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Spontaneously broken Lorentz invariance from the dynamics of a heavy sterile neutrino

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A relativistic theory for neutrino superluminality is presented (in principle, the same mechanism applies also to other fermions). The theory involves the standard-model particles and one additional heavy sterile neutrino with an energy-scale close to or above the electroweak one, all particles propagating in the usual $3+1$ spacetime dimensions. Lorentz violation results from spontaneous symmetry breaking in the sterile-neutrino sector. The theory tries, as far as possible, to be consistent with the existing experimental data from neutrino physics and to keep the number of assumptions minimal. There are clear experimental predictions which can be tested.

1. Introduction. The time-of-flight results of the MINOS [1] and OPERA [2, 3] long-baseline neutrino experiments are, for the moment, inconclusive as to the existence of a superluminal muon-neutrino velocity. At the same level of accuracy, the ICARUS Collaboration [4] reports finding a standard (luminal) value for the muon-neutrino velocity. Awaiting further experimental results, theory may benefit by performing a "fire drill", best made as realistic and as difficult as possible.

Let us assume, for the sake of argument, that there is indeed neutrino superluminality: $v/c-1>0$ with c the speed of light in vacuum. And take the superluminality of a 10 GeV neutrino to be at the 10^{-6} level, that is, relatively large but a factor 25 below the original OPERA claim [2] and compatible with the result of ICARUS [4].

The challenge, then, is to find a theoretical explanation that is both consistent and convincing. The qualification "consistent" refers to all other experimental facts established by elementary particle physics to date. The qualification "convincing" refers to the desire to introduce as few additional assumptions as possible [5].

Before presenting one possible theoretical explanation, let us give a list of experimental facts:

(i) The qualitative (low-statistics) ICARUS bound [4] on the time-of-flight muon-neutrino velocity at an energy of order 10 GeV: $|v_{\nu_\mu}(10\text{ GeV})-c|/c\lesssim 5\cdot 10^{-6}$. Our theoretical "fire drill" takes the fiducial value $[v_{\nu_\mu}(10\text{ GeV})-c]/c=10^{-6}$.

(ii) The supernova SN1987a bound on the time-of-flight electron-antineutrino velocity [6, 7]: $|v_{\overline{\nu}_e}(10\text{ MeV})-c|/c\lesssim 10^{-9}$. (Bunching may even suggest a tighter bound at the 10^{-13} level.).

(iii) The existence of coherent mass-difference neutrino oscillations, which requires nearly equal maximum velocity of the three known flavors ($f=e,\mu,\tau$) of neutrinos [8].

(iv) The absence of significant energy losses [9] from the vacuum-Cherenkov-type process $\nu_\mu\to\nu_\mu+Z^0\to\nu_\mu+e^-+e^+$ at tree-level, in particular, for the CERN–GranSasso (CNGS) neutrinos detected by OPERA and ICARUS.

(v) The negligible leakage [10] of Lorentz violation from the neutrino sector into the charged-lepton sector by quantum effects (e.g., loop corrections to the electron propagator).

There may be other experimental facts to explain, but the five listed above (with the relatively large fiducial value of point (i) at the 10^{-6} level) suffice for the moment as they are already difficult enough to understand.

Points (i) and (iv) are especially difficult to reconcile, that is, having superluminality but no Cherenkov-type energy losses. If we do not wish to introduce new light particles (such as light sterile neutrinos with or without extra spacetime dimensions [10–14]), the simplest approach may be to seek a *reduction* of the vacuum-Cherenkov rate, instead of the complete absence of the process. It has, indeed, been pointed out that a strong energy dependence of the neutrino velocity excess may lead to a significant reduction of the vacuum-Cherenkov rate [15]. The details of the calculation of Ref. [15] still need to be verified, but, as will be explained later on, it is also possible to understand the reduction factor analytically [16, 17]. It, therefore, appears to be attractive phenomenologically to consider an energy-dependent modification of the neutrino dispersion relation which leads to superluminality.

