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The analytical singlet α_s^4 QCD contributions into the e^+e^--annihilation Adler function and the generalized Crewther relations

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The generalized Crewther relations in the channels of the non-singlet and vector quark currents are considered. These relations follow from the double application of the operator product expansion approach to the same axial vector–vector–vector triangle amplitude in two regions, adjoining to the angle sides (x, y) (or p^2, q^2). We assume that the generalized Crewther relations in these two kinematic regimes result in the existence of the same perturbation expression for two products of the coefficient functions of annihilation and deep-inelastic scattering processes in the non-singlet and vector channels. This feature explains the conformal symmetry motivated cancellations between the singlet α_s^3 corrections to the Gross–Llewellyn Smith sum rule $S_{\rm GLS}$ of νN deep inelastic scattering and the singlet α_s^3 correction to the e^+e^--annihilation Adler function D_A^V in the product of the corresponding perturbative series. Taking into account the Baikov–Chetyrkin–Kuhn 4-th order result for $S_{\rm GLS}$ and the perturbative effects of the violation of the conformal symmetry in the generalized Crewther relation, we obtain the analytical contribution to the singlet α_s^4 correction to the D_A^V-function. Its a-posteriori comparison with the recent result of direct diagram-by-diagram evaluation of the singlet 4-th order corrections to D_A^V- function demonstrates the coincidence of the predicted and obtained ζ_3^2-contributions to the singlet term. They can be obtained in the conformal invariant limit from the original Crewther relation. Therefore, on the contrary to previous belief, the appearance of ζ_3-terms in the perurbative series in quantum field theory gauge models does not contradict to the property of the the conformal symmetry and can be considered as regular feature. The Banks–Zaks motivated relation between our predicted and the obtained directly 4-th order corrections is mentioned. It confirms the expectation, previously made by Baikov–Chetykin–Kuhn, that at the 5-loop level the generalized Crewther relation in the channel of vector currents may receive additional singlet contribution, which in this order of perturbation theory is proportional to the first coefficient of the QCD β-function.

Quite recently the 4-th order perturbative coefficients to the flavour non-singlet (NS) part of the e^+e^--annihilation Adler D-function $D_A^{\rm NS}$ and to the NS Bjorken sum rule S_{Bjp} of the deep-inelastic scattering (DIS) process of polarised leptons on nucleons were evaluated symbolically within general $SU(N_c)$ colour gauge group [1]. The explicit analytical expression for the 4-th perturbative coefficient of S_{Bjp} in the $\overline{MS}$-scheme was obtained in [2] by inverting the order a_s^4 analytical results for $1/S_{Bjp}$, presented in [1]. Apart of the phenomenological applications to the analysis of the experimental data for the semi-hadronic width of τ-lepton [3][2)] and of the experimental data for the Bjorken polarised sum rule [5], the $\overline{MS}$-scheme results of [1] have also important theoretical consequences. As was emphasised in [1, 2], the analytical $SU(N_c)$-group expressions of [1] play essential role in the verification of the discovered in [6] β-function factorisable representations for the conformal symmetry breaking term in the QCD-generalization of the original quark-parton model Crewther relation [7]. The validity of this discovery of [6] was proved in the $\overline{MS}$-scheme in all orders of perturbation theory [8] (for some additional discussions see the review of [9]).

It is worth to stress, that to apply self-consistently the 4-th order perturbative results of [1] in the analysis of the $e^+e^- \to$ *hadrons* data above thresholds of charm quarks production and in the fits of the precise LEP-data for $Z^0 \to$ *hadrons* decay width as well, it is necessary to find still unknown 4-th order singlet (SI) contribution to the Adler functions D_A of vector quark currents. In the previous a_s^3 order of perturbation theory these types of corrections were analytically evaluated in [10] and confirmed later in [11] and [12].

