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Restriction on the neutron-antineutron oscillations from the SNO-data on the deuteron stability[1)]

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Restriction on the neutron-antineutron oscillation time in vacuum is obtained from latest SNO-data on the deuteron stability, $\tau_D > 3.01 \cdot 10^{31}$ years. Calculation performed within the quantum field theory based diagram technique reproduces satisfactorily results of the potential approach previously developed. The dependence of the obtained restriction on the total spin structure of the annihilating $N\bar{N}$-system and the deuteron wave function modifications is discussed.

1. Searches for the baryon number violating processes predicted by Grand Unified Theories (GUT) remain to be an actual experimental task during many years. The neutron-antineutron transition induced by the baryon number violating interaction ($\Delta B = 2$) predicted within some variants of GUT has been discussed in many papers since 1970 [1], see [2–7]. Experimental results of searches for such transition are available, in vacuum (reactor experiments [8], and references therein), in nucleus ^{16}O [9] and in Fe nucleus [10], in neutron magnetic trap [11], see [3, 12].

Here we derive a restriction on the neutron-antineutron transition time in vacuum using new data on the deuteron stability obtained in Sudbury Neutrino Observatory (SNO) [13] and our former result on the suppression of $n{-}\bar{n}$-transition in deuterium [14]. Restrictions on the $n{-}\bar{n}$ transition time in vacuum which follow from data on nuclei stability have been obtained previously in a number of papers by different methods, beginning with the potential approach of [4–6], see [15–21].

To introduce notations, let us consider first the $n{-}\bar{n}$-transition in vacuum which is described by the baryon number violating interaction (see, e.g. [2, 16, 18]) $V = \mu_{n\bar{n}}\sigma_1/2$, σ_1 being the Pauli matrix; $\mu_{n\bar{n}}$ is the parameter which has the dimension of mass, to be predicted by grand unified theories and to be defined experimentally[3)]. As usually, a point-like $n - \bar{n}$ coupling is assumed here. The $n{-}\bar{n}$-state is described by the 2-component spinor Ψ, lower component being the starting neutron, the upper one – the appearing antineutron. The evolution equation is

$$i\frac{d\Psi}{dt} = (V_0 + V)\Psi, \tag{1}$$

where the diagonal matrix V_0 has matrix elements $V_0^{11} = V_0^{22} = m_N - i\gamma_n/2$ in the rest frame of the neutron (m_N is the neutron (antineutron) mass, γ_n – the (anti)neutron normal weak interaction decay width, and we take $\gamma_{\bar{n}} = \gamma_n$, as it follows from CPT-invariance of strong and weak interactions). Eq. (1) has solution

$$\Psi(t) = \exp\left[-i\left(\mu_{n\bar{n}} t\, \sigma_1/2 + V_0 t\right)\right]\Psi_0 =$$

$$= \left(\cos\frac{\mu_{n\bar{n}}t}{2} - i\sigma_1 \sin\frac{\mu_{n\bar{n}}t}{2}\right)\exp(-iV_0 t)\Psi_0. \tag{2}$$

Here Ψ_0 is the starting wave function, e.g. for the neutron in the initial state $\Psi_0 = (0, 1)^{\mathrm{T}}$. In this case we have for the wave functions at arbitrary time t

$$\begin{aligned}\Psi(\bar{n}, t) &= -i\sin\frac{\mu_{n\bar{n}}t}{2}\exp(-iV_0 t),\\ \Psi(n, t) &= \cos\frac{\mu_{n\bar{n}}t}{2}\exp(-iV_0 t),\end{aligned} \tag{3}$$

which describe oscillation $n{-}\bar{n}$: in vacuum the neutron goes over into antineutron, also the discrete localized in space state, which can go over again to the neutron, so the oscillation neutron to antineutron and back takes place indeed.

