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Form factors in the Bullough–Dodd related models: The Ising model in a magnetic field

O. V. Alekseev[1)]

Landau Institute for Theoretical Physics of the RAS, 117940 Moscow, Russia

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We consider particular modification of the free-field representation of the form factors in the Bullough–Dodd model. The two-particles minimal form factors are excluded from the construction. As a consequence, we obtain convenient representation for the multi-particle form factors, establish recurrence relations between them and study their properties. The proposed construction is used to obtain the free-field representation of the lightest particles form factors in the $\Phi_{1,2}$ perturbed minimal models. As a significant example we consider the Ising model in a magnetic field. We check that the results obtained in the framework of the proposed free-field representation are in agreement with the corresponding results obtained by solving the bootstrap equations.

1. Introduction. As it is known two-dimensional statistical models in their critical points are described by the so called minimal models of the conformal field theory [1]. Away from critical points the scaling region can be described by relevant perturbations of the fixed point action. The corresponding models can be referred to as the perturbed minimal models. The off-shell behavior of these models can be studied in the framework of the form factor approach [2]. The non-integrable field theories can be treated as a particular perturbations of the integrable ones [3] and multi-particle form factors are significant in the study of these models. A particular example is provided by the Ising model whose off-critical behavior can be considered by using the two different integrable models.

The mentioned relation between different integrable models provides an effective method of the form factors calculation in the perturbed minimal models. In this paper we consider a very particular class of the form factors which can be obtained from the form factors of the Bullough–Dodd model [4–6]. The convenient method of the multi-particle form factor calculation is provided in the framework of the free-field representation [7]. The free-field representation for the Bullough–Dodd model was proposed in [8, 9]. However in some cases particular modification of the free-field representation is more convenient. In this representation the two-particle minimal form- factors are excluded from the construction [10, 11]. This representation possesses simple analytical properties and can be used to obtain convenient free-field representation for the lightest particles form factors in the $\Phi_{1,2}$ perturbed minimal models.

The S-matrix of the lightest particles in these models [12] correspond the S-matrix of the Bullough–Dodd model being analytically continued to the imaginary values of the coupling constant. This allows us to propose the free-field representation for the lightest particles form factors in the $\Phi_{1,2}$ perturbed minimal models. As a significant example we consider the Ising model in a magnetic field (IMMF). This model is known to be related with the E_8 algebra [13]. It is a challenging problem to obtain the free-field representations associated with this algebra directly. The remarkable connection between IMMF and the Bullough–Dodd model was established in [8, 9]. We present the free-field representation for the lightest particle of the IMMF-explicitly.

2. The Bullough–Dodd model. Let briefly describe a scattering theory of the Bullough–Dodd model [4]. This model is a two-dimensional integrable quantum field theory defined by the Euclidean action

$$S_{BD} = \int d^2x \left[\frac{1}{16\pi}(\partial_\nu\varphi)^2 + \mu(e^{\sqrt{2}b\varphi} + 2e^{-\frac{b}{\sqrt{2}}\varphi}) \right], \tag{1}$$

where b is the coupling constant and μ is the regularized mass parameter which is related to the mass m of the single bosonic particle A in the spectrum of the model. Let us introduce the convenient notation

$$Q = b^{-1} + b. \tag{2}$$

The integrability of the model implies that the n-particle S-matrix factorizes into the $n(n-1)/2$ two-particle scattering amplitudes,

[1)]e-mail: alekseev@itp.ac.ru

$$S(\theta)=\frac{\tanh\frac{1}{2}(\theta+\frac{2i\pi}{3})}{\tanh\frac{1}{2}(\theta-\frac{2i\pi}{3})}\times$$
$$\times\frac{\tanh\frac{1}{2}(\theta-\frac{2i\pi}{3bQ})\tanh\frac{1}{2}(\theta-\frac{2i\pi b}{3Q})}{\tanh\frac{1}{2}(\theta+\frac{2i\pi}{3Qb})\tanh\frac{1}{2}(\theta+\frac{2i\pi b}{3Q})}, \qquad (3)$$

where θ is a rapidity. For real values of the coupling constant b the S-matrix has a simple pole at $\theta=2i\pi/3$ corresponding to the bound state represented by the particle A itself in the scattering processes. The space of local operators of the model consist of the exponential operators $V_a(x)=\exp[a\varphi(x)]$ and their descendants. We are interested in the form factors of the exponential operators only. Let us label them in the following,

$$F^a(\theta_1,\dots,\theta_N)\equiv\langle e^{a\varphi}\rangle f^a(\theta_1,\dots,\theta_N)=$$
$$=\langle\mathrm{vac}|V_a(x)|\theta_1,\dots,\theta_N\rangle, \qquad (4)$$

where θ-independent factor $\langle e^{a\varphi}\rangle$ is the vacuum expectation value of the exponential operator in the Bullough–Dodd model [14] and $f^a(\theta_1,\dots,\theta_N)$ is non-normalized form factor. One can show that the non-normalized form factors possess the following parametrization,

$$f^a(\theta_1,\dots,\theta_n)=$$
$$=\rho^N J_{N,a}(x_1,\dots,x_N)\prod_{1\le i<j\le N}R(\theta_i-\theta_j). \qquad (5)$$