In this work the symbolical expressions for the 4-th order SI contributions to D_A-function are predicted. Three important theoretical inputs will be used:

- the universality of the original Crewther relation [7], which is valid in the conformal invariant limit not only for the product of the coefficient functions of D_A^{NS}-function and NS DIS Bjorken sum rule,

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[2)]Note, that in this analysis the $SU(3)$ variant [4] of the $SU(N_c)$ expression for $D_A^{\rm NS}$ [1] was used.

but for the product of the coefficient functions D_A^V of the Adler D-function of vector currents and of the coefficient function of the Gross–Llewellyn DIS sum rule for neutrino-nucleon DIS as well [13];

- the statement of [14] that the product of the normalised perturbative coefficient functions of the vector currents D-function and of the Gross–Llewellyn sum rule is identical to the $\overline{MS}$-scheme QCD expression of the generalized Crewther relation in the NS-case;
- the obtained in the case of the general $SU(N_c)$-group analytical expressions for the $O(a_s^4)$ flavour SI perturbative contributions to the Gross–Llewellyn Smith sum rule in the $\overline{MS}$-scheme [15] and the previous $O(a_s^3)$ order scheme-independent SI correction to this sum rule which was analytically evaluated in [16].

Consider now physical quantities to be analysed in this work. Perturbative expression for the e^+e^- Adler D-function in the vector channel has the following form

$$D_A^V(a_s) = Q^2\int_0^\infty \frac{R^{e^+e^-}(s)}{(s+Q^2)^2}ds = D_A^{\rm NS}(a_s) + D_A^{\rm SI}(a_s), \quad (1)$$

where the functions in the r.h.s. of Eq. (1) are defined as

$$D_A^{\rm NS}(a_s) = d_R\left(\sum_F Q_F^2\right)C_A^{\rm NS}(a_s),$$
$$D_A^{\rm SI}(a_s) = \left(\sum_F Q_F\right)^2 C_A^{\rm SI}(a_s). \quad (2)$$

Here, Q_F are the electric charges of quarks and d_R is the dimension of $SU(N_c)$-group representation (for detailed definitions and studies see e.g. Ref. [17]). It enters in the normalisation factor of the NS contribution to the Adler vector function and do not enter in the normalisation factor of the SI-part of Eq. (1).

Within the fundamental representation of $SU(N_c)$-group one has $d_R = N_c$. The coefficient functions $C_A^{\rm NS}$ and $C_A^{\rm SI}$ have the following perturbation expansions

$$C_A^{\rm NS}(a_s) = 1 + \sum_{n\geq 1} d_n^{\rm NS}a_s^n, \quad C_A^{\rm SI}(a_s) = \sum_{n\geq 3} d_n^{\rm SI}a_s^n. \quad (3)$$

Within perturbation theory the polarised Bjorken DIS sum rule is defined as

$$S_{Bjp}^{\rm NS}(a_s) = \int_0^1 g_1^{lp-ln}(x,Q^2)dx = \frac{g_a}{6}C_{Bjp}^{\rm NS}(a_s), \quad (4)$$

where the coefficient function can be expressed as

$$C_{Bjp}^{\rm NS}(a_s) = 1 + \sum_{m\geq 1} c_m^{NS}a_s^m. \quad (5)$$

The known coefficients d_1–d_4 and c_1–c_4 contain the powers of $SU(N_c)$ colour group structures C_F, C_A, T_FN_F and d^{abcd} [1], where C_F and C_A are the Casimir operators, N_F is the number of flavours, T_F is the normalisation factor of the trace of product of two $SU(N_c)$ generators and d^{abcd} are the structure constants of $SU(N_C)$-group.

