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Non-adiabatic electron charge pumping in coupled semiconductor quantum dots

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The possibility of non-adiabatic electron pumping in the system of three coupled quantum dots (QDs) attached to the leads is discussed. We have found out that periodical changing of energy level position in the middle QD results in non-zero mean tunneling current appeared due to non-adiabatic non-equilibrium processes. The same principle can be used for fabrication of a new class of semiconductor electronic devices based on non-stationary non-equilibrium currents. As an example we propose a nanometer quantum emitter with non-stationary inverse level occupation achieved by electron pumping.

1. Introduction. Electron pumping in nanoscale structures attracts much attention nowadays [1–6]. A great deal of the previous research works have been devoted to adiabatic electron pumping, the idea discussed by Thouless [7] rather long time ago. The first (to our knowledge) experiment on electron pumping in single electron device was described in [2]. Then experiments in this direction were continued in a three-junction geometry by Pothier et al. [8]. Two phase shifted rf signals were used to realize a single electron pump: a device with current $I = ef$ at zero bias voltage (f – frequency of the rf signal). Non-adiabatic charge pumping based on periodical variation of the potential barriers formed by the finger gates was also recently investigated in [1]. In these systems quantized current is connected with periodic changing of the QD population.

Proposed in a number of papers photo-assisted tunneling through coupled QDs [9–12] is also an example of an electron pump. Pumping effect is achieved by applying an oscillating signal to the gate electrode or by irradiating the structure by monochromatic [12] and pulsed [13] microwaves.

It was understood that for practical realization of quantized electron pump the phenomenon of Coulomb blockade is very important [14]. General approach to the pumping through the interacting QDs in this regime is based on supposition that the charge relates to instantaneous chemical potential of a dot [15]. Using Coulomb blockade ideas another class of non-adiabatic quantized pumping of electrons in hybrid normal metal-superconductor structures was proposed in [16, 17]. These systems are more like a "turnstile" rather than a "pump" because quantized current directly proportional to gate frequency appears at finite (nonzero) value of applied bias. But at low temperatures these systems are very promising as current standards [18].

Adiabatic charge pumping through three tunnel-coupled QDs attached to electron leads in the regime of strong Coulomb blockade was investigated in [19]. Slow variations of coupling strength between the dots lead to adiabatic changes of energy levels in the system. Time dependent charge redistribution caused by energy levels changes results in non-stationary adiabatic tunneling current.

In the present paper we suggest a new type of electron pumping also in a system with three QDs, but based on non-equilibrium non-stationary tunneling currents. The proposed device requires only a single ac gate signal. Mean current appears in our model due to non-adiabatic changing of electron level in a single QD.

2. Three dot model of electron pump. We investigate non-stationary currents which flow in a three dot system shown in Fig. 1. The left and right dots have

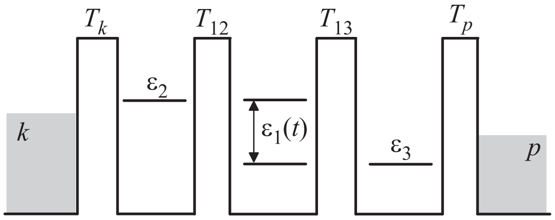

Fig. 1. Schematic diagram of the three coupled QDs with energy level position in the middle QD depending on the time

energy levels ε_2 and ε_3 constant in time. And the level position of the middle dot ε_1 is modulated by external gate voltage.

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QDs with energy levels ε_2 and ε_3 are also coupled to continuous spectrum states – massive leads. Hamiltonian of the system under investigation has the form:

$$\hat{H} = \sum_{i=1}^{3} \varepsilon_i c_i^+ c_i + \sum_k \varepsilon_k c_k^+ c_k + \\ + T_{12}(c_1^+ c_2 + c_2^+ c_1) + T_{13}(c_1^+ c_3 + c_3^+ c_1) + \\ + \sum_k T_k(c_k^+ c_2 + c_2^+ c_k) + \sum_p T_p(c_p^+ c_3 + c_3^+ c_p), \quad (1)$$

where T_{12}, T_{13} are tunneling transfer amplitudes between the QDs and amplitudes T_k and T_p correspond to the tunneling processes between the QDs and continuous spectrum states, c_i^+/c_i and $c_{k(p)}^+/c_{k(p)}$ – electrons creation/annihilation operators in the QDs localized states and in the continuous spectrum states correspondingly. Energy values satisfy the following ratios: $\varepsilon_2 > \varepsilon_{\mathrm{F}}$ and $\varepsilon_3 < \varepsilon_{\mathrm{F}}$. Pumping effect appears if gate voltage switches the level ε_1 between energies ε_2 and ε_3.