Since the parameter $\mu_{n\bar{n}}$ is small, the expansion of sin and cos can be made in Eq. (3) at not too large times. In this case the average (over the time $t^{\mathrm{obs}} \ll 1/\mu_{n\bar{n}}$) change of the probability of appearance of antineutron in vacuum is (for the sake of brevity we do not take into

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[3)]There is relation $\mu_{n\bar{n}} = 2\delta m$ with the parameter δm introduced in [2]. The neutron-antineutron oscillation time in vacuum is $\tau_{n\bar{n}} = 1/\delta m = 2/\mu_{n\bar{n}}$, see also [21] and references in this paper.

account the (anti)neutron natural instability which has obvious consequences) is

$$W(\bar{n};t^{\rm obs})/t^{\rm obs} = |\Psi(\bar{n},t^{\rm obs})|^2/t^{\rm obs} \simeq \frac{\mu_{n\bar{n}}^2 t^{\rm obs}}{4}, \quad (4)$$

which has, obviously, dimension of the width Γ. From existing data obtained with free neutrons from reactor the oscillation time is greater than $0.86\cdot 10^8\,{\rm s} \simeq 2.7$ years [8], therefore,

$$\mu_{n\bar{n}} < 1.5\cdot 10^{-23}\ {\rm eV}. \quad (5)$$

2. Recalculation of the quantity $\mu_{n\bar{n}}$ or $\tau_{n\bar{n}}$ from existing data on nuclei stability [9, 10, 13] is somewhat model dependent, and different authors obtained somewhat different results, within about one order of magnitude, see e.g. discussion in [18, 20, 21]. In the case of nuclei the $n{-}\bar{n}$-line with the transition amplitude $\mu_{n\bar{n}}$ is the element of any amplitude describing the nucleus decay $A \to (A-2) +$ mesons, where $A-2$ denotes a nucleus or some system of baryons with baryonic number $A-2$. The decay probability is therefore proportional to $\mu_{n\bar{n}}^2$, and we can write by dimension arguments

$$\Gamma(A \to (A-2)\ +\ {\rm mesons}) \sim \frac{\mu_{n\bar{n}}^2}{m_0}, \quad (6)$$

where m_0 is some energy (mass) scale.

It was argued in [14] that m_0 is of the order of normal hadronic or nuclear scale, $m_0 \sim m_{\rm hadr} \sim (10{-}100)$ MeV. The dimensionless suppression factor is therefore

$$F_S = \frac{\mu_{n\bar{n}}}{m_0} < 10^{-30}. \quad (7)$$

We can obtain the same result from the above vacuum formula (4), if we take the observation time $t^{\rm obs} \sim \sim 1/m_{\rm hadr}$.

The physical reason of such suppression is quite transparent and has been discussed in the literature long ago (see e.g. [2, 16, 19]): it is the localization of the neutron inside the nucleus, whereas no localization takes place in the vacuum case.

3. The case of the deuteron, which is most simple and instructive, can be treated using the standard diagram technique. Such technique or its modifications have been used in [17, 18] and [20] [4)].

The point is that in this case there is no final state containing antineutron – it could be only the $p{-}\bar{n}$-state, by the charge conservation. But this state is forbidden by energy conservation, since the deuteron mass is smaller than the sum of masses of the proton and antineutron. Therefore, if the $n{-}\bar{n}$-transition took place within the deuteron, the final state could be only some amount of mesons. The amplitude of the process is described by the diagrams of the type shown in Figure and is equal to

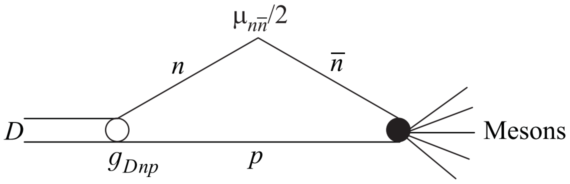
$$T(D \to {\rm mesons}) = 2ig_{Dnp}m_N^2\mu_{n\bar{n}}\times$$

$$\times\int \frac{T(\bar{n}p \to {\rm mesons})}{(p^2-m_N^2)[(d-p)^2-m_N^2]^2}\frac{d^4p}{(2\pi)^4}. \quad (8)$$