The Gross–Llewellyn Smith sum rule of νN DIS can be defined as

$$S_{\rm GLS}(a_s) = \frac{1}{2}\int_0^1 F_3^{\nu p+\overline{\nu}p}(x,Q^2)dx =$$
$$= 3C_{\rm GLS}(a_s) + O\left(\frac{1}{Q^2}\right), \quad (6)$$

where its pure perturbative coefficient function can be decomposed to the NS- and SI-parts as

$$C_{\rm GLS}(a_s) = C_{\rm GLS}^{\rm NS}(a_s) + C_{\rm GLS}^{\rm SI}(a_s). \quad (7)$$

Note, that for the NS-part the following identity holds:

$$C_{\rm GLS}^{\rm NS}(a_s) = C_{Bjp}^{\rm NS}(a_s) \quad (8)$$

The validity of Eq. (8) was first proved within dimensional regularization in [18] by demonstration that the l.h.s and r.h.s. of Eq. (8) coincide after proper definition of axial Ward identities in QCD. The necessity of performing extra finite renormalizations in the axial-vector channel were understood in the process of studies of 2-loop corrections to dimensionally regularized [19] Adler D-functions for the case of axial-vector and vector quark currents in the independent, but simultaneous works of Refs. [20, 21].

As was demonstrated in Ref. [16], at the 3-rd order of perturbation theory the coefficient function of the Gross–Llewellyn Smith sum rule of Eq. (7) starts to differ from $C_{Bjp}^{\rm NS}(a_s)$ due to the appearance of the additional SI contributions. They enter into the expression of the following coefficient function:

$$C_{\rm GLS}^{\rm SI}(a_s) = \sum_{n\geq 3} c_n^{\rm SI}a_s^n. \quad (9)$$

Its first scheme-independent term has the following analytical form [16]

$$c_3^{\rm SI} = \frac{d^{abc}d^{abc}N_F}{d_R}c_{3,1}^{\rm SI} = \frac{d^{abc}d^{abc}N_F}{d_R}\left(-\frac{11}{192}+\frac{1}{8}\zeta_3\right). \quad (10)$$

Notice, the appearance of $SU(N_c)$-group structure $d^{abc}d^{abc}$. Next coefficient was analytically evaluated in the $\overline{MS}$ scheme in [15]. It has the following representation:

$$c_4^{\rm SI} = \frac{d^{abc}d^{abc}N_F}{d_R}\left(C_F\ c_{4,1}^{\rm SI} + C_A\ c_{4,2}^{\rm SI} + T_F N_F\ c_{4,3}^{\rm SI}\right), \quad (11)$$

where all coefficients were analytically evaluated in [15].

Consider now the concrete consequences of the conformal symmetry and of the effects of its violation in the theory of strong interactions. The validity of the conformal invariance in the quark-parton model leads to the existence of the Crewther relation [7]. In the renormalizable gauge quantum field models (say QED or QCD) this relation can be written down as

$$C_A^{\rm NS}(a_s) \times C_{Bjp}^{\rm NS}(a_s)|_{c-i} = 1. \quad (12)$$

Here, due to the property of the the conformal invariance, a_s does not depend on transferred momentum Q^2.

Note that the similar Crewther relation is also valid for the product of the D_A^V and the Gross–Llewellyn Smith sum rule coefficient functions:

$$C_A^V(a_s) \times C_{\rm GLS}(a_s)|_{c-i} = 1. \quad (13)$$

It is based on exact x-space derivation of [13] (compare Eq. (2.9a) with Eq. (2.9b) in [13]).

Originally Crewther relation in the NS and vector channels was obtained in [7] and analysed in more details in the work of [13] by means of double application of the operator product expansion (OPE) approach for the vector–vector–axial–vector 3-point function

$$T_{\mu\alpha\beta}^{abc}(x,0,y) = \langle 0|T[V_\mu^a(x)V_\alpha^b(0)A_\beta^c(y)]|0\rangle, \quad (14)$$

where $V_\mu^a = \overline{\psi}(x)\gamma_\mu(\lambda^a/2)\psi(x)$ and $A_\beta^c = \overline{\psi}(x)\gamma_5\gamma_\beta(\lambda^c/2)\psi(x)$. In the momentum space the amplitude of Eq. (14) can be written down as

$$T_{\mu\alpha\beta}^{abc}(p,q) = $$
$$= i\int\langle 0|TV_\mu^a(x)V_\alpha^b(0)A_\alpha^c(y)|0\rangle e^{ipx+iqy}dxdy = $$
$$= d^{abc}T_{\mu\alpha\beta}(p,q). \quad (15)$$