Though systems with varying parameters can be described by scattering matrix approach for non-stationary quantum transport [20] we apply more straightforward method based on Keldysh diagram technique. In order to develop a theory for non-stationary current let us first describe in details charge relaxation processes in the system if we assume that at the initial moment all charge density in the system is localized in the first (middle) QD and has the value $n_1(0)$. At the first step we need to calculate exact retarded Green functions of the system. In the absence of tunneling between the three QDs Green functions $G_{11}^R(t-t')$, $G_{22}^R(t-t')$, and $G_{33}^R(t-t')$ are equal to:

$$G_{11}^R(t-t') = -i\Theta(t-t')e^{-i\varepsilon_1(t-t')}, \\ G_{22}^R(t-t') = -i\Theta(t-t')e^{-i\varepsilon_2(t-t')-\gamma_2(t-t')}, \\ G_{33}^R(t-t') = -i\Theta(t-t')e^{-i\varepsilon_3(t-t')-\gamma_3(t-t')}, \quad (2)$$

where $\gamma_2 = \pi\nu_k^0 T_k^2$ and $\gamma_3 = \pi\nu_p^0 T_p^2$ are tunneling relaxation rates from leftmost and rightmost dots respectively to the leads. Exact retarded electron Green's function G_{11}^R in the first QD can be found from the integral equation:

$$G_{11}^R = G_{11}^{0R} + G_{11}^{0R} T_{12}^2 G_{22}^R G_{11}^R + G_{11}^{0R} T_{13}^2 G_{33}^R G_{11}^R. \quad (3)$$

Acting in turn by inverse operators $G_{11}^{0R-1}, G_{22}^{R-1}, G_{33}^{R-1}$ this integral equation can be also presented in the equivalent differential form (except for the point $t = t'$):

$$\left[\left(i\frac{\partial}{\partial t} - \varepsilon_3 + i\gamma_3\right)\left(i\frac{\partial}{\partial t} - \varepsilon_2 + i\gamma_2\right) \times \\ \times \left(i\frac{\partial}{\partial t} - \varepsilon_1\right) - T_{12}^2\left(i\frac{\partial}{\partial t} - \varepsilon_3 + i\gamma_3\right) - \\ - T_{13}^2\left(i\frac{\partial}{\partial t} - \varepsilon_2 + i\gamma_2\right)\right] G_{11}^R(t,t') = 0. \quad (4)$$

Consequently, retarded Green's function which determine spectrum re-normalization due to the tunneling between the QDs can be written in the following form:

$$G_{11}^R(t,t') = i\Theta(t-t')[A_1 e^{-iE_1(t-t')} + A_2 e^{-iE_2(t-t')} + \\ + A_3 e^{-iE_3(t-t')}]. \quad (5)$$

Where eigenfrequencies $E_{1,2,3}$ can be found from equation (see (4)):

$$(E-\varepsilon_1)(E-\varepsilon_2+i\gamma_2)(E-\varepsilon_3+i\gamma_3) - \\ - T_{12}^2(E-\varepsilon_3+i\gamma_3) - T_{13}^2(E-\varepsilon_2+i\gamma_2) = 0. \quad (6)$$

And coefficients A_i can be evaluated using integral equation for G_{11}^R:

$$A_1 = \frac{E_1(E_1+E_3-\tilde{\varepsilon}_2-\tilde{\varepsilon}_3) - E_1E_3 + \tilde{\varepsilon}_2\tilde{\varepsilon}_3}{(E_1-E_2)(E_1-E_3)}, \\ A_2 = \frac{E_2(E_2+E_3-\tilde{\varepsilon}_2-\tilde{\varepsilon}_3) - E_2E_3 + \tilde{\varepsilon}_2\tilde{\varepsilon}_3}{(E_2-E_3)(E_2-E_1)}, \\ A_3 = \frac{E_3(E_2+E_3-\tilde{\varepsilon}_2-\tilde{\varepsilon}_3) - E_2E_3 + \tilde{\varepsilon}_2\tilde{\varepsilon}_3}{(E_3-E_1)(E_3-E_2)}, \quad (7)$$

where $\tilde{\varepsilon}_i = \varepsilon_i - i\gamma_i$.