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The fundamental problem, however, is to give an explanation for the superluminal maximum velocity of these neutrinos. A relatively simple explanation relies on spontaneous breaking of Lorentz invariance [18, 19]. In the following, we give one possible realization of such a 4-dimensional superluminal-neutrino model. Different from the previous discussion in Ref. [18], special attention is paid to the issue of gauge invariance.

2. Theory. Let us start at the phenomenological level and then work our way down to the underlying theory. In terms of the 3-momentum norm $p \equiv |\mathbf{p}|$, the following model dispersion relations of the three neutrino mass eigenstates ($n = 1, 2, 3$) are considered:

$$\left[E_n(p)\right]^2 \sim c^2\,p^2 + \left(m_n\,c^2\right)^2 + c^{-4}\,(b^0)^8\,M^{-6}\,p^8\,, \quad (1)$$

with an identical eighth-order term for all three dispersion relations in terms of a dimensionless constant $b^0 \in \mathbb{R}$ and a mass scale M, both resulting from a fermion condensate to be discussed shortly. The single eighth-order term in (1) is only an example and is supposed to hold for sufficiently low values of E_n/Mc^2, as will be discussed further in Sec. 3.

Henceforth, we set $c = \hbar = 1$ and use the Minkowski metric,

$$g_{\alpha\beta}(x) = \eta_{\alpha\beta} \equiv \mathrm{diag}(1,\,-1,\,-1,\,-1)\,, \quad (2)$$

together with the standard notation ∂_α for the partial derivative $\partial/\partial x^\alpha$ (operating to the right, as usual). The theory without Lorentz violation is defined by the Lagrange density $\mathcal{L}_{\mathrm{SM}}$ of the standard model (SM) [20], to which are added three neutrino mass terms ($n = 1, 2, 3$),

$$\mathcal{L}_{m_n,\,\mathrm{kin}} = -m_n\,\overline{\psi}_n\,\psi_n + \overline{\psi}_{nR}\,i\gamma^\alpha\partial_\alpha\,\psi_{nR}, \quad (3)$$

in terms of neutrino Dirac fields $\psi_n(x)$. Also included in (3) are the kinetic terms for the noninteracting right-handed components, as these terms are absent in $\mathcal{L}_{\mathrm{SM}}$.

The modified dispersion relations (1) now come from additional Lorentz-violating (LV) Lagrange-density terms which take the form of standard mass terms $\mathcal{L}_{m_n}$ with derivative operators inserted between the Dirac spinors [18]. Specifically, the relevant higher-derivative term is

$$\mathcal{L}_{\nu-\mathrm{LV}} = -M^{-3}\,\sum_{n=1}^{3}\,\overline{\psi}_n\,[b^\alpha\,\partial_\alpha]^4\,\psi_n\,, \quad (4a)$$

where the background vector is assumed to be purely timelike,

$$(b^\alpha) = (b^0,\,0,\,0,\,0)\,. \quad (4b)$$

The dispersion relation (1) then results for small enough values of the mass and energy, $\max[m_n,\,E] \ll M$ for $|b^0| \sim 1$. Terms of the type (4a) may also provide for superluminality of fermionic particles other than neutrinos, but here the focus is on neutrinos.

Again following Ref. [18], the dimensionless background vector b^α arises as a fermion condensate,

$$b^\alpha = M^{-4}\,\eta^{\alpha\beta}\,\langle\overline{\chi}_S\,(-i\,\partial_\beta)\,\chi_S\rangle, \quad (5)$$

where $\chi_S(x)$ is the Dirac field of a heavy sterile neutrino with mass scale $M \gg \max(|m_1|,\,|m_2|,\,|m_3|)$. Dynamically, the condensate (5) may come from the following multi-fermion interaction term:

$$\mathcal{L}_{S,\,\mathrm{int}} = -\lambda\,M^4\,\left(X - B^2\right)^2, \quad (6a)$$