Thanks to the studies of Ref. [22] it was understood that in the conformal-invariant limit this 3-index tensor is proportional to the fermion triangle one-loop graph, constructed from massless fermions, namely

$$T_{\mu\alpha\beta}^{abc}(x,0,y) = d^{abc}N\Delta^{1-loop}(x,0,y). \quad (16)$$

Here N is a number, which is proportional to anomalous constant, related to $\pi^0 \to \gamma\gamma$ decay [23, 24]. Fixing $N = 1$ and keeping in mind this physical relation to the amplitude of $\pi^0 \to \gamma\gamma$ decay, we define the coefficient function of the Adler D-function in the vector channel by taking the limit of equal charges of all quarks flavours and re-writing Eq. (1) as

$$D_A^V(a_s) = d_R\left(\sum_F Q_F^2\right)C_A^V(a_s), \quad (17)$$

where

$$C_A^V(a_s) = C_A^{\rm NS}(a_s) + \frac{\left(\sum_F Q_F\right)^2}{d_R\left(\sum_F Q_F^2\right)}\sum_{n\geq 3} d_n^{\rm SI}a_s^n = $$
$$= C_A^{\rm NS}(a_s) + \frac{N_F}{d_R}\sum_{n\geq 3} d_n^{\rm SI}a_s^n. \quad (18)$$

The coefficient $d_3^{\rm SI}$ is known from the analytical calculations of [10–12] and reads

$$d_3^{\rm SI} = d^{abc}d^{abc}d_{3,1}^{\rm SI} = d^{abc}d^{abc}\left(\frac{11}{192} - \frac{1}{8}\zeta_3\right). \quad (19)$$

Consider now the generalizations of Crewther relation in the channels of vector and NS quark currents. They can be defined as

$$C_A^V[a_s(Q^2)] \times C_{\rm GLS}[a_s(Q^2)] = 1 + \Delta_{csb}^{V,\rm GLS}[a_s(Q^2)] \quad (20)$$

and

$$C_A^{\rm NS}[a_s(Q^2)] \times C_{Bjp}^{\rm NS}[a_s(Q^2)] = 1 + \Delta_{csb}^{\rm NS}[a_s(Q^2)]. \quad (21)$$

The expression for the conformal symmetry breaking contribution into Eq. (21) can be presented in the following form

$$\Delta_{csb}^{\rm NS}(a_s) = $$
$$= \frac{\beta(a_s)}{a_s}\left[K_1^{\overline{MS}}a_s + K_2^{\overline{MS}}a_s^2 + K_3^{\overline{MS}}(a_s^3) + O(a_s^4)\right], \quad (22)$$

where the coefficients $K_1^{\overline{MS}}$ and $K_2^{\overline{MS}}$ were first determined in [6], while the analytical expression for $K_3^{\overline{MS}}$ was obtained only recently [1]. Note, that the possibility that the factorisation of the renormalization group β-function in Eq. (22) is valid in all orders of perturbation theory was first studied in Ref. [25] in the momentum space. These studies were made by double application of OPE approach to the axial vector–vector–vector triangle amplitude of Eq. (15) in the kinematic limit $|p^2| \gg |q^2| \to \infty$ [25]. The validity of this factorizable feature of the generalization of Crewther relation was proved in all orders of perturbation theory in the coordinate space [8] by means of double application of the OPE to this axial–vector–vector–vector amplitude of Eq. (14) in the limit $x \ll y,\ x, y \to 0$.

