

The low-temperature collapse of the Fermi surface and phase transitions in correlated Fermi systems

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A topological crossover, associated with the collapse of the Fermi surface in strongly correlated Fermi systems, is examined. It is demonstrated that in these systems, the temperature domain where standard Fermi liquid results hold dramatically narrows, because the Landau regime is replaced by a classical one. The impact of the collapse of the Fermi surface on pairing correlations is analyzed. In the domain of the Lifshitz phase diagram where the Fermi surface collapses, splitting of the BCS superconducting phase transition into two different ones of the same symmetry is shown to occur.

Introduction. For the past few years, the investigation of topological transitions in correlated Fermi systems that dates back to a pioneer work by I. M. Lifshitz, published in 1960 [1], has become one of hot topics in condensed matter physics [2–15]. These transitions are responsible for non-Fermi-liquid (NFL) behavior that manifests itself in singularities of thermodynamic characteristics of Fermi systems. E.g. in FL-theory, the spin susceptibility $\chi(T)$ remains unchanged at $T \rightarrow 0$. Contrariwise, at the transition point, $\chi(T \rightarrow 0)$ diverges as $T^{-\alpha}$, with the critical index $\alpha > 0$, somewhat depending on the shape of the single-particle spectrum $\epsilon(\mathbf{p})$ [16, 17, 11]. In case the function $\epsilon(\mathbf{p})$ is measured from the chemical potential μ , the topological rearrangement of the Landau state is associated with the change in the number of its zeroes [1, 18]. In conventional nonsuperfluid homogeneous Fermi systems, whose Fermi surfaces are singly connected, equation

$$\epsilon(p) = 0 \quad (1)$$

has the single *real* root, the Fermi momentum p_F . In the original article, Lifshitz analyzed topological transitions, occurring in noninteracting electron gas of metals at high pressures. Scenarios for such transitions, entailed by interactions between quasiparticles, one of which is addressed in this article, have emerged thirty years later [19–23], (for a recent review, see Ref. [24]). NFL behavior of Fermi systems near one of the critical points of Eq. (1) associated with the divergence of the effective mass M^* (this point is called the quantum critical point (QCP)), has been studied extensively in recent years [25, 26].

Usually Eq. (1) is analyzed at zero temperature. However, the spectrum $\epsilon(p)$ depends on T , being a functional of the quasiparticle momentum distribution $n(p, T)$ that has the standard Fermi-like form

$$n(p, T) = \left(1 + e^{\epsilon(p)/T}\right)^{-1}, \quad (2)$$

normalized in 3D by ordinary condition

$$\int n(p) dv = \rho \equiv \frac{p_F^3}{3\pi^2}, \quad (3)$$

with the volume element in 3D momentum space $dv = p^2 dp / \pi^2$.

The key point of this article is that at $T > 0$ where topological transitions are replaced by crossovers, there is a different route of the topological rearrangement, associated with the collapse of the Fermi surface. The collapse occurs in case Eq. (1) has no real roots at all. A classical Maxwell reconstruction of the $T = 0$ ideal-Fermi-gas momentum distribution is the simplest example of such a topological rearrangement. In this case where $\epsilon(p) = p^2/2M - \mu(T)$, the trajectory of the single root of Eq. (1), denoted further $p_{1/2}$ due to relation $n(p_{1/2}) = 1/2$, stemming from Eq. (2) at $\epsilon(p) = 0$, is easily traced. At $T \rightarrow 0$, one has $p_{1/2}(0) = p_F$. With the temperature rise, $p_{1/2}(T)$ moves toward the origin, attaining it at temperature where $\mu(T)$ changes sign. At higher T , real roots of Eq. (1) no longer exist, and the function $\epsilon(p)$ becomes positive defined, the gap in the spectrum $\epsilon(p)$ increasing with the further increase of T that entails rapid spread of the quasiparticle momentum distribution $n(p, T)$.