$$X \equiv M^{-8}\,\eta^{\alpha\beta}\left[\overline{\chi}_S\,(-i\,\partial_\alpha)\,\chi_S\right]\left[\overline{\chi}_S\,(-i\,\partial_\beta)\,\chi_S\right], \quad (6b)$$

with real coupling constants λ and B (alternatively, λ can be considered to be a Lagrange multiplier; cf. Ref. [19]). Suitable boundary conditions and small symmetry-breaking perturbations (later taken to zero, the standard procedure for the study of spontaneous symmetry breaking) pick out a condensate vector (5) which is purely timelike (4b) with time-component

$$b^0 = \pm\,B. \quad (7)$$

Remark that, with the chosen signature (2), the interaction term (6) is only able to select a nonvanishing timelike condensate vector, not a spacelike one.

An appropriate higher-derivative interaction term produces the effective momentum-dependent mass terms (4a) for the propagators of the light (active) neutrinos. Using the notation of Ref. [21] and correcting a typo in its Eq. (13.69), one possible gauge-invariant relativistic interaction term reads

$$\mathcal{L}_{\nu S,\,\mathrm{int}} = \left(M^{19}\,v/\sqrt{2}\right)^{-1} \sum_{f=e,\,\mu,\,\tau}\left\{\overline{L}_f\cdot\widetilde{\Phi}\;[\left(\overline{\chi}_S\,\partial^\alpha\chi_S\right)\partial_\alpha]^4\,\nu_{R,f} + \mathrm{h.c.}\right\}, \quad (8)$$

with the usual left-handed lepton isodoublets $L_f(x)$ and Higgs isodoublet $\Phi(x)$ of the standard model, together with three (ultra)heavy right-handed sterile neutrinos $\nu_{R,f}(x)$ (which may or may not be related to the right-handed projection of our previous field $\chi_S(x)$). The Higgs vacuum expectation value v is defined by $\langle\Phi^\dagger\cdot\Phi\rangle \equiv v^2/2$. With $\widetilde{\Phi} \equiv (i\tau_2)\Phi^*$ and $\overline{L}_f\cdot\langle\widetilde{\Phi}\rangle = \overline{\nu}_{L,f}\;\;v/\sqrt{2}$, expression (8) corresponds to a sum of identical Dirac-mass-type terms, $\sum_f[\overline{\nu}_{L,f}\,\mathcal{M}\,\nu_{R,f} + \overline{\nu}_{R,f}\,\mathcal{M}^\dagger\,\nu_{L,f}]$, which ultimately gives (4a) by use of (5).

All in all, the underlying relativistic theory of the model dispersion relations (1) has the following Lagrange density:

$$\mathcal{L} = \mathcal{L}_{\mathrm{LI}} + \mathcal{L}_{\mathrm{SSB}}, \tag{9a}$$

$$\mathcal{L}_{\mathrm{LI}} = \mathcal{L}_{\mathrm{SM}} + \sum_n \mathcal{L}'_{m_n,\,\mathrm{kin}} + \mathcal{L}_{M,\,\mathrm{kin}}\,, \tag{9b}$$

$$\mathcal{L}_{\mathrm{SSB}} = \mathcal{L}_{S,\,\mathrm{int}} + \mathcal{L}_{\nu S,\,\mathrm{int}}\,, \tag{9c}$$

$$\mathcal{L}_{M,\,\mathrm{kin}} = \overline{\chi}_S \left(i\,\gamma^\alpha \partial_\alpha - M\right)\chi_S\,, \tag{9d}$$

where (9d) is the standard kinetic Dirac term for the heavy sterile neutrino and the other parts of the standard Lorentz-invariant (LI) term (9b) have been discussed in the second paragraph of this section. The prime on the light-neutrino mass term in (9b) indicates that it is gauge invariant and written in terms of the interacting SM fields [21], having additional derivative terms for the noninteracting fields as discussed below (3).

