ was reported. This ratio is extremely surprising. In Ref. [24] charged ions (Na^+) have been drifted to the Si/SiO_2-interface and CIS was claimed to be relevant. A ratio $\tau_t/\tau_s < 1$ implies (i) the importance of many-body effects if CIS is relevant (see Fig. 3) or/and the importance of multiple scattering effects (see Fig. 4). However, for CIS at the high densities of the experiment one expects from theory $\tau_t^{(0)}/\tau_s^{(0)} > 1$. Multiple scattering effects should not be relevant because it was argued that $1/\tau_s \propto N_i$. But we don't know if the relation $1/\tau_t \propto N_i$ was really tested. A density dependence of the effective mass was reported in the density range $1 \cdot 10^{11}\,\mathrm{cm}^{-2} \leq N \leq 5 \cdot 10^{12}\,\mathrm{cm}^{-2}$ [26]. More recently [27], the low density range $1 \cdot 10^{11}\,\mathrm{cm}^{-2} < N \leq 4 \cdot 10^{11}\,\mathrm{cm}^{-2}$ with a mass divergence near $N = N^* \approx 0.8 \cdot 10^{11}\,\mathrm{cm}^{-2}$ and with $1 < \tau_t/\tau_s < 1.8$ was studied. Apparently, this mass divergence was not seen in Ref. [26]. We conclude that for the 2DEG at the $(100)Si/SiO_2$-interface the ratio τ_t/τ_s in the density range $4 \cdot 10^{11}\,\mathrm{cm}^{-2} \leq N \leq 2 \cdot 10^{12}\,\mathrm{cm}^{-2}$ was not yet really explored in experiment. For the 2DEG at the $(111)Si/SiO_2$-interface the ratio $0.3 < \tau_t/\tau_s < 1.5$ was determined in the density range $3 \cdot 10^{11}\,\mathrm{cm}^{-2} < N < 3 \cdot 10^{12}\,\mathrm{cm}^{-2}$, also with an apparent mass divergence at $N = N^* \approx 3 \cdot 10^{11}\,\mathrm{cm}^{-2}$ [28]. The data on τ_t/τ_s versus density have been explained in a calculation [29] taking into account many-body effects via the Hubbard approximation, a strongly density dependent effective mass, and CIS together with IRS. It is the interplay of CIS and IRS which makes the comparison between theory and experiment so difficult for the 2DEG in Si/SiO_2. Only measurements in a large density range allow to determine the parameters of disorder. We mention that IRS is not important in QW with width $L > 75$ Å [18]. In the present paper we considered QW with $L = 100$ Å (Fig. 1–3) and with $L = 200$ Å (Fig. 4).

Recently, the single-particle relaxation time also was studied for the 2DEG in δ-doped $SrTiO_3$ [30] and for the 2DEG at the $LaAlO_3/SrTiO_3$ interface [31, 32] and a ratio τ_t/τ_s was reported. Electron densities are too high to observe many-body effects in these structures. But this might change in the future and the measurements show that the Dingle temperature is an important material parameter of the 2DEG and a trustable theory must be available for τ_s.

In conclusion we studied for the 2DEG as realized in SiGe/Si/SiGe-quantum wells the density dependence and the remote-doping dependence of the single-particle relaxation time when many-body effects are important. Charged-impurity scattering was considered. The range of applicability of the Fermi liquid theory including the local-field correction, used in the present paper, is not fully understood and the form of many-body effects is not yet known from experiment. We predict that with high mobility samples these many-body effects can be studied by measuring the Dingle temperature.

1. T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).
2. S. Das Sarma and F. Stern, Phys. Rev. B **32**, 8442 (1985).
3. A. Gold, Phys. Rev. B **38**, 10798 (1988).
4. M. A. Palaanen, D. C. Tsui, and J. C. M. Hwang, Phys. Rev. Lett. **51**, 2226 (1983).
5. U. Bockelmann, G. Abstreiter, G. Weimann, and W. Schlapp, Phys. Rev. B **41**, 7864 (1990).
6. D. Többen, F. Schäffler, A. Zrenner, and G. Abstreiter, Phys. Rev. B **46**, 4344 (1992).
7. G. Stöger, G. Brunthaler, G. Bauer et al., Phys. Rev. B **49**, 10417 (1994).
8. K. Ismail, M. Arafa, K. L. Saenger et al., App. Phy. Lett. **66**, 1077 (1995).
9. Z. Wilamowski, N. Sandersfeld, W. Jantsch et al., Phys. Rev. Lett. **87**, 026401 (2001).
10. M. J. Mantra, S. H. Simon, K. W. Baldwin et al., Appl. Phys. Lett. **85**, 5278 (2004).
11. F. Stern and S. E. Laux, App. Phys. Lett. **61**, 1110 (1992).
12. V. T. Dolgopolov, E. V. Deviatov, A. A. Shashkin et al., Superlattices and Microstructures **33**, 271 (2003).
13. A. Gold, J. Appl. Phys. **108**, 063710 (2010).
14. T. M. Lu, D. C. Tsui, C.-H. Lee, and C.Ẇ. Liu, App. Phys. Lett. **94**, 182102 (2009); **97**, 059904 (2010) (Erratum).
15. G. Abstreiter, H. Brugger, T. Wolf et al., Phys. Rev. Lett. **54**, 2441 (1985).

16. D. Pines and P. Nozières, *The Theory of Quantum Liquids*, Benjamin, New York, 1966.
17. K. S. Singwi and M. P. Tosi, Solid State Phys. **36**, 177 (1981).
18. A. Gold, Phys. Rev. B **35**, 723 (1987).
19. A. Gold and L. Calmels, Phys. Rev. B **48**, 11622 (1993); A. Gold, Phys. Rev. B **50**, 4297 (1994); Z. Phys. B **103**, 491 (1997).
20. A. Gold, Phys. Rev. B **35**, 8818 (1991).
21. A. Gold and W. Götze, Solid State Commun. **47**, 627 (1983); Phys. Rev. B **33**, 2495 (1986).
22. A. A. Shashkin, Phys. Usp. **48**, 129 (2005).
23. A. Gold, Europhys. Lett. **92**, 67002 (2010).
24. A. Hartstein and F. F. Fang, Phys. Rev. B **18**, 5502 (1978).
25. F. F. Fang, A. B. Fowler, and A. Hartstein, Phys. Rev. B **16**, 4446 (1977).
26. V. M. Pudalov, M. E. Gershenson, H. Kojima et al., Phys. Rev. Lett. **88**, 196404 (2002).
27. A. A. Shashkin, M. Rahimi, S. Anissimova et al., Phys. Rev. Lett. **91**, 046403 (2003).
28. A. A. Shashkin, A. A. Kapustin, E. V. Deviatov et al., Phys. Rev. B **76**, 241302(R) (2007).
29. A. Gold, J. Phys.: Condensed Matter **19**, 506214 (2007).
30. B. Jalam, S. Stemmer, S. Mack, and S. J. Allen, Phys. Rev. B **82**, 081103(R) (2010).
31. M. Ben Shalom, A. Ron, A. Palevski, and Y. Dagan, Phys. Rev. Lett. **105**, 206401 (2010).
32. A. D. Caviglia, S. Gariglio, C. Cancellieri et al., Phys. Rev. Lett. **105**, 236802 (2010).