In systems with weak and/or moderate correlations, such an option is of no interest, since temperature T_M ,

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at which the Maxwell-like crossover takes place, is comparable with the Fermi energy $\epsilon_F^0 = p_F^2/2M$. However, with strengthening correlations, the effective mass M^* increases, and correspondingly, the FL-spectrum $\epsilon(p) = p_F(p - p_F)/M^*$ becomes flatter and flatter. As a result, temperature T_M goes down that leads, in its turn, to the dramatic shrinkage of the temperature region where the standard FL behaviors: $C(T) \propto T$, for the specific heat, and $\chi(T) = \text{const}$, for the spin susceptibility, hold. Importantly, at $T \simeq T_M \ll \epsilon_F^0$, the Landau–Migdal quasiparticle pattern remains applicable to evaluation of *thermodynamic* properties. This allows one to apply the quasiparticle formalism to the investigation of relevant problems, like splitting of the BCS superconducting phase transition into two ones, (see below), notwithstanding discrepancies between experimental data and standard FL-predictions.

Simple model of the collapse of the Fermi surface at low T . Within the quasiparticle picture, all the thermodynamic properties of correlated Fermi systems are evaluated in terms of the single-particle spectrum $\epsilon(p)$. This spectrum can be calculated on the base of FL-equation, connecting $\epsilon(p)$ with the FL quasiparticle momentum distribution (2) in terms of the first harmonic f_1 of the interaction function f , taken as a phenomenological input. In the 3D case, this equation has the form:

$$\frac{\partial \epsilon(p)}{\partial p} = \frac{p}{M} + \frac{1}{3} \int f_1(p, p_1) \frac{\partial n(p_1)}{\partial p_1} dv_1. \quad (4)$$

The interaction function $f(\mathbf{p}, \mathbf{p}_1)$ is known to coincide with a specific limit of the scattering amplitude $\Gamma(\mathbf{p}_1, \mathbf{p}_2; \mathbf{q}, \omega)$ where $\mathbf{q} = \mathbf{p} - \mathbf{p}_1 \rightarrow 0, \omega \rightarrow 0; q/\omega \rightarrow 0$. Generally Γ contains two different components. The first that prevails in the vicinity of second-order phase transitions changes rapidly near the Fermi surface. The second varies smoothly until momenta reach values much larger than p_F . Neglecting irregular components, the scattering amplitude Γ can be written in the standard form $\Gamma \propto 1/(a^{-1} - r_e q^2/2)$ where a is the scattering length, and r_e , the effective range. This form has to be supplemented by an exchange term to yield

$$\Gamma(\mathbf{p}, \mathbf{p}_1, \mathbf{q}) = \frac{4\pi}{M} \left[\frac{a}{1 - ar_e q^2/2} - \frac{a/2}{1 - ar_e (\mathbf{p} - \mathbf{p}_1 + \mathbf{q})^2/2} \right]. \quad (5)$$

To facilitate the analysis we expand this expression into the Taylor series. Retaining only two first terms in this expansion, inherent in an effective mass approximation, where

$$f(\mathbf{p}, \mathbf{p}_1) = \Gamma(\mathbf{p}, \mathbf{p}_1, \mathbf{q} = 0) = \frac{2\pi a}{M} \left[1 - \frac{ar_e}{2} (\mathbf{p} - \mathbf{p}_1)^2 \right], \quad (6)$$

the group velocity $d\epsilon(p)/dp$ is evaluated from Eq. (4) in the closed form:

$$\frac{d\epsilon(p)}{dp} = \frac{p}{M} (1 - 2\pi a^2 r_e \rho). \quad (7)$$

The spectrum $\epsilon(p)$ is then calculated straightforwardly. In doing so an immaterial constant, associated with the first term in Eq. (6) that contains the scattering length a , is absorbed into the chemical potential μ to yield

$$\epsilon(p) = \frac{p^2}{2M^*} - \mu, \quad \frac{M}{M^*} = 1 - 2\pi a^2 r_e \rho. \quad (8)$$

Upon accounting for a contribution σ from long-wavelength spin fluctuations, relevant e.g. in the case of 3D liquid ^3He discussed below, these formulas change. Nevertheless, as it was shown in microscopic calculations of the spectrum $\epsilon(p)$, performed in Ref. [27], the effective mass approximation remains adequate, merely the effective mass is modified to

$$\frac{M}{M^*} = 1 - 2\pi a^2 r_e \rho - \sigma, \quad (9)$$

with $\sigma \simeq 0.5$. With these results, the temperature evolution of the single root $p_{1/2}(T)$ turns out to be identical to that in ideal Fermi gas.