The spontaneous symmetry breaking (SSB) term (9c) involves the interaction terms (6) and (8), which are manifestly gauge-invariant because the heavy-neutrino field χ_S is a gauge singlet (normal derivatives suffice). As these interaction terms are non-renormalizable, it can be expected that a different, more fundamental theory applies for center-of-mass scattering energies $\sqrt{s} \gtrsim M$.

This completes the presentation of one particular relativistic theory with spontaneous breaking of Lorentz invariance giving rise to superluminal neutrinos propagating in the usual $3+1$ spacetime dimensions.

3. Discussion. Return to the phenomenological level (1) of our relativistic theory (9). Then, the neutrino group velocity $v_{n,gr}$ has a relative superluminality proportional to $B^8\,(E_n)^6/M^6$, in terms of the heavy-neutrino scale M and the coupling constant B entering the heavy-neutrino self-interaction term (6) and giving the dispersion-relation parameter b^0 by (7). A hypothetical superluminality at the 10^{-6} level of a 10 GeV muon-neutrino implies a mass scale $M \sim 10^2$ GeV for $|b^0| = |B| \sim 1$.

Let us now go through the list of experimental facts from Sec. 1. The ICARUS bound from point (i) is obviously satisfied. The supernova bound from point (ii) is also satisfied because, according to (1), the modification of the group velocity $v_{n,gr}$ has a sixth-order energy dependence, so that a decrease of the energy from 10 GeV to 10 MeV reduces superluminality effects by an additional factor of 10^{-18}. (As to the actual energy dependence of $v_{n,gr}(E)$, it has been argued [10] that these functions must peak at $E \sim 10$ GeV or reach a plateau for $E \gtrsim 10$ GeV [22]. In our case, this may require further higher-derivative terms of the type (4a). Needless to say, the discussion of the present article is at the level of an "existence proof", leaving aside all questions of naturalness.) Point (iii) holds because the theory is constructed to give an identical eighth-order term for all three dispersion relations (1). The last two points, (iv) and (v), are more subtle.

The vacuum-Cherenkov-type process of point (iv) is not forbidden but has a significantly reduced rate [15]. In fact, a heuristic argument based on the concept of effective-mass-squares [8, 9, 16] gives an extra factor of order $\left(1/\sqrt{7}\right)^5 \approx 10^{-2}$ for the tree-level decay rate $\Gamma(\nu_\mu \to \nu_\mu + e^- + e^+) \propto (G_F)^2\,(m_{\mathrm{eff}})^5$ in our theory (9) compared to the rate in the theory with an identical Lorentz-violating p^2 term in the three neutrino dispersion relations. Incidentally, the Lorentz-violating vacuum-Cherenkov-type process is similar to the standard Cherenkov-radiation process but not identical, as discussed in Secs. III and IV of Ref. [17].

Point (v) regarding the Lorentz-violation leakage into the charged-lepton sector remains problematic [10]. It is all the more important to find a good theoretical explanation for point (v), as the experimental bounds of the maximum velocities of certain standard-model particles are extremely tight [24].

In addition, it remains for us to better understand the dynamic origin of the particular Lorentz-violating term (4a), because the required higher-derivative interaction terms were introduced in a more or less *ad hoc* fashion. See, e.g., Refs. [25–28] for further discussion on spontaneous breaking of Lorentz invariance.

Leaving these fundamental properties of the theory aside, it is evident that the superluminal-neutrino model (1) as it stands leads to direct predictions for experiment. With the CNGS setup, for example, a narrow symmetric pulse of nearly mono-energetic muon-neutrinos produced at CERN (cf. Sec. 9 of Ref. [2]) would give a nearly equal pulse shape for the final muon-neutrinos to be detected by OPERA and ICARUS. The final pulse profile would be not exactly identical to the initial one, as a percent or so of the muon-neutrinos would have lost energy due to vacuum-Cherenkov electron-positron-pair emission [15], which would have reduced the speed of these neutrinos somewhat according to (1). The superluminal-sterile-neutrino hypothesis [10–14] typically predicts a substantial broadening of the pulse profile.