Here, $J_{N,a}(x_1,\dots,x_N)$ is a symmetric rational function in the variables $x_i=e^{\theta_i}$ with proper kinematical and dynamical poles. The function $R(\theta_i-\theta_j)$ represents the so called minimal two-particle form factor [5, 6] and ρ is defined in [9].

Let us show that the function $J_{N,a}(x_1,\dots,x_N)$ admits the free-field representation. We follow the guideline of Lukyanov's construction of the free-field representation [9]. Let us specify the differences. We consider the Heisenberg algebra generated by a countable set of the elements a_n^s, $n\in\mathbb{Z}$, $n\neq 0$, and $s=\pm,0$, with the following commutation relations,

$$[a_n^s,a_m^{s'}]=-4n\sin\Big[\frac{(n-1)\pi b}{3Q}+$$
$$+\frac{\pi}{6}-\frac{n\pi}{3}\Big]\sin\Big[\frac{(n-1)\pi b}{3Q}+\frac{\pi}{6}\Big]A_n^{s,s'}\delta_{n,-m}, \qquad (6)$$

and the matrix $A^{\sigma,\sigma'}$ is given by

$$A_n^{s,s'}=\begin{pmatrix}0 & \omega^n(1+\omega^n) & \omega^n\\ \omega^{-n}(1+\omega^{-n}) & 0 & \omega^{-n}\\ \omega^{-n} & \omega^n & 1\end{pmatrix}, \qquad (7)$$

where $\omega=\exp(i\pi/3)$, the raw $s=+,-,0$, and the column $s'=+,-,0$. Let us define the exponential operators

$$\lambda^s(z)=:\exp\sum_{i\neq 0}\frac{a_n^s}{n}z^n:, \quad s=\pm,0. \qquad (8)$$

Here the normal ordering is not needed for the operators $\lambda^{\pm}(z)$ since the corresponding generators commute. Following [9] we can determine the proper Lukyanov's generators by using the operators (8) and identify the exponential fields with the projectors on the corresponding Fock modules. It is convenient to formulate the result as a following prescription. The function $J_{N,a}(x_1,\dots,x_N)$ can be represented as a matrix element,

$$J_{N,a}(x_1,\dots,x_N)=\langle t(x_1)\dots t(x_N)\rangle, \qquad (9)$$

where

$$t(z)=\gamma\lambda^+(z)+\gamma^{-1}\lambda^-(z)+h\lambda^0(z)\ . \qquad (10)$$

Here we introduce the functions γ and h defined by

$$h=2\sin\frac{\pi(b-b^{-1})}{6Q},$$
$$\gamma=-i\exp\Big[\frac{i\pi}{6Q}(4\sqrt{2}a-b+b^{-1})\Big]. \qquad (11)$$

In (9) we use the notation $\langle\dots\rangle=\langle 0|\dots|0\rangle$ for the corresponding matrix elements and the vacuums are defined by the relations

$$a_n^s|0\rangle=0,\quad \langle 0|a_{-n}^s=0,\quad s=\pm,0,\quad \text{for } n>0. \quad (12)$$

The calculation of the matrix elements multi-particle matrix elements is performed using Wick's averaging procedure with the following rules,

$$\begin{aligned}\lambda^{\pm}(x)\lambda^{\pm}(x') &= :\lambda^{\pm}(x)\lambda^{\pm}(x'):,\\ \lambda^0(x)\lambda^0(x') &= f\Big(\frac{x}{x'}\Big):\lambda^0(x)\lambda^0(x'):,\\ \lambda^0(x')\lambda^{\pm}(x) &= \lambda^{\pm}(x)\lambda^0(x') =\\ &= f\Big(\frac{x}{x'}\omega^{\pm1}\Big):\lambda^{\pm}(x)\lambda^0(x'):,\\ \lambda^-(x')\lambda^+(x) &= \lambda^+(x)\lambda^-(x') =\\ &= f\Big(\frac{x}{x'}\omega\Big)f\Big(\frac{x}{x'}\omega^2\Big):\lambda^+(x)\lambda^-(x'):,\end{aligned} \qquad (13)$$

where we introduce the function $f(x)=(x+x^{-1}+h^2-2)/(x+x^{-1}-1)$.