Temperature T_M , at which the chemical potential changes sign, is straightforwardly evaluated from the normalization condition (3) and formula (8) to yield

$$T_M \simeq \frac{p_F^2}{2M^*(\rho)}. \quad (10)$$

As correlations are strengthening, the ratio M^*/M increases and consequently, temperature T_M goes down.

Unfortunately, in the vicinity of the QCP where the effective mass M^* diverges, the effective mass approximation begins to fail, because other terms of the Taylor expansion come into play. For illustration, let us retain only the next term $\propto (\mathbf{p}_1 - \mathbf{p}_2)^4$. In this case, the group velocity $d\epsilon/dp$ acquires the form

$$\frac{d\epsilon(p)}{dp} = \alpha \frac{p}{M} + \nu \frac{p^3}{p_F^2 M} \quad (11)$$

with $\nu > 0$ and α , being dimensionless quantities, whose values are supposed to be small. As long as α keeps positive sign, the bifurcation momentum p_b equals 0. At the critical point where α vanishes, one has $d\epsilon(p \rightarrow p_F)/dp \equiv p_F/M^* = \nu p_F/M$, so that $M/M^* = \nu \ll 1$. Repeating the same manipulations that give rise to Eq. (10), one then finds $T_M \propto \nu p_F^2/M$, and therefore $T_M/\epsilon_F^0 \ll 1$.

When α changes sign, the single-particle spectrum, found with the aid of simple integration of Eq. (11), can be conveniently rewritten in the form:

$$\epsilon(x) = \nu \frac{p_F^2}{4M} (x - x_b)^2 - \mu(T), \quad (12)$$

where $x = p^2/p_F^2$ and $x_b = |\alpha|/\nu$. Again the effective mass M^* turns out to be enhanced: $M^*/M \simeq \nu^{-1}$.

The single root p_F of $T = 0$ equation (1) persists as long as $x_b < 1/2$, otherwise, this equation acquires the second root $p_{<}(0) = p_F(2x_b - 1)$, and the Landau state is rearranged. The single-particle states with $p_{<} < p < p_F$, where p_F is a new Fermi momentum, determined by the normalization condition (3), remain filled, while states with $p < p_{<}$ and $p > p_F$ turn out to be empty. With the rise of temperature, both the roots of Eq. (1) move to meet each other at the bifurcation momentum $p_b = p_F \sqrt{x_b}$. Critical temperature T_M is straightforwardly evaluated from Eq. (3). In the relevant case $x_b \simeq 1$, one finds

$$T_M \simeq \nu p_F^2 / M \ll \epsilon_F^0, \quad (13)$$

implying again that the elevation of temperature results in the rapid shrinkage of the FL-domain.

Discussion. All the analytically solved models, addressed above, have a common feature: strengthening interactions results in the shrinkage of the temperature domain where standard FL results hold. This conclusion remains valid even if flattening of the single-particle spectrum $\epsilon(p)$ takes place only for occupied states, the situation, inherent in many strongly correlated systems where the spectrum $\epsilon(p)$ has a more complicated structure, (see e.g. Ref. [4]). The shrinkage becomes especially pronounced beyond the QCP [22, 23, 4], where as mentioned above, the Fermi surface becomes multi-connected. For illustration, let Eq. (1) have n roots at $T = 0$, implying that the Fermi surface has n sheets. Assuming the single-particle spectrum to be a parabolic function between two neighbour roots of Eq. (1), one finds

$$\epsilon_{\max} \simeq \frac{\epsilon_{\text{QCP}}}{n^2}, \quad (14)$$

where $\epsilon_{\max} = \max|\epsilon(p)|$ inside the Fermi volume, i.e. for occupied states, and ϵ_{QCP} is the QCP Fermi energy. Since, as seen from Eq. (2), the critical collapse condition $n(p) < 1/2$ is met at $T_M \simeq \epsilon_{\max}$, we infer that beyond the QCP, critical temperature T_M rapidly falls with increasing the number n of sheets of the Fermi surface.