Further on we assume for simplicity that $T_{12} = T_{13} = T$. If $\varepsilon_1 = \varepsilon_2$ and $\varepsilon_1 - \varepsilon_3 \gg T, \gamma$ coefficients A_i has the form:

$$A_1 = A_1^0\left(1 + \frac{E_3-\tilde{\varepsilon}_3}{E_1-E_3}\right) = A_1^0\left[1 + \frac{T^2}{(\varepsilon_1-\varepsilon_3)^2}\right], \\ A_2 = A_2^0\left(1 + \frac{E_3-\tilde{\varepsilon}_3}{E_1-E_3}\right) = A_2^0\left[1 + \frac{T^2}{(\varepsilon_1-\varepsilon_3)^2}\right], \\ A_3 = -\frac{T^2}{(\varepsilon_1-\varepsilon_3)^2}, \quad (8)$$

where $A_1^0 = \frac{E_1-\tilde{\varepsilon}_2}{E_1-E_2}$, $A_2^0 = -\frac{E_2-\tilde{\varepsilon}_2}{E_1-E_2}$.

If we disconnect the third QD then coefficients A_1^0 and A_2^0 give an exact solution for a system of two coupled QDs for all energy values ε_1 and ε_2.

Time evolution of electron density in the middle dot is determined by the Keldysh Green function $G^<$ [21]:

$$G_{11}^<(t,t') = in_1(t). \quad (9)$$

Equations for the Green functions $G_{ii}^<$ has the form:

$$(G_{11}^{0-1} - T_{12}^2 G_{22}^R - T_{13}^2 G_{33}^R) G_{11}^< = \\ = T_{12}^2 G_{22}^< G_{11}^A + T_{13}^2 G_{33}^< G_{11}^A,$$

$$\left(G_{22}^{0-1} - T_{12}^2 G_{11}^R - \sum_k T_k^2 G_{kk}^R \right) G_{22}^< = \\ = T_{12}^2 G_{11}^< G_{22}^A + \sum_k T_k^2 G_{kk}^< G_{22}^A,$$

$$\left(G_{33}^{0-1} - T_{13}^2 G_{11}^R - \sum_p T_p^2 G_{pp}^R \right) G_{33}^< = \\ = T_{13}^2 G_{11}^< G_{33}^A + \sum_p T_p^2 G_{pp}^< G_{33}^A. \quad (10)$$

If $G_{22}^<(0,0) = in_F(\varepsilon_2) \simeq 0$, $G_{33}^<(0,0) = in_F(\varepsilon_3) \simeq 1$, and $G_{11}^<(0,0) = n_1(0)$ then Green function $G_{11}^<(t,t)$ is determined by the sum of homogeneous and inhomogeneous solutions. Inhomogeneous solution of the equation can be written in the following way:

$$G_{11}^<(t,t') = iT_{13}^2 \int_0^t dt_1 \int_0^{t'} dt_2 G_{11}^R(t-t_1) \times \\ \times n_F(\varepsilon_3) e^{-i\tilde{\varepsilon}_3(t_1-t_2)} G_{11}^A(t_2 - t'). \quad (11)$$

Homogeneous solution of the differential equation has the form:

$$-iG_{11}^<(t,t') = f_1(t') e^{-iE_1 t} + \\ + f_2(t') e^{-iE_2 t} + f_3(t') e^{-iE_3 t}. \quad (12)$$