The free-field representation (9) is very similar to Lukyanov's representation. Its main advantage is that redundant factors of the two-particles minimal form factors are excluded from the construction. Therefore, calculation of the matrix elements involve Wick's averaging represented by rational functions in the variables x_i. Notice that the averaging procedure in the N-particle matrix elements generates 3^N terms. The particular terms may possess poles at the points $x_i = x_j\omega^{\pm 2}$ corresponding to bound state poles, at $x_i = -x_j$ corresponding to kinematical poles, and redundant ones at $x_i = x_j\omega^{\pm 1}$. The presence of the redundant poles complicates calculation of the form factors. One can show that their contributions in the whole expression for the matrix elements vanish.

The simple analytic structure of the matrix elements allows us to obtain recurrence relations between them. Consider the function $J_{N+1,a}(z, x_1, \ldots, x_N)$ as an analytic function in the variable z depending on the parameters $x_1, \ldots, x_N$. One can separate the contribution of the poles from the regular part. Since the residues of this function can be evaluated explicitly the only unknown is the regular part. However, for the exponential operators the regular part of this function can be determined using the factorization property [15]. As a result we can formulate the following proposition.

Proposition 1. *The recurrence relations*

$$\begin{aligned} J_{N+1,a}(z, x_1, \ldots, x_N) &= J_{1,a}J_{N,a}(x_1, \ldots, x_N) + \\ &+ \sum_{n=1}^{N} \frac{x_n}{z+x_n} K_n J_{N-1,a}(\ldots, \not{x}_n, \ldots) + \\ &+ \sum_{n=1}^{N} \frac{x_n\omega^2}{z - x_n\omega^2} B_n^+ J_{N,a}(\ldots, x_n\omega, \ldots) - \\ &- \sum_{n=1}^{N} \frac{x_n\omega^{-2}}{z - x_n\omega^{-2}} B_n^- J_{N,a}(\ldots, x_n\omega^{-1}, \ldots) \end{aligned} \quad (14)$$

together with the initial condition

$$J_{1,a} = 4\sin\Big(\frac{\pi\sqrt{2}a}{3Q}\Big)\cos\Big[\frac{\pi}{6Q}(2\sqrt{2}a - b + b^{-1})\Big], \quad (15)$$

uniquely define the set of functions $J_{N,a}(x_1, \ldots, x_N)$ of exponential operators. The functions K_n and $B_n^{\pm}$ depend on the set $x_1, \ldots, x_N$ and are given by

$$\begin{aligned} K_n(x_1, \ldots, x_N) &= \frac{(h^2-3)(h^2-1)}{2(\omega - \omega^{-1})} \times \\ &\times \Big[\prod_{i\neq n} f\Big(\frac{x_n}{x_i}\omega^2\Big) f\Big(\frac{x_n}{x_i}\omega\Big) - \prod_{i\neq n} f\Big(\frac{x_n}{x_i}\omega^{-2}\Big) f\Big(\frac{x_n}{x_i}\omega^{-1}\Big)\Big], \\ B_n^{\pm}(x_1, \ldots, x_N) &= \frac{h(h^2-1)}{\omega - \omega^{-1}} \prod_{i\neq n} f\Big(\frac{x_n}{x_i}\omega^{\pm 1}\Big). \end{aligned} \quad (16)$$

As an example of possible applications of the recurrence relations (14) one can prove that the form factors of the exponential operators satisfy the quantum equation of motion and the reflection relations similar to those ones presented in [14].

From the representation (9) it follows that the functions $J_{N,a}(x_1, \ldots, x_N)$ are symmetric and $2\pi i$-periodic functions in the variables $x_i = e^{\theta_i}$. Consequently, these functions can be expressed in the convenient form by means of symmetric functions. A basis in the space of the symmetric functions in N variables is provided by the elementary symmetric polynomials $\sigma_N(x_1, \ldots, x_N)$. It is convenient to separate the poles contribution prescribed by the recurrence relation (14) into an overall factor. Then the functions $J_{N,a}(x_1, \ldots, x_N)$ can be parametrized as

$$\begin{aligned} J_{N,a}(x_1, \ldots, x_N) &= \Lambda_N(\sigma_1, \ldots, \sigma_N) \times \\ &\times \prod_{1\le i<j\le N} \frac{1}{(x_i + x_j)(x_i^2 + x_i x_j + x_j^2)}, \end{aligned} \quad (17)$$