Let us now briefly discuss the collapse of the Fermi surface in strongly correlated systems, where at $T = 0$,

there exist flat bands. A different name for this dispersionless portion of the single-particle spectrum $\epsilon(p)$ is the fermion condensate (FC) [19]. As long as the FC-density is small, the collapse is insensitive to the presence of the FC. Although with further increasing the FC-fraction, temperature T_M rapidly falls, it keeps a nonzero value, even if all the quasiparticles get to the FC. This conclusion holds until f_1 attains a critical value, at which the inequality $n(p) < 1/2$ is met for any momentum p . In this situation, Eq. (1) has no roots, and hence, such Fermi systems have *no Fermi surface even at $T = 0$* .

It is instructive to trace how the standard FL-regime that operates in the $T \rightarrow 0$ limit gives way to a different one at $T > T_M$ where Eq. (1) has no roots at all. For illustration, let us address the spin susceptibility $\chi(T)$, neglecting for a while spin-spin correlations. In this case, $\chi(T)$ is given by the standard FL-formula

$$\chi(T) \propto \frac{1}{T} \int n(p)[1 - n(p)] dv. \quad (15)$$

In the FL-regime, overwhelming contributions to this integral come from a domain $|\epsilon(p)| \leq T$, adjacent to the Fermi surface, and after simple algebra with employing the FL-formula $dp/d\epsilon = M^*/p_F$, we arrive at the Pauli result: $\chi(T) = \text{const} \propto M^*$.

The collapse of the Fermi surface leads to the reconstruction of the FL-regime. As T increases away from T_M , the chemical potential μ becomes more and more negative, that triggers exponential suppression of the integral with the $n^2(p)$ term on the r.h.s. of Eq. (15). Eventually this contribution becomes negligible, and the integral (15) is then evaluated in the closed form with the aid of the normalization condition (3), producing the classical Curie law: $\chi(T) \propto \rho/T$. Accounting for the spin-spin correlations introduces a correction, associated with the Weiss temperature Θ_W , to yield $\chi(T) \propto \rho/(T - \Theta_W)$. Thus from the analysis of the spin susceptibility, we infer that temperature T_M , at which the Fermi surface collapses, does specify the topological crossover from the standard FL-regime to a classical-like one. The verification of this conclusion on the base of experimental data on the specific heat $C(T)$ encounters difficulties, requiring information on the spectrum $\epsilon(p, T)$ at $T > T_M$ over a wide momentum range that remains scarce yet.

3D-liquid ^3He is one of the systems that experiences the topological crossover, discussed in this article. The Fermi energy of this liquid is $\epsilon_F^0 \simeq 5$ K, while the effective mass $M^*(\rho) \geq 3M$. At very low temperatures, behaviors of both $C(T)$ and $\chi(T)$ are known to be consistent with FL-theory. However, already at $T \simeq 0.2$ K, the spin susceptibility $\chi(T)$ begins to decline, and at

$T \geq p_F^2/2M^* \simeq 1.5$ K its behavior is, indeed, obeys the classical Curie formula $\chi(T) \propto \rho/T$. Meanwhile conventionally, the departure of experimental data from the FL result $\chi(T) = \text{const}$ is attributed to damping effects [28]. Alas, how accounting for damping may lead to the Curie behavior of $\chi(T)$ in liquid ^3He , is hardly explicable.

Let us now turn to strongly correlated electron systems where the effective electron mass M^* is enhanced as well. To clear up confusion, caused by the presence of the lattice field in solids, it should be pointed out that it results in a modification of Eq. (4) that can be done on the base of FL-relation (see e.g. Ref. [29]),

$$z\mathcal{T}^k(\mathbf{p}) = z\mathcal{T}^\omega(\mathbf{p}) + \int f(\mathbf{p}, \mathbf{p}_1) \frac{\partial n(p_1)}{\partial \mathbf{p}_1} \frac{d^3 p_1}{(2\pi)^3}, \quad (16)$$

connecting ω - and k -limits of the vertex part $\mathcal{T}(\mathbf{p})$, independently of whether the external field presents or not. In this equation, z stands for the renormalization factor. The l.h.s. of this equation is expressed in the closed form: $z\mathcal{T}^k(\mathbf{p}) = -M\partial G^{-1}(\epsilon = 0, \mathbf{p})/\partial \mathbf{p} = M\partial \epsilon(p)/\partial \mathbf{p}$. As for the quantity $z\mathcal{T}^\omega(\mathbf{p})$, it can be written in the Migdal form:

$$z\mathcal{T}^\omega(\mathbf{p}) = e_q \mathbf{p}, \quad (17)$$

where e_q is the quasiparticle effective charge [29] that equals unity in homogeneous matter due to momentum conservation. In solids, momentum conservation breaks down, and therefore $e_q \neq 1$. Additionally, in solids, the electron mass M is replaced by the so-called LDA electron mass M_{LDA} , which cannot be evaluated with great precision because of the complicated structure of the lattice field. Therefore the effective charge e_q can be absorbed into M_{LDA} . With this specification, the above analysis can be applied to data on the magnetic susceptibility of the heavy-fermion alloy $\text{Yb}(\text{Rh}_{1-x}\text{Co}_x)_2\text{Si}_2$, obtained at different dopings x in Ref. [30]. Experimental results evidence for *gradual* flattening out of the product $T\chi(T)$, like in 3D-liquid ^3He . However, the authors [30] attribute such a behavior to the localization of $4f$ -electrons, notwithstanding the localization is a phase transition, rather than crossover, and therefore the localization scenario fails to explain gradual flattening of the function $T\chi(T)$.

Impact of the collapse of the Fermi surface on pairing correlations. Phase transitions, associated with pairing correlations, are traditionally investigated on the base of the BCS gap equation. In the case of s -pairing, this equation reads

$$\Delta(p, T) = - \int \mathcal{V}(p, p_1) \frac{\tanh E(p_1)/[2T]}{2E(p_1)} \Delta(p_1, T) dv_1. \quad (18)$$

Here $\Delta(p)$ is the gap function, $E(p) = \sqrt{\epsilon^2(p) + \Delta^2(p)}$ is the Bogoliubov quasiparticle energy, and $\mathcal{V}(p, p_1)$ is the zero harmonic of the pairing interaction $\mathcal{V}(\mathbf{p}, \mathbf{p}_1)$, presented by a set of Feynman diagrams, irreducible in the particle-particle channel. Upon neglecting its momentum dependence we are led to the well-known BCS-result:

$$\Delta(0) = \Omega_D e^{-2/\lambda}, \quad (19)$$

with the dimensionless pairing constant $\lambda = p_F M^* |\mathcal{V}_L(p_F, p_F)|/\pi^2$ and Ω_D , the Debye frequency.

Critical temperature T^* , at which pairing correlations die out, is found from homogeneous equation

$$D(p) = - \int \mathcal{V}(p, p_1) \frac{\tanh[\epsilon(p_1, T^*)]/[2T^*]}{2\epsilon(p_1, T^*)} D(p_1) dv_1, \quad (20)$$

determining the location of the pole of the two-particle Green function at the total momentum $\mathbf{P} = 0$ (Thouless criterion). Conventional wisdom reads that T^* is critical temperature of termination of BCS-superconductivity. Indeed, in the weak-coupling limit, Eq. (20) is derived from Eq. (18), setting there $\Delta \rightarrow 0$, and therefore temperature T^* coincides with BCS critical temperature T_c . In this case, T_c is expressed in terms of the gap magnitude $\Delta(0)$ as

$$T_c = 0.57\Delta(0). \quad (21)$$

However, with strengthening correlations, the density of states, proportional to the effective mass M^* , increases, and both the quantities $\Delta(0)$ and T_c soar up, while T_M goes down. Eventually, when inequality $T_M < T_c = 0.57\Delta(0)$ is met, the coincidence between T^* and T_c is destroyed. Indeed, at $T > T_M$, the Gor'kov term $\Delta^2/[\epsilon + \epsilon(p)]$ in the mass operator $\Sigma(p, \epsilon)$ has no longer pole at the Fermi surface, or equivalently at $\epsilon \rightarrow 0$, due to the fact that $\epsilon(p, T) > 0$ at any momentum p . Therefore at $T > T_M$ the Gor'kov term has to be treated on equal footing with regular contributions to Σ that results in recovering the Dyson-like form of the quasiparticle Green function $G(p, \epsilon) = [\epsilon - \epsilon(p)]^{-1}$. In this situation, the Meissner effect no longer exists, and superconductivity is terminated.