Let us return to the problem of determination of the a_s^4-coefficient in Eq. (3). Its obvious diagrammatic representation allowed to express it in the following form [15]:

$$d_4^{\rm SI} = d^{abc}d^{abc}\left(C_F\ d_{4,1}^{\rm SI} + C_A\ d_{4,2}^{\rm SI} + T_F N_F\ d_{4,3}^{\rm SI}\right). \tag{23}$$

It is known, that when the proportional to $C_F^k a_s^k$ (with $k \geq 1$) contributions to C_A^V and $C_{\rm GLS}$ are only considered, the corresponding expression for Eq. (13) is valid in all orders of perturbation theory [13, 26]. Indeed, when charge renormalization effects, and thus the conformal symmetry breaking contribution $\Delta_{csb}[a_s(Q^2)]$ in the r.h.s. of Eq. (20) and Eq. (21) are not taken into account, the conformal symmetry is effectively restored and the cancellation of these special contributions to Adler functions and DIS sum rules holds in all orders of perturbation theory [13, 26]. In the case of Eq. (13), the cancellations of the similar contributions to the SI-parts of C_A^V and $C_{\rm GLS}$ were already observed at the a_s^3-level [6]. In [15] the conformal-invariant limit and Eq. (13) were non-obviously used to get the expression for $d_{4,1}^{\rm SI}$ term from the SI order a_s^3 and a_s^4 contributions to Eq. (9). Indeed, re-writing the variant of expression of Eq. (13) as

$$C_A^V(a_s)|_{c-i} = 1/C_{\rm GLS}(a_s)|_{c-i}, \tag{24}$$

one can get the prediction for $d_{4,1}^{\rm SI}$ from Ref. [15], namely

$$d_{4,1}^{\rm SI} = -\frac{3}{2}c_{3,1}^{\rm SI} - c_{4,1}^{\rm SI} = -\frac{13}{64} - \frac{1}{4}\zeta_3 + \frac{5}{8}\zeta_5, \tag{25}$$

where the factor $-3/2$ before $c_{3,1}^{\rm SI}$ comes from the well-known term $c_1 = -(3/4)C_F$, which enters into the contribution $2c_1c_{3,1}^{\rm SI}$ to the $O(a_s^4)$-coefficient of $1/C_{\rm GLS}(a_s)$-function in the r.h.s. of Eq. (24).

Let us now consider the status of the statement of [14] that for the generalizations of Crewther relation of Eq. (21) and Eq. (20) following identity holds

$$C_A^V[a_s(Q^2)] \times C_{\rm GLS}[a_s(Q^2)] = \\ = C_A^{\rm NS}[a_s(Q^2)] \times C_{Bjp}^{\rm NS}[a_s(Q^2)] = 1 + \Delta_{csb}^{\rm NS}[a_s(Q^2)]. \tag{26}$$

The second product in Eq. (26) was identified in [14] with the product of the functions, which appeared after application to this triangle amplitude of OPE in the limit $|q^2| \gg |p^2| \to \infty$. Note, that during more rigorous coordinate-space studies, performed in [8], it was mentioned, that the expression for $\Delta_{csb}^{V,\rm GLS}$ in Eq. (20) should have the same all-order structure as Eq. (22), but with unfixed from theory coefficients $K_i^{\overline{MS}}$. This statement was obtained after application of the OPE-formalism to the triangle Green function of Eq. (14), in the kinematic regime $y \ll x,\ y, x \to 0$.

In spite of the careful assumption of Ref. [15], that the analogy of the expression of Eq. (22) in Eq. (20) may contain additional singlet-type contribution to $K_3^{\overline{MS}}$, namely $K_3 = K_3^{\overline{MS}} + K_3^{\rm SI}$, we expected that the changes of limits in applications of OPE to the same Green function will not lead to modification of the concrete expression of the $\overline{MS}$-scheme coefficients from Eq. (22) for the generalized Crewther relation of Eq. (20).