Assuming neutrino superluminality to exist in the first place, detailed measurements of the final pulse profile may thus provide information about the underlying mechanism. The main point of this article, however, has been to show that it is possible to obtain spontaneously broken Lorentz invariance from a four-dimensional gauge-invariant relativistic theory if there exists a heavy sterile neutrino with appropriate higher-derivative self-interactions.

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1. P. Adamson et al. (MINOS Collaboration), Phys. Rev. D **76**, 072005 (2007); arXiv:0706.0437.
2. T. Adam et al. (OPERA Collaboration), arXiv:1109.4897v2.
3. A CERN press release from the OPERA Collaboration (dated February 23, 2012) states that two possible errors have been found and that new short-pulse measurements are scheduled for May, 2012.
4. M. Antonello et al. (ICARUS Collaboration), arXiv:1203.3433v3.
5. Many papers on superluminal neutrinos have appeared since September 23, 2011 (see http://inspirehep.net/ for all papers quoting OPERA's first preprint [2]), but, here, we only refer to those of direct relevance to our discussion.
6. K. Hirata et al. (KAMIOKANDE-II Collaboration), Phys. Rev. Lett. **58**, 1490 (1987); R.M. Bionta et al., Phys. Rev. Lett. **58**, 1494 (1987).
7. M.J. Longo, Phys. Rev. D **36**, 3276 (1987); L. Stodolsky, Phys. Lett. B **201**, 353 (1988).
8. S.R. Coleman and S.L. Glashow, Phys. Rev. D **59**, 116008 (1999); arXiv:hep-ph/9812418.
9. A.G. Cohen and S.L. Glashow, Phys. Rev. Lett. **107**, 181803 (2011); arXiv:1109.6562.
10. G.F. Giudice, S. Sibiryakov, and A. Strumia, Nucl. Phys. B **861**, 1 (2012); arXiv:1109.5682.
11. S. Hannestad and M.S. Sloth, arXiv:1109.6282.
12. A. Nicolaidis, arXiv:1109.6354.
13. W. Winter, Phys. Rev. D **85**, 017301 (2012); arXiv:1110.0424.
14. F.R. Klinkhamer, Phys. Rev. D **85**, 016011 (2012); arXiv:1110.2146.
15. S. Mohanty and S. Rao, arXiv:1111.2725v4.
16. C. Kaufhold and F.R. Klinkhamer, Nucl. Phys. B **734**, 1 (2006); arXiv:hep-th/0508074.
17. C. Kaufhold and F.R. Klinkhamer, Phys. Rev. D **76**, 025024 (2007); arXiv:0704.3255.
18. F.R. Klinkhamer and G.E. Volovik, JETP Lett. **94**, 673 (2011); arXiv:1109.6624.
19. S. Nojiri and S.D. Odintsov, Eur. Phys. J. C **71**, 1801 (2011); arXiv:1110.0889.
20. M. Veltman, *Diagrammatica – The Path to Feynman Rules*, Cambridge Univ. Press, Cambridge, England, 1994, App. E.
21. T.P. Cheng and L.F. Li, *Gauge Theory of Elementary Particle Physics*, Clarendon Press, Oxford, England, 1984.
22. See, in particular, Table II of Ref. [23] with experimental upper bounds on $|v_\nu - c|/c$ at the 10^{-4} level for neutrino energies up to 200 GeV.
23. G.R. Kalbfleisch, N. Baggett, E.C. Fowler, and J. Alspector, Phys. Rev. Lett. **43**, 1361 (1979).
24. B. Altschul, Astropart. Phys. **28**, 380 (2007); arXiv:hep-ph/0610324.
25. J.D. Bjorken, Annals Phys. **24**, 174 (1963).
26. A. Jenkins, Phys. Rev. D **69**, 105007 (2004); arXiv:hep-th/0311127.
27. V.A. Kostelecky, Phys. Rev. D **69**, 105009 (2004); arXiv:hep-th/0312310.
28. F.R. Klinkhamer, arXiv:1202.0531.