To clarify the structure of the pairing correlations in the domain $T_M \leq T < T^*$ where, according to Eq. (20), these correlations still persist, it is worth remembering the familiar situation with low-density symmetric nuclear matter at $T = 0$ where the deuteron pole in the n - p scattering exhibits itself in full force. In this case, analyzed in numerous theoretical studies [31–35], the pairing correction to the chemical potential $\mu(0)$ turns out to be

in great excess of the Fermi energy $p_F^2/2M$. As a result, the Cooper condensate transforms into a Bose condensate of bound states of quasiparticle pairs, the boson radius r_B being much less than the interparticle distance r_0 (the BCS–BEC-crossover). Analogous results have been obtained in the model of bipolaronic superconductivity [36], where the strength of the electron-phonon attraction is supposed to be large: $\lambda \gg 1$. With decreasing density ρ or increasing λ , the chemical potential $\mu(0)$ diminishes and eventually becomes negative, as in the classical situation: $\mu(0) \rightarrow -\epsilon_0/2$ where ϵ_0 stands for the quasimolecule binding energy.

Above critical temperature T_B , at which the Bose condensate dies out, a conglomerate of moving bound pairs remains that coexists with ordinary quasiparticles, the well-known feature of hot low-density nuclear matter. This coexistence manifests itself in the enhancement of deuteron formation in deep inelastic nuclear reactions. In unison with this fact, a large diamagnetic response, observed in high- T_c superconductors, is explained, attributing it to the normal-state Landau diamagnetism of the bipolaron system [37].

In contrast to the case of low-density matter, in dealing with strongly correlated Fermi systems, the energy ϵ_0 , determined by the Bethe–Salpeter equation

$$D(p) = - \int \mathcal{V}(p, p_1) \frac{\tanh \epsilon(p_1, T)/(2T)}{2\epsilon(p_1, T) - \epsilon_0(T)} D(p_1) dv_1, \quad (22)$$

depends on temperature, and as stems from comparison of Eqs. (20) and (22), $\epsilon_0(T)$ vanishes at $T = T^*$.

Our first goal is to evaluate a critical point of the Lifshitz phase diagram for systems with pairing correlations, where both the Cooper condensate and the conglomerate of bound pairs disappear simultaneously that occurs provided temperatures T_M and T^* coincide with each other. The corresponding critical value λ_c is found with the aid of the Thouless criterion (20). In doing so we employ the above model with the single-particle spectrum (12) where the root $p_{1/2}(T_M)$ is assumed to be located quite far from the origin: $p_{1/2}(T_M)/p_F \simeq 1$. Remembering that at $T = T_M \propto 1/M^*$, the chemical potential $\mu(T)$ vanishes, we arrive at the following equation

$$1 \propto \lambda_c \sqrt{\frac{\epsilon_F^0}{M^* T^*}} \int_0^\infty \frac{\tanh z}{z^{3/2}} dz, \quad (23)$$

to find that in the case $T^* = T_M$,

$$\lambda_c \simeq O(1). \quad (24)$$

This result holds beyond the QCP as well. Indeed, by definition, all new pockets of the Fermi surface that emerge beyond the QCP disappear at $T = T_M$.

Near the bifurcation momentum $p_{1/2}(T_M)$, the single-particle spectrum $\epsilon(p, T_M)$ has the same parabolic form: $\epsilon(p, T_M) \propto w[p - p_{1/2}(T_M)]^2$ as in Eq. (12), with the prefactor $w \propto 1/M^*$. Evidently, upon inserting these relations into Eq. (20) the result (24) is recovered.

Evidently, in case M^* increases, temperature T_M drops, while T^* grows. Thus in the domain of the phase diagram where $\lambda \simeq 1$, the BCS superconducting phase transition is split into two ones. The lower transition continues to be associated with termination of superconductivity, while the upper one turns out to be related to the extinguishment of the bound pairs.

The excess $T^* - T_M$ can be shown to obey a linear relation

$$T^* - T_M = T_M(\lambda - \lambda_c), \quad (25)$$

determining the range of the region $T_M < T < T^*$ where the quasiparticles coexist with the quasimolecules, while the BCS-correlations are completely suppressed.