Function $G^<(t,t')$ satisfies the symmetry relations:

$$[G_{11}^<(t,t')]^* = -G_{11}^<(t',t). \quad (13)$$

Then coefficients $f_i(t')$ can be written as:

$$f_1(t') = Ae^{iE_1^* t'} + Be^{iE_2^* t'} + Xe^{iE_3^* t'}, \\ f_2(t') = Ce^{iE_2^* t'} + B^* e^{iE_1^* t'} + De^{iE_3^* t'}, \\ f_3(t') = X^* e^{iE_1^* t'} + D^* e^{iE_2^* t'} + Ze^{iE_3^* t'}. \quad (14)$$

Since the solution has to satisfy homogeneous integro-differential equation, we are able to determine all coefficients. After some calculations we obtain that the following proportionality takes place:

$$f_2(t') = F_{21} f_1(t'), \\ f_3(t') = F_{31} f_1(t')$$

with coefficients F_{21}, F_{31}:

$$F_{21} = -\left\{(E_2 - \tilde{\varepsilon}_2)(E_2 - \tilde{\varepsilon}_3)[(E_1 - \tilde{\varepsilon}_2)(E_3 - \tilde{\varepsilon}_2) + \\ + (E_1 - \tilde{\varepsilon}_3)(\tilde{\varepsilon}_2 - E_3)]\right\} \times \\ \times \left\{(E_1 - \tilde{\varepsilon}_2)(E_1 - \tilde{\varepsilon}_3)[(E_3 - \tilde{\varepsilon}_3)(E_2 - \tilde{\varepsilon}_2) + \\ + (\tilde{\varepsilon}_2 - E_3)(E_2 - \tilde{\varepsilon}_2)]\right\}^{-1}, \\ F_{31} = \frac{(E_3 - \tilde{\varepsilon}_2)[E_2 - \tilde{\varepsilon}_2 + (E_1 - \tilde{\varepsilon}_2) F_{21}]}{(E_2 - \tilde{\varepsilon}_2)(E_1 - \tilde{\varepsilon}_2)}. \quad (15)$$

Now we can find all coefficients in (14):

$$A = \frac{n_1(0)}{1 + \left|F_{21} + F_{31}\right|^2 + 2\mathrm{Re}F_{21} + 2\mathrm{Re}F_{31}}, \\ B = F_{21}^* A, \quad C = |F_{21}|^2 A, \\ D = F_{31}^* F_{21} A, \quad Z = |F_{31}|^2 A, \quad X = F_{31}^* A. \quad (16)$$

Finally, time dependence of the filling number in the middle QD $n_1(t)$ can be written as:

$$n_1(t) = n_1^0 [Ae^{-i(E_1 - E_1^*)t} + Ce^{-i(E_2 - E_2^*)t} + \\ + Ze^{-i(E_3 - E_3^*)t}] + 2\mathrm{Re}[Be^{-i(E_1 - E_2^*)t}] + \\ + 2\mathrm{Re}[Xe^{-i(E_1 - E_3^*)t}] + 2\mathrm{Re}[De^{-i(E_2 - E_3^*)t}]. \quad (17)$$

We see that there are six typical time scales in the considered system, which are described by the expression (17). Three of them we can identify as three relaxation modes with rates $2\mathrm{Im}E_1$, $2\mathrm{Im}E_2$, and $2\mathrm{Im}E_3$. Three other time scales are determined by the expressions $\mathrm{Re}(E_1 - E_2^*)$, $\mathrm{Re}(E_1 - E_3^*)$, and $\mathrm{Re}(E_2 - E_3^*)$. These time scales are related with charge density oscillations between quantum dots, if the following ratio between T_{ij} and γ takes place: $T_{ij}/\gamma > 1/\sqrt{2}$.

If we neglect for a moment the tunneling from the middle dot to the right one, then for the system of two coupled QDs ($T_{13} = 0$) only three time scales appear and the equations (15)–(17) are transformed in the following way:

$$\frac{f_1(t')}{f_2(t')} = -\frac{\varepsilon_2 - E_1 - i\gamma}{\varepsilon_2 - E_2 - i\gamma}. \quad (18)$$

Time dependence of the filling numbers in the first QD $n_1(t)$ can be written as:

$$n_1(t) = n_1^0 \left\{ Ae^{-i(E_1 - E_1^*)t} + 2\mathrm{Re}[Be^{-i(E_1 - E_2^*)t}] + \\ + Ce^{-i(E_2 - E_2^*)t} \right\}, \quad (19)$$

where coefficients A, B, and C are determined as:

$$A = \frac{|E_2 - \varepsilon_1|^2}{|E_2 - E_1|^2}, \\ C = \frac{|E_1 - \varepsilon_1|^2}{|E_2 - E_1|^2}, \\ B = -\frac{(E_2 - \varepsilon_1)(E_1^* - \varepsilon_1)}{|E_2 - E_1|^2}, \quad (20)$$

and coefficients X, Z, and D are equal to zero.