where $\Lambda_N(\sigma_1, \ldots, \sigma_N)$ are symmetric polynomials in the variables $\sigma_i(x_1, \ldots, x_N)$. One can show that for spinless operators the total degree of this polynomial is equal to $3N(N-1)/2$ while the partial degree in the each variable σ_i is bounded by the the condition [16] and could not exceed $3(N-1)$, for example,

$$\Lambda_1 = J_{1,a},$$
$$\Lambda_2(\sigma_1, \sigma_2) = J_{1,a}^2\sigma_1^2 - J_{1,a}(J_{1,a} + h - h^3)\sigma_2.$$

Using this method also we computed polynomials Λ_3 and Λ_4. As a simple check of the proposed expressions let us consider form factors of the exponential operators which appear in the action (1). The general expression of the stress- energy tensors trace $\Theta(x)$ which is compatible with the quantum equation of motion is given by the linear combination of these operators [17]. Taking into account the stress-energy tensor conservation law one can show that the N-particle form factors of $\Theta(x)$ has to be proportional to the combination of symmetric polynomials $\sigma_1\sigma_{N-1}$ for $N > 2$. Using explicit expressions of the three- and the four- particle form factors we checked that the form factors of the corresponding exponential operators indeed possess this property.

3. The $\Phi_{1,2}$ perturbed minimal models. Consider an imaginary coupling affine Toda theory based on the twisted affine Kac-Moody algebra $A_2^{(2)}$, which is obtained from (1) by the substitutions $b \to i\beta$, $\mu \to -\mu$,

$$S_{cBD} = \int \Big[\frac{1}{16\pi}(\partial\varphi)^2 - \mu(e^{i\sqrt{2}\beta\varphi} - 2e^{-i\frac{\beta}{\sqrt{2}}\varphi})\Big]. \quad (18)$$

This model can be referred to as the Zhiber–Mikhailov-Shabat or the imaginary coupling Bullough–Dodd model. It is not unitary theory since its Hamiltonian is not hermitian. However in [18] it was shown that the model is reducible for certain values of its coupling constant. Let us briefly describe the restriction of the complex Bullough–Dodd model. By using the action (18) it is possible to determine non-local conserved charges that commute with the Hamiltonian. These charges generate the quantum affine algebra $U_q(A_2^{(2)})$ with $q = \exp(i\pi/2\beta^2)$ and fix the S-matrix of the model up to a scalar factor [18–20]. The restriction proceeds in the following way. The quantum algebra $U_q(A_2^{(2)})$ contains two subalgebras $U_q(sl_2)$ and $U_{q^4}(sl_2)$ and both of them can be used to obtain quantum group restrictions of (18). Consider the first possibility. Let the parameter $2\beta^2$ takes the following rational values $2\beta^2 = p/p'$, where p and p' are relatively primary integers such that $p' > p > 1$. In this case the Hilbert space of the imaginary coupling Bullough–Dodd model can be consistently truncated to the representations of $U_q(sl_2)$ [18, 20]. The restricted theory is known to coincide with the $\Phi_{1,2}$ perturbations of minimal models $\mathcal{M}_{p,p'}$ [18]. Let us briefly introduce conventional notation used in the conformal field theory,

$$c = 1 + 6Q_L^2, \quad Q_L = \frac{1}{\sqrt{2}b} + \sqrt{2}b, \tag{19}$$

and the substitution $b = i\beta$ is assumed. The primary operator content of the minimal model $\mathcal{M}_{p,p'}$ is given by a set of the degenerate fields $\Phi_{m,n}$ whose conformal dimensions can be parametrized in the following,

$$\Delta_{m,n} = a_{m,n}(Q_L - a_{m,n}), \tag{20}$$

where

$$a_{m,n} = -\frac{(n-1)}{2}\sqrt{2}i\beta - \frac{(m-1)}{2}\frac{1}{\sqrt{2}i\beta}. \tag{21}$$

.

The primary operators of the perturbed minimal models relates with the specific exponential fields of the imaginary coupling Bullough–Dodd model [19, 21]. Namely, the following exponential operators, $V_{a_{1,n}}(x) = \exp[a_{1,n}\varphi(x)]$ commute with particular generators of the $U_q(sl_2)$ and can be identified with the primary operators $\Phi_{1,n}$ of the perturbed minimal models.