It is instructive to evaluate the binding energy $\epsilon_0(\lambda, T)$ at critical temperature T_M . At small difference $(T^* - T_M) \ll T^*$, this energy is calculated from a set of two equations, one of which is Eq. (23), while the second reads

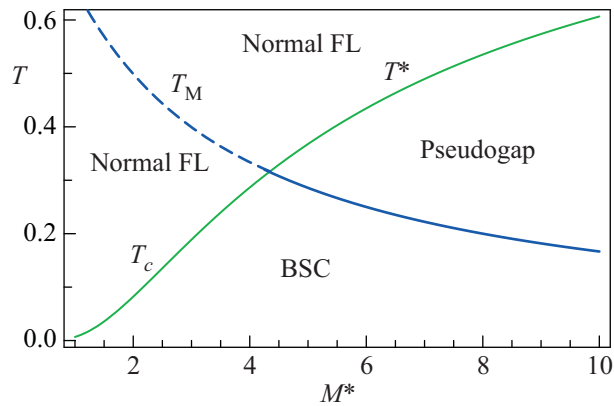
$$1 \propto \lambda \sqrt{\frac{\epsilon_F^0}{M^* T_M}} \int_0^\infty \frac{\tanh z}{z^{1/2}(z + \epsilon_0/T_M)} dz. \quad (26)$$

Straightforward calculations then yield

$$\epsilon_0(\lambda, T_M) \propto T_M(\lambda - \lambda_c)^2. \quad (27)$$

One of consequences of this result is that the pairing correlations cannot regain its original BCS-form at $T = T_M$, and therefore the BCS critical temperature T_c must be lower than T_M . The analysis of the interplay between the two types of these correlations at low $T < T_M$, dating back to Ref. [31], is not properly performed yet. I hope to revert to this question in the future. These results are summarized in the Lifshitz phase diagram of a Fermi system with pairing correlations, drawn in figure where the effective mass M^* changes, while the pairing interaction \mathcal{V} remains unchanged.

Let us now turn to pairing correlations in systems with flat bands, being nowadays the subject of wide speculation because of the huge density of states of these systems [6, 7, 9]. As we have seen, if in a system with the flat band, there exists the point $p_{1/2}$ where $n(p) = 1/2$, such a system possesses the Fermi surface, and one can expect that its superconductivity is described within BCS-theory. In systems without the Fermi surface where inequality $n(p) < 1/2$ is met at any point of momentum



Color online. The phase diagram of a Fermi system with pairing correlations vs. the effective mass M^* . The strength of pairing interaction \mathcal{V} is supposed to be small enough to avoid the regime of bipolaronic superconductivity. The line, separated the state with pairing correlations from the conventional FL-state, is drawn in grey (green online). The pseudogap phase is separated from the BCS one by the solid black (blue online) line where the collapse of the Fermi surface occurs. The dashed black (blue online) line separates the Landau states from the normal state without the Fermi surface. Both temperature T and the effective mass M^* are given in arbitrary units

space, the $T = 0$ ground state of the system with the flat band, being superfluid, is a condensate of the bound quasiparticle pairs.

In conclusion, in this article, topological crossover in homogeneous matter, associated with the collapse of the Fermi surface, is analyzed. It is shown that this collapse leads to the replacement of the standard FL-regime by a classical Maxwell-like one. Temperature behavior of the spin susceptibility of 3D-liquid ^3He is demonstrated to be properly explained within this scenario that implies the substantial extension of the temperature region where the quasiparticle pattern of phenomena in 3D-liquid ^3He holds. The impact of the collapse of the Fermi surface on pairing correlations in a domain of the Lifshitz phase diagram is examined. The topological crossover, connected with the collapse, is shown to be responsible for splitting of the BCS-superconducting phase transition into two ones of *the same symmetry*. The lower one is related to termination of BCS-superconductivity, while the upper one is associated with the disappearance of the conglomerate of the bound pairs, a key feature of the pseudogap phenomenon [38].