In this situation we can distinguish two relaxation rates γ_{res} and γ_{nonres} which characterises charge relaxation through an intermediate QD in resonant and non-resonant cases:

$$\gamma_{\text{res}} = 2T^2/\gamma \quad \gamma_{\text{nonres}} = \gamma_{\text{res}}\gamma^2/(\varepsilon_1 - \varepsilon_2)^2. \tag{21}$$

As we consider $\varepsilon_1 - \varepsilon_3 \gg T, \gamma$ then $\gamma_{\text{res}} \gg \gamma_{\text{nonres}}$ and small parameter $\gamma_{\text{nonres}}/\gamma_{\text{res}}$ exists in the theory. This allows us to write the following approximate relations for the system of three coupled QDs in the case $\varepsilon_1 \simeq \varepsilon_2$ valid in the first order of the small parameter $\gamma^2/(\varepsilon_1 - \varepsilon_3)^2$

$$\begin{aligned}
E_1 - E_1^* &= -i\gamma_{\text{res}}\left[1 + \frac{\gamma^2}{(\varepsilon_1 - \varepsilon_3)^2}\right],\\
E_2 - E_2^* &= -2i\gamma\left[1 - \frac{T^2}{\gamma^2} + \frac{T^2}{\gamma^2}\frac{T^2}{(\varepsilon_1 - \varepsilon_3)^2}\right],\\
E_3 - E_3^* &= 2i\gamma\left[1 - \frac{T^2}{(\varepsilon_1 - \varepsilon_3)^2}\right], \qquad (22)\\
E_2 - E_3^* &= \varepsilon_1 - \varepsilon_3 - 2i\gamma\left[1 + \frac{T^2}{2\gamma^2} - \frac{T^2}{2(\varepsilon_1 - \varepsilon_3)^2}\right],\\
E_1 - E_3^* &= \varepsilon_1 - \varepsilon_3 - i\gamma_{\text{res}}\frac{\gamma^2}{(\varepsilon_1 - \varepsilon_3)^2} - \\
&- i\gamma\left[1 - \frac{T^2}{(\varepsilon_1 - \varepsilon_3)^2}\right],\\
E_1 - E_2^* &= i\gamma + \frac{2T^2}{\varepsilon_1 - \varepsilon_3} - \\
&- i\frac{T^2}{\gamma}\left[\frac{\gamma^2}{(\varepsilon_1 - \varepsilon_3)^2} - \frac{T^2}{(\varepsilon_1 - \varepsilon_3)^2}\right].
\end{aligned}$$

When $\varepsilon_1 = \varepsilon_2$ and $\varepsilon_1 - \varepsilon_3 \gg T, \gamma$ the exact equations (15) can be transformed in the following way

$$\begin{aligned}
F_{21} &\simeq -\frac{T^2}{\gamma^2}\left[1 + \frac{\gamma}{\varepsilon_1 - \varepsilon_3} + \frac{T^2}{\gamma^2} - \frac{T^2}{\gamma(\varepsilon_1 - \varepsilon_3)}\right],\\
F_{31} &\simeq \frac{T^2}{\gamma^2}\left[\frac{T^2}{(\varepsilon_1 - \varepsilon_3)^2} + i\frac{T^2}{(\varepsilon_1 - \varepsilon_3)^2}\right]. \qquad (23)
\end{aligned}$$

So, coefficients D, Z and X which are resposible for the "reverse" current to the right lead are much smaller than A, B, and C, which correspond to "direct" current to the left, due to the appearance of the parameter $\frac{T^2}{(\varepsilon_1 - \varepsilon_3)^2}$. For simplicity we omit the terms with coefficients D, Z, and X in equation (17) which determine time evolution of localized charge in the middle QD. For any concrete system the accuracy of this approximation can be easily estimated from the exact equations.