Here p and d are the 4-momenta of the virtual proton and deuteron. The constant g_{Dnp} is normalized by the condition

$$\frac{g_{Dnp}^2}{16\pi} = \frac{\kappa}{m_N} = \sqrt{\frac{\epsilon_d}{m_N}} \simeq 0.049, \quad (9)$$

which follows, e.g. from the deuteron charge formfactor normalization $F_D^{ch}(t=0)=1$, see discussion and references in [14]; $\kappa = \sqrt{m_N\epsilon_D} \simeq 46$ Mev, $\epsilon_D \simeq 2.23$ MeV being the binding energy of the deuteron. In the non-



The Feynman diagram describing $n{-}\bar{n}$-oscillation in the deuteron with subsequent annihilation of antineutron and proton to mesons

relativistic reduction of Feynman diagram (Figure) we should write for the vertex $D \to np$ $2m_Ng_{Dnp}$ and $m_N\mu_{n\bar{n}}$ for the $n \to \bar{n}$ transition amplitude, to ensure the correct dimension of the whole amplitude (here we correct some inaccuracy of our former consideration in [14]).

Presence of the second order pole in the energy variable of intermediate nucleon is a characteristic feature of diagrams describing the $n{-}\bar{n}$-transition in nuclei, as discussed in [14]. This does not lead, however, to any dramatic consequences, and the integration over internal 4-momentum d^4p in (8) can be made easily taking into account the nearest singularities in the energy $p_0 = E$, in the nonrelativistic approximation for nucleons. The integral over d^3p converges at small $p \sim \kappa$ which corresponds to large distances between nucleons in the deuteron, $r \sim 1/\kappa$. By this reason the annihilation amplitude can be taken out of the integration on

[4)] Later the author of [17] tried to develope a field theory motivated approch to this problem which was criticized by scientific community [15, 16, 20, 14].

the mass shell in some average point, and we obtain the approximate equality [14]

$$T(D \to \text{mesons}) = 2g_{Dnp} m_N^2 \mu_{n\bar{n}} I_{\text{DNN}} T(\bar{n}p \to \text{mesons}), \tag{10}$$

with the integral

$$I_{\text{DNN}} = \frac{i}{(2\pi)^4} \int \frac{d^4p}{(p^2 - m_N^2)[(d-p)^2 - m_N^2]^2} \simeq$$

$$\simeq \frac{i}{(2\pi)^4 (2m)^3} \times$$

$$\int \frac{d^4p}{\left[p_0 - m_N - \frac{\mathbf{p}^2}{(2m_N)} + i\delta\right]\left[m_D - m_N - p_0 - \frac{\mathbf{p}^2}{(2m_N)} - i\delta\right]^2} =$$

$$= \int \frac{d^3p}{(2\pi)^3 \cdot 8m_N[\kappa^2 + \mathbf{p}^2]^2} = \frac{1}{64\pi m_N \kappa}. \tag{11}$$

There is close connection between the amplitude of the $n{-}\bar{n}$-transition in deuteron and the deuteron charge formfactor at zero momentum transfer, which contains the integral I_{DNN} as well [14].

Using the standard technique, we obtain for the decay width (probability):

$$\Gamma(D \to \text{mesons}) \simeq \mu_{n\bar{n}}^2 g_{Dnp}^2 I_{\text{DNN}}^2 m_N^3 \times$$

$$\times \int |T(\bar{n}p \to \text{mesons})|^2 d\Phi(\text{mesons}), \tag{12}$$

$\Phi(\text{mesons})$ is the final states phase space. Our final result for the width of the deuteron decay into mesons is

$$\Gamma_{D\to\text{mesons}} \simeq \frac{\mu_{n\bar{n}}^2}{64\pi\kappa} m_N^2 \left[v_0 \sigma^{\text{ann}}(\bar{n}p)\right]_{v_0\to 0} \simeq$$

$$\simeq \frac{\mu_{n\bar{n}}^2}{32\pi\kappa} m_N \left[p_{\text{c.m.}} \sigma_{\bar{n}p}^{\text{ann}}\right]_{p_{\text{c.m.}}\to 0}, \tag{13}$$

where $p_{\text{c.m.}}$ is the (anti)nucleon momentum in the center of mass system. This result is close to that obtained in [17] and in [18] where it was obtained using the induced $\bar{n}p$ wave function.[5)].