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1. I. M. Lifshitz, Sov. Phys. JETP **11**, 1130 (1960).
2. N. Doiron-Leyrand et al., Nature **447**, 545 (2007).
3. D. LeBoeuf et al., Nature **450**, 533 (2007).
4. V. A. Khodel, J. W. Clark, and M. V. Zverev, Phys. Rev. B **78**, 075120 (2008); and references therein.
5. M. R. Norman, J. Lin, and A. J. Millis, Phys. Rev. B **81**, 180513 (2010).
6. G. E. Volovik, JETP Lett. **93**, 66 (2011).
7. N. B. Kopnin, T. T. Heikkilä, and G. E. Volovik, Phys. Rev. B **83**, 220503(R) (2011).
8. N. B. Kopnin, T. T. Heikkilä, and G. E. Volovik, Pis'ma ZhETF **94**, 252 (2011).
9. N. B. Kopnin, Pis'ma ZhETF **94**, 81 (2011); arXiv:1105.1883;
10. D. LeBoeuf et al., Phys. Rev. B **83**, 054506 (2011).
11. V. R. Shaginyan, A. Z. Mzezane, and K. G. Popov, Phys. Rev. B **84**, 060401(R) (2011).
12. S. S. Pankratov, M. V. Zverev, and M. Baldo, JETP Lett. **93**, 591 (2011).
13. V. A. Khodel, J. W. Clark, and M. V. Zverev, JETP Lett. **94**, 73 (2011).
14. A. P. Schnyder and S. Ryu, Phys. Rev. B **84**, 060504 (2011).
15. P. M. R. Brydon, A. P. Schnyder, and C. Timm, Phys. Rev. B **84**, 020501 (2011).
16. J. W. Clark, V. A. Khodel, and M. V. Zverev, Phys. Rev. B **71**, 012401 (2005).
17. V. A. Khodel, M. V. Zverev, and V. M. Yakovenko, Phys. Rev. Lett. **95**, 236402 (2005).
18. G. E. Volovik, Springer Lecture Notes in Physics **718**, 31 (2007); arXiv:0601372, and references cited therein.
19. V. A. Khodel and V. R. Shaginyan, JETP Lett. **51**, 553 (1990).
20. G. E. Volovik, JETP Lett. **53**, 222 (1991).
21. P. Nozières, J. Phys. I France **2**, 443 (1992).
22. M. V. Zverev and M. Baldo, JETP **87**, 1129 (1998); J. Phys.: Condens. Matter **11**, 2059 (1999).
23. S. A. Artamonov, V. R. Shaginyan, and Yu. G. Pogorelov, JETP Lett. **68**, 942 (1998).
24. V. R. Shaginyan et al., Phys. Rep. **492**, 31 (2010) and references therein.
25. H. v. Löhneysen et al., Rev. Mod. Phys. **79**, 1015 (2007).
26. P. Gegenwart, Q. Si, and F. Steglich, Nature Phys. **4**, 186 (2008).
27. E. Krotscheck, J. Springer, JLTP **132**, 281 (2003).
28. D. Pines and P. Nozières, *Theory of Quantum Liquids*, v. 1, p. 91, W. A. Benjamin, New York-Amsterdam, 1966.

29. A. B. Migdal, *Theory of Finite Fermi Systems and Applications to Atomic Nuclei*, Wiley, N.Y., 1967.
30. C. Klingner et al., Phys. Rev. B **83**, 144405 (2011).
31. D. M. Eagles, Phys. Rev. **186**, 456 (1969).
32. A. J. Leggett, in *Modern Trends in the Theory of Condensed Matter* (eds. A. Pekalski and J. Przystawa), Spriger-Verlag, Berlin, 1980.
33. P. Nozières and S. Schmitt-Rink, JLTP **59**, 980 (1985).
34. M. Schmidt, G. Röpke, and H. Schulz, Ann. Phys. (NY) **202**, 57 (1990).
35. M. Baldo, U. Lombardo, and P. Schuck, Phys. Rev. C **52**, 975 (1995).
36. A. S. Alexandrov and N. Mott, *Polarons and Bipolarons*, World Scientific, Singapore, 1996, and references therein.
37. A. S. Alexandrov, Phys. Rev. Lett. **96**, 147003 (2006); J. Phys. Condens. Matter **22**, 426004 (2010).
38. R.-H. He et al., Science **331**, 1579 (2011) and references therein.