Pumping of electrons takes place if energy level $\varepsilon_1(t)$ is a function of time and changes periodically (Fig. 1). We shall describe the most favorable case with $T \ll \gamma$. For current calculation we consider the situation of periodically switching the position of level ε_1 by external gate:

$\varepsilon_1(t) = \varepsilon_3$ in the interval $0 < t < t_0$ it means resonant tunneling between energy levels ε_1 and ε_3,

$\varepsilon_1(t) = \varepsilon_2$ in the interval $t_0 < t < 2t_0$ – resonance between energy levels ε_1 and ε_2.

Time evolution of local electron density $n_1(t)$ in the central QD can be determined from equation (17) (Fig. 2).

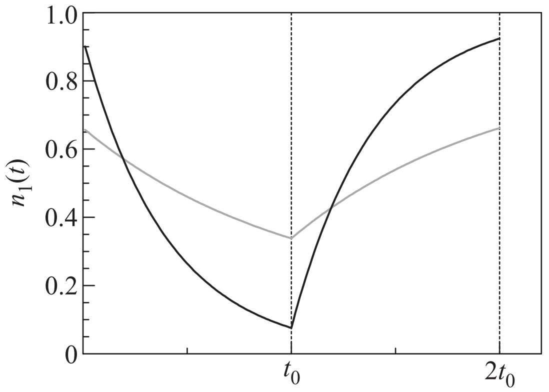

Fig. 2. Time evolution of local electron density $n_1(t)$ in the central QD in the system of three coupled QDs if energy level position in the middle QD depends on time. Black line corresponds to the case when frequency of energy level $\varepsilon_1(t)$ switching is lower than tunneling rates and grey line corresponds to the case when frequency is higher than tunneling rates

When $0 < t < t_0$

$$n_1(t) = n_1^0\left[\left(1 + \frac{\gamma_{\text{res}}}{\gamma}\right)e^{-\gamma_{\text{res}}t} - \frac{\gamma_{\text{res}}}{\gamma}e^{-\gamma t}\right] \tag{24}$$

and when $t_0 < t < 2t_0$

$$\begin{aligned}
n_1(t) = n_1^0&\Big[\Big(1 + \frac{\gamma_{\text{res}}}{\gamma}\Big)e^{-\gamma_{\text{res}}(t - t_0)} - \\
&- \frac{\gamma_{\text{res}}}{\gamma}(e^{-\gamma(t - t_0)}\Big] + \\
&+ \Big[1 - \Big(1 + \frac{\gamma_{\text{res}}}{\gamma}\Big)e^{-\gamma_{\text{res}}(t - t_0)} + \\
&+ \frac{\gamma_{\text{res}}}{\gamma}e^{-\gamma(t - t_0)}\Big]. \qquad (25)
\end{aligned}$$

During first half period ($0 < t < t_0$) localized state is emptying due to the tunneling to the left lead and for the second half period ($t_0 < t < 2t_0$) it becomes occupied due to the tunneling from the right lead. Taking into account periodicity condition $n_1(2t_0) = n_1^0$, one can find n_1^0:

$$n_1^0 = \frac{1}{1 + \left(1 + \frac{\gamma_{\rm res}}{\gamma}\right) e^{-\gamma_{\rm res} t_0} - \frac{\gamma_{\rm res}}{\gamma} e^{-\gamma t_0}}. \qquad (26)$$

Results for $n_1(t)$ are shown in Fig. 2. Situation when frequency $\Omega \equiv 1/2t_0$ of the level $\varepsilon_1(t)$ switching is higher than tunneling rates $\gamma_{\rm res}, \gamma$ is depicted by grey line. Black line corresponds to the case when frequency Ω is lower than $\gamma_{\rm res}, \gamma$. It is clear that with the increasing of frequency the value of $n_1(t)$ always tends to the value $1/2$ and is almost independent on time at high gate frequencies.