The annihilation cross section of the antineutron with velocity v_0 on the proton at rest equals

$$\sigma(\bar{n}p \to \text{mesons}) =$$

$$= \frac{1}{4m_N^2 v_0} \int |T(\bar{n}p \to \text{mesons})|^2 d\Phi(\text{mesons}). \tag{14}$$

[5)]The result of [18] given by Eq. (17) can be rewritten in our notations as

$$\Gamma_{D\to\text{mesons}} \simeq 0.01 \mu_{n\bar{n}}^2 \frac{m_N^2}{\kappa} \left[v_0 \sigma^{\text{ann}} \bar{n}p\right]_{v_0\to 0}, \tag{17'}$$

which is abot twice greater than our result.

According to PDG at small v_0, roughly, $\left[v_0 \bar{\sigma}_{\bar{n}p}^{\text{ann}}\right]_{v_0\to 0} \simeq$ $\simeq$ (50–55)mb $\simeq$ (130–140) GeV^{-2}. Taking this into account, we obtain from Eq. (13) for the deuteron life time

$$\tau_D = \frac{4}{\mu_{n\bar{n}}^2} \frac{8\pi\kappa}{m_N \left[p_{c.m.} \sigma_{\bar{n}p}^{\text{ann},S=1}\right]_{p_{\text{c.m.}}\to 0}} =$$

$$= \tau_{n\bar{n}}^2 \frac{16\pi\kappa}{m_N^2 \left[v_0 \sigma_{\bar{n}p}^{\text{ann},S=1}\right]_{v_0\to 0}}, \tag{15}$$

v_0 is the antiproton velocity in the laboratory frame where proton is at rest.

If we define the suppression factor (dimensional) R_D as usually,

$$\tau_D = \tau_{n\bar{n}}^2 R_D, \tag{16}$$

with $\tau_{n\bar{n}} = 2/\mu_{n\bar{n}}$, then we have from Eq. (15) that

$$R_D = \frac{16\pi\kappa}{m_N^2 \left[v_0 \bar{\sigma}_{\bar{n}p}^{\text{ann},S=1}\right]_{v_0\to 0}}. \tag{17)}$$

Quite naturally suppression increases with increasing binding energy of the deuteron or κ. Numerically we have $R_D \simeq 2.94 \cdot 10^{22}\,\text{s}^{-1}$ if we neglect the influence of the spin structure of annihilation cross section. This is in agreement with pioneer papers [6], where R_D was found to be in the interval between $2.4 \cdot 10^{22}$ and $2.75 \cdot 10^{22}\,\text{s}^{-1}$ for different variants of the potential model.

To get an idea what happens for heavier nuclei, we can use the formula (17) with greater value of ϵ, about ~ 8 MeV, the binding energy per one nucleon in heavy nucleus. This leads to the value $R_A \sim 5.5 \cdot 10^{22} \cdot \text{s}^{-1}$, in qualitative agreement with [6]. We obtain same estimate if we replace the deuteron size $r_D = 1/\sqrt{\epsilon m_N}$ by average internucleon distance in heavy nucleus, $r_{\text{inter}-N} \sim$ $\sim 2\,Fm$.

4. This result can be refined taking into account spin dependence of the $N\bar{N}$ annihilation amplitudes and going beyond the zero range approximation for the deuteron wave function.

Within the deuteron neutron and proton are in triplet state, and after $n \to \bar{n}$-transition antineutron and proton remain in triplet state, therefore, only part of the total annihilation cross section corresponding to triplet state works in our case.

We can write for the annihilation cross section averaged over spin states

$$\bar{\sigma}_{\bar{n}p}^{\text{ann}} = \frac{1}{4}(3\sigma_{\bar{n}p}^{\text{ann},S=1} + \sigma_{\bar{n}p}^{\text{ann},S=0}) = \frac{3r+1}{4r} \sigma_{\bar{n}p}^{\text{ann},S=1}, \tag{18}$$

where $\sigma_{\bar{n}p}^{\mathrm{ann},S=1}$ and $\sigma_{\bar{n}p}^{\mathrm{ann},S=0}$ are the triplet and singlet annihilation cross sections, $r = \sigma_{\bar{n}p}^{\mathrm{ann},S=1}/\sigma_{\bar{n}p}^{\mathrm{ann},S=0}$, or $\sigma_{\bar{n}p}^{\mathrm{ann},S=1} = 4r\bar{\sigma}_{\bar{n}p}^{\mathrm{ann}}/(3r+1)$.