The fundamental particle of the complex Bullough–Dodd model is a kink triplet. Under the action $U_q(sl_2)$ the fundamental kinks form either singlet bound states generating a series of breathers or triplet ones corresponding to higher kinks. After analytical continuation to the imaginary values of the coupling constant and quantum group restriction the S-matrix of the Bullough–Dodd model can be treated as the lightest particles S-matrix in the $\Phi_{1,2}$ perturbed minimal models [12]. It develops two additional poles in the "physical strip" corresponding to heavier particles. We propose an identification between the form factors of the Bullough-Dodd model and the lightest particle form factors of the corresponding minimal models.

Further, we restrict our attention to the non-normalized form factors only. In the free-field representation the correct normalization is fixed by the vacuum expectation value of the corresponding operator. However, perturbed minimal models possess several vacuums and calculation of their contribution to the vacuum expectation values is an open problem. As a result let us propose the following representation for the lightest particles form factors of the $\Phi_{1,n}$ primary operators in the $\Phi_{1,2}$ perturbed minimal models $\mathcal{M}_{p,p'}$,

$$f_{1\ldots1}^{a_{1,n}}(\theta_1,\ldots,\theta_N) = f^{a_{1,n}}(\theta_1,\ldots,\theta_N)\Big|_{2b^2=-p/p'}, \tag{22}$$

where that the form factor in the right hand side of this relation admits the free-field representation (9). Being analytically continued to the imaginary values of the coupling constant the minimal two-particle form factors develop additional poles. Therefore, the poles emerge in Lukyanov's representation due to Wick's averaging procedure. However, in the proposed free-field representation (9) these redundant factors are excluded from the construction. The averaging procedure generates functions with the same analytic structure as in the Bullough–Dodd model.

The minimal model $\mathcal{M}_{3,4}$ perturbed by the operator $\Phi_{1,2}$ describes the Ising model at critical temperature in non-zero magnetic field (IMMF). It is characterized by central charge $c = 1/2$ and contains two non-trivial primary operators $\Phi_{1,2}$ and $\Phi_{1,3}$ which can be identified with the energy density and spin density operators respectively. The spectrum of the model consist of eight different species of self-conjugated particles A_i, $i = 1,\ldots,8$.

Under analytical continuation to imaginary value of the coupling constant, $2b^2 = -3/4$, the S-matrix of the Bullough–Dodd becomes the S-matrix of the lightest particles in the IMMF. The additional poles that emerges in the "physical strip" after analytical continuation of the S-matrix correspond to heavier particles. As shown in [13] the full set of the two-particle amplitudes $S_{ab}(\theta)$, where $a, b = 1,\ldots,8$, can be reconstructed starting from the two-particle amplitude $S_{11}(\theta)$. Under analytical continuation of the Bullough–Dodd model and quantum group restriction not all integral of motion survive. Their spins are exactly the exponents of the Lie algebra E_8, repeated modulo 30. In addition the number of particles is exactly the rank of E_8.

The form factors of the IMMF can be calculated in the framework of the proposed free-field representation. One can show that in the parametrization (5) the lightest particle form factors in the IMMF can be represented in the following form,

$$f^{a_{1,n}}_{1\ldots 1}(\theta_1,\ldots,\theta_N) = \rho^N J_{N,a_{1,n}}(x_1,\ldots,x_N)\times \times \prod_{1\leq i<j\leq N}\frac{R_{11}(\theta_{ij})}{\mathcal{P}_{2/5}(\theta_{ij})\mathcal{P}_{1/15}(\theta_{ij})}, \qquad (23)$$

where we introduce the notation $R_{11}(\theta)$ for the minimal two-particle form factor of the lightest particles in the IMMF. The poles representing by the functions $\mathcal{P}_\alpha(\theta)$,

$$\mathcal{P}_\alpha(\theta) = \frac{\cos\pi\alpha - \cosh\theta}{2\cos^2(\pi\alpha/2)}, \qquad (24)$$

correspond to the A_2 and A_3 bound state poles.

Notice that the functions $J_{N,a_{1,n}}(x_1,\ldots,x_N)$ in (23) are given by the corresponding matrix elements (9). Therefore, one can obtain multi-particle form factors of the lightest particles either by using Wick's averaging procedure or by means of the proposed recurrence relations. We checked that both these methods up to four-particle form factors reproduces the results of [22], where the form factors were obtained by solving the bootstrap equations. Due to bootstrap structure of the model the form factors involving different species of particles can be obtained from (23) by means of the residue equation. Notice that for this model the correct normalization of the form factors of primary operators can be obtained by using the vacuum expectation values of the corresponding exponential operators in the Bullough–Dodd model [14]. As a result we show that the proposed construction of the free-field representation can be used for the multi-particle form factors computation in the IMMF.

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