For low frequencies if $\gamma_{\rm res} t_0 \gg 1$ the value n_1^0 is almost equal to 1. The energy level ε_1 is filled up almost completely during the pumping cycle (for considered situation when energy level ε_2 is well above and energy level ε_3 is well below the Fermi energy). Non-stationary tunneling current through the system appears for zero applied bias. It can be calculated for any half of the period using the equation:

$$e \frac{\partial}{\partial t} n_1(t) = I(t). \qquad (27)$$

One can find for $0 < t < t_0$

$$I(t) = e n_1^0 \gamma_{\rm res} \left[\left(1 + \frac{\gamma_{\rm res}}{\gamma}\right) e^{-\gamma_{\rm res} t} - e^{-\gamma t} \right]. \qquad (28)$$

Mean tunneling current value can be found as:

$$\langle I \rangle = e \frac{1}{2t_0} \int_0^{t_0} I(t) dt = e \frac{1}{2t_0} n_1^0 \times$$
$$\times \left[1 - \left(1 + \frac{\gamma_{\rm res}}{\gamma}\right) e^{-\gamma_{\rm res} t_0} + \frac{\gamma_{\rm res}}{\gamma} e^{-\gamma t_0} \right]. \qquad (29)$$

If $\Omega \equiv 1/2t_0 \ll \gamma_{\rm res}$ tunneling current mean value can be written as $\langle I \rangle = e\Omega$ since $n_1^0 = 1$ for such frequencies. This is the regime, when the device operates like a current standart: the current is directly proportional to the gate frequency. This regime has exponential accuracy which is governed by the second and third terms in the square brackets in expression (29). Note, that even for very unsuitable case if $|\varepsilon_2 - \varepsilon_3| \simeq \gamma \simeq T$ the pumping effect still remains, and the current is proportional to the frequency, though it's value is suppressed compared to the ideal relation $\langle I \rangle = e\Omega$.

For high frequencies of the gate voltage in the region $\gamma \gg \Omega \equiv 1/2t_0 \gg \gamma_{\rm res}$ tunneling current average value is almost independent on the frequency and equal to: $\langle I \rangle = e\gamma_{\rm res}/4$ $(\gamma_{\rm res} = 2T^2/\gamma)$. With further frequency increase $(\Omega \gg \gamma \gg \gamma_{\rm res})$ mean current value decreases to $\langle I \rangle = e\gamma_{\rm res}^2/4\gamma$.

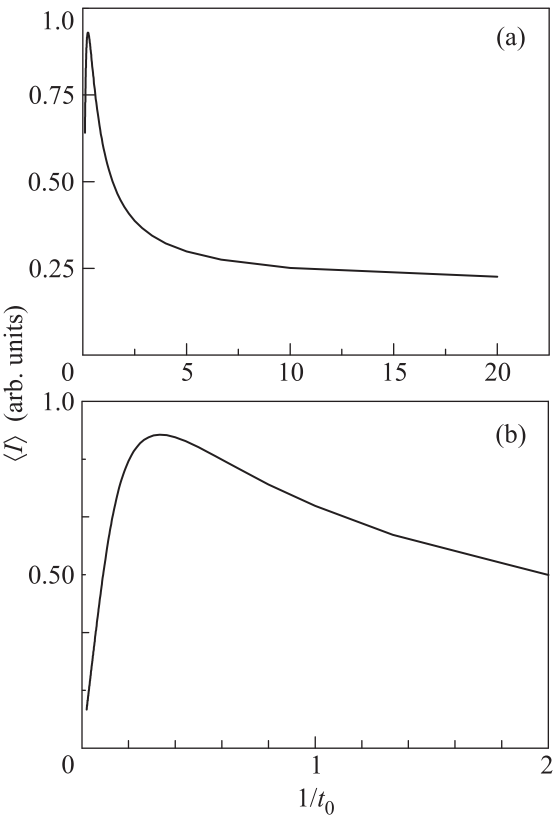

Fig. 3. Frequency dependence of the mean tunneling current for the system of three coupled QDs (gate frequency $\Omega = 1/2t_0$). Different frequency scales are presented

The non-stationary mean tunneling current value has non-monotonic dependence on the gate frequency with maximum at $\Omega \simeq \gamma_{\rm res}$ (Fig. 3). This effect can be used for frequency stabilization in nanoelectronics.