From these results and definitions we obtain the final formula

$$\tau_{n\bar{n}} = \left[\tau_D \frac{r\, m_N^2 \left[v_0\sigma_{\bar{n}p}^{\mathrm{ann}}\right]_{v_0\to 0}}{4\pi\kappa(3r+1)}\right]^{1/2}. \qquad (19)$$

The supression factor is

$$R_D = \frac{4(3r+1)\pi\kappa}{r\, m_N^2 \left[v_0\bar{\sigma}_{\bar{n}p}^{\mathrm{ann}}\right]_{v_0\to 0}}. \qquad (20)$$

Numerically $R_D \simeq 2.94\cdot 10^{22}(3r+1)/4r$ s^{-1}, and from new SNO-data [13] $\tau_D > 3.01\cdot 10^{31}$ years, we obtain

$$\tau_{n\bar{n}} > 1.8\cdot 10^8\,[4r/(3r+1)]^{1/2}\ \mathrm{s}. \qquad (21)$$

At $r = 1$ we recover our former result for the spinless case, if $r \gg 1$

$$\tau_{n\bar{n}} > 2.1\cdot 10^8\ \mathrm{s}. \qquad (22)$$

In the paper [22] it was obtained from the combined analysis of the $\bar{p}p$ atom data and results of OBELIX scattering experiments at LEAR that for the antiproton-proton interactions $r_{\bar{p}p} \simeq 0.42$. The $\bar{p}p$-state is a mixture of isoscalar and isovector, whereas $\bar{n}p$ is pure isovector, but if we take the same ratio for the $\bar{n}p$-interaction, we obtain from the SNO-data [13] that $\tau_{n\bar{n}} > 1.55\cdot 10^8$ s.

5. To go beyond the zero range approximation for the deuteron wave function we can use e.g. the Hulthen deuteron wave function in the form

$$\Psi_{D,H}(\mathbf{r}) = \frac{1}{\sqrt{2\pi}}\frac{\sqrt{\alpha\kappa(\alpha+\kappa)}}{\alpha-\kappa}\frac{[\exp(-\kappa r)-\exp(-\alpha r)]}{r}, \qquad (23)$$

where $\alpha \simeq 270$ Mev is the Hulthen parameter [23]. In the limit $\alpha \to \infty$ we obtain the zero range deuteron wave function $\Psi_D = \sqrt{\kappa/2\pi}\exp(-\kappa r)/r$.

The additional formfactor appears in the integrand of the amplitude in Eq. (11) when we are using the Hulthen wave function. Instead of the integral

$$\int \frac{d^3p}{(\kappa^2+\mathbf{p}^2)^2} = \frac{\pi^2}{\kappa}$$

we obtain now

$$\sqrt{\alpha(\alpha+\kappa)^3}\int \frac{d^3p}{[\kappa^2+\mathbf{p}^2]^2\,(\alpha^2+\mathbf{p}^2)} = \frac{\pi^2}{\kappa}\sqrt{\frac{\alpha}{\alpha+\kappa}}. \qquad (24)$$

So, using more realistic deuteron wave function leads to additional factor $\sqrt{\alpha/(\alpha+\kappa)} \simeq 0.92$ which does not change substantionally our results. This result means also that possible modifications of the deuteron wave function at small internucleon distancies will not change our results considerably.

To conclude, the results presented here are in satisfactory agreement with previously obtained results of the potential approach by Dover, Gal and Richard [6], and also in crude agreement with [18, 17]. Our method opens a way to study possible relativistic effects and corrections. Restriction on the $n-\bar{n}$-transition time in vacuum obtained from the SNO-data [13] is comparable with that from data on heavier nuclei stability. The best restriction which comes from data on oxygen stability, is $\tau_{n\bar{n}} > 3.3\cdot 10^8$ s [21].

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