Using this expectation and the results for $c_{4,2}^{\rm SI}$ and $c_{4,3}^{\rm SI}$ obtained in Ref. [15], we get the prediction for two remaining still unknown contributions to $d_4^{\rm SI}$ in Eq. (23). Taking into account the obtained in Ref. [15] prediction of Eq. (25), which is based on the concept of the conformal symmetry, we get the following prediction for the a_s^4-coefficient in the expression for the SI-part of Adler function, namely:

$$d_4^{\rm SI} = d^{abc}d^{abc}\Big[C_F\left(-\frac{13}{64} - \frac{\zeta_3}{4} + \frac{5\zeta_5}{8}\right) + \\ + C_A\left(\frac{481}{1152} - \frac{971}{1152}\zeta_3 + \frac{295}{576}\zeta_5 - \frac{11}{32}\zeta_3^2\right) + \\ + T_F N_F\left(-\frac{119}{1152} + \frac{67}{288}\zeta_3 - \frac{35}{144}\zeta_5 + \frac{1}{8}\zeta_3^2\right)\Big]. \tag{27}$$

It is necessary to stress once more, that our prediction is based on the expectations that Crewther relations in the NS and vector quark channels may have the same structure. Both follow from the same axial vector–vector–vector triangle Green function after change of limits of applications of OPE approach to the same amplitude, which depends from two conjugated sets of variables (either (x, y) or (p, q)).

Therefore, the direct analytical evaluation of $d_4^{\rm SI}$-coefficient is very important for getting better understanding of the theoretical status of Crewther relation in different channels and for the clarification of the fundamental theoretical features of applications of both OPE and renormalization-group formalism for the triangle amplitudes with two variables.

After the results were presented and discussed at ACAT2011 Workshop (Brunel Univ., London, UK, 5–9 September) and this work was submitted for publication, I became aware that the direct diagram-by-diagram calculations of $d_4^{\rm SI}$-term were completed and presented at RADCOR2011 Workshop [27]. The results of these calculations confirmed the predicted in Eq. (27) coefficients of the ζ_3^2-terms, which enter into $d_{4,2}^{\rm SI}$- and $d_{4,3}^{\rm SI}$-terms of Eq. (23). It should be stressed, that the analytical expressions for these contributions result from from the similar expressions for $c_{4,2}^{\rm SI}$ and $c_{4,3}^{\rm SI}$, obtained in [15], and from the original Crewther relation, obtained from Eq. (20) in the conformal invariant limit after nullification of $\Delta_{csb}^{V,{\rm GLS}}[a_s(Q^2)]$-term. This is the second example, when the proportional to ζ_3 transcendental term enters into the respecting conformal symmetry parts of order a_s^4 contributions into the Adler D_A^V-function (the first similar ζ_3 contribution was discovered by direct analytical calculations in [1]). This fact demonstrates once more, that on the contrary to previous belief (see e.g. Ref. [28]) the proportional to ζ_3-terms can appear in respecting conformal symmetry contributions to perturbative series in realistic gauge models like QED and QCD and is not the accident (for complementary discussions see [29]). Note also, that the difference between other analytical contributions into the result of Ref. [27] and the ones, which enter into Eq. (27), are nullified after application of the proposed in Ref. [2] test, based on the application of Banks–Zaks anzatz $T_F N_F = (11/4)C_A$ [30]. It comes from the special condition $\beta_0(N_F) = 0$ for the first coefficient of the QCD β-function. Thus, the Baikov–Chetyrkin–Kuhn assumption, that the generalized Crewther relation in the channel of vector currents may receive additional singlet contribution [15], which in this order of perturbation theory is proportional to the first coefficient of the QCD β-function and has the form $\beta_0 K_3^{\rm SI} a_s^4$, is correct.