The parameters of devices based on QDs depend on the size of quantum dots, tunneling transfer rates, energy levels positions and distances between them. Estimation of tunneling parameters for achievable setup gives us characteristic frequencies and currents for such devices:

$$T \simeq 1\,\text{meV}, \quad \gamma \simeq (1\text{–}10)\,\text{meV} \Rightarrow$$
$$\Rightarrow \gamma_{\rm res} = 2T^2/\gamma \simeq (0.1\text{–}1)\,\text{meV} \simeq (10^{10}\text{–}10^{11})\,1/\text{s},$$

where parameters T and γ are determined by the widths and heights of the barriers. For such values of the tunneling rates we need to have QDs of tens of nanometers size, for which quantum size quantization energies are not less than the tunneling width of levels. Such devices could operate at gigahertz and subgigahertz frequencies at nano and subnanoampere currents ($1\,\text{nA} \simeq 6 \cdot 10^9\,e/\text{s}$).

The difference between tunneling rates in resonant and non resonant cases can be used also for creation of inverse occupation in quantum emitter based on the system of coupled QDs. An example of such device is shown in Fig. 4. In the system of two coupled QDs we

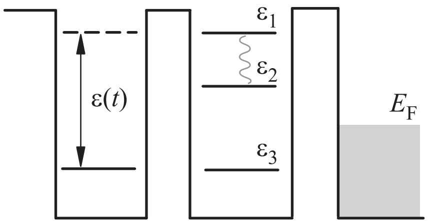

Fig. 4. Schematic diagram of two coupled QDs which operate as an emitter due to nonstationary inverse occupation of levels

switch the level ε by applying gate voltage to the second QD between two levels of the first QD directly coupled to the lead. If the third non-resonant level exists in the same QD between the lowest and the highest levels, then charge pumping from the initially filled state ε_3 to the highest level ε_1 creates inverse occupation of the states ε_2 and ε_1. So some nanometer quantum generators can be fabricated on this principle.

3. Conclusion. We investigated electron pumping ability of a system of three tunnel coupled QDs attached to the leads. Periodical changing of energy level position in the middle QD by gate voltage leads to nonzero tunneling current even if applied to the structure bias is equal to zero. Physical mechanism for the pumping is the strong difference of tunneling rates through intermediate QDs in resonant and non-resonant cases. This leads to periodical switching of the middle QD by fast resonant tunneling to the left and to the right leads in turn.

Our calculations of the mean current are based on accurate analysis of relaxation processes in QDs in non-adiabatic regime. Exact equations allows to investigate various regimes of the device and estimate the mean current value and accuracy of it's operation as a current standard. For very small dots with pronounced size effect a possibility of room temperature electron pumping is opened. We should like to stress that the ideas discussed in the paper can be used for fabrication of a new type of electronic devices based on non-equilibrium non-stationary tunneling currents. As an example we proposed in the paper nanometer quantum emitter based on two coupled QDs.

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1. M. D. Blumenthal, B. Kaestner, L. Li et al., Nature Physics **3**, 343 (2007).
2. L. G. Geerligs, V. F. Anderegg, P. A. M. Holweg et al., Phys. Rev. Lett. **64** , 2691 (1990).
3. V. E. Anderegg, L. G. Geerligs, J. E. Mood et al., Physica B **61**, 165 (1990).
4. M. W. Keller, J. M. Martins, N. M. Zimmerman, and A. H. Steinbach, Appl. Phys. Lett. **69**, 1804 (1996).
5. M. Switkes, C. M. Markus, K. Campman et al., Science **283**, 1905 (1999).
6. B. Kaestner, V. Kashcheyevs, S. Amakawa et al., Phys. Rev. B **77**, 153301 (2008).
7. D. J. Thouless, Phys. Rev. B **27**, 6083 (1983).
8. H. Pothier, Europhys. Lett. **17**, 249 (1992).
9. H. Stoof and Yu. Nazarov, Phys. Rev. B **53**, 1050 (1996).
10. M. Covington, M. W. Keller, R. L. Kautz, and J. M. Martins, Phys. Rev. Lett. **84