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1. P. A. Baikov, K. G. Chetyrkin, and J. H. Kuhn, Phys. Rev. Lett. **104**, 132004 (2010); arXiv:1001.3606 [hep-ph].
2. A. L. Kataev and S. V. Mikhailov, arXiv:1011.5248 [hep-ph], to be published in Teor. Mat. Fiz. **170** (2012).
3. M. Davier, S. Descotes-Genon, A. Hocker et al., Eur. Phys. J. C **56**, 305 (2008); arXiv:0803.0979 [hep-ph].
4. P. A. Baikov, K. G. Chetyrkin, and J. H. Kuhn, Phys. Rev. Lett. **101**, 012002 (2008); arXiv:0801.1821 [hep-ph].
5. V. L. Khandramai, R. S. Pasechnik, D. V. Shirkov et al., arXiv:1106.6352 [hep-ph].
6. D. J. Broadhurst and A. L. Kataev, Phys. Lett. B **315**, 179 (1993); arXiv:hep-ph/9308274.
7. R. J. Crewther, Phys. Rev. Lett. **28**, 1421 (1972).
8. R. J. Crewther, Phys. Lett. B **397**, 137 (1997); arXiv:hep-ph/9701321.
9. V. M. Braun, G. P. Korchemsky, and D. Mueller, Prog. Part. Nucl. Phys. **51**, 311 (2003); arXiv:hep-ph/0306057.
10. S. G. Gorishny, A. L. Kataev, and S. A. Larin, Phys. Lett. B **259**, 144 (1991).
11. L. R. Surguladze and M. A. Samuel, Phys. Rev. Lett. **66**, 560 (1991); Erratum-ibid. **66**, 2416 (1991).
12. K. G. Chetyrkin, Phys. Lett. B **391**, 402 (1997); arXiv:hep-ph/9608480.
13. S. L. Adler, C. G. Callan, D. J. Gross, and R. Jackiw, Phys. Rev. D **6**, 2982 (1972).
14. A. L. Kataev, *The generalized Crewther relation: The peculiar aspects of the analytical perturbative QCD calculations*, Preprint INR-0926-96 in *Proceedings of the 2nd Workshop on Continuous Advances in QCD*, Minneapolis, MN, 28–31 March, 1996, World Scientific, 107 (ed. M. I. Polikarpov); arXiv:hep-ph/9607426.
15. P. A. Baikov, K. G. Chetyrkin, and J. H. Kuhn, Nucl. Phys. Proc. Suppl. **205–206**, 237 (2010); arXiv:1007.0478 [hep-ph].
16. S. A. Larin and J. A. M. Vermaseren, Phys. Lett. B **259**, 345 (1991).
17. T. van Ritbergen, A. N. Schellekens, and J. A. M. Vermaseren, Int. J. Mod. Phys. A **14**, 41 (1999); arXiv:hep-ph/9802376.
18. S. G. Gorishny and S. A. Larin, Phys. Lett. B **172**, 109 (1986).
19. G. 't Hooft and M. J. G. Veltman, Nucl. Phys. B **44**, 189 (1972).
20. I. Antoniadis, Phys. Lett. B **84**, 223 (1979).
21. T. L. Trueman, Phys. Lett. B **88**, 331 (1979).
22. E. J. Schreier, Phys. Rev. D **3**, 980 (1971).
23. J. S. Bell and R. Jackiw, Nuovo Cim. A **60**, 47 (1969).
24. S. L. Adler, Phys. Rev. **177**, 2426 (1969).
25. G. T. Gabadadze and A. L. Kataev, JETP Lett. **61**, 448 (1995) [Pisma Zh. Eksp. Teor. Fiz. **61**, 439 (1995)]; arXiv:hep-ph/9502384.
26. A. L. Kataev, Phys. Lett. B **668**, 350 (2008); arXiv:0808.3121 [hep-ph].
27. P. A. Baikov, K. G. Chetyrkin, J. H. Kuhn, and J. Rittenger, work presented by K. G. Chetyrkin at 10th International Symposuim RADCOR2011 on Radiative Corrections, *Applications to Quantum Field Theory to Phenomenology*, Mamallapuram, India, September 29, 2011.
28. C. M. Bender, R. W. Keener, and R. E. Zippel, Phys. Rev. D **15**, 1572 (1977).
29. A. L. Kataev, Phys. Lett. B **691**, 82 (2010); arXiv:1005.2058 [hep-th].
30. T. Banks and A. Zaks, Nucl. Phys. B **196**, 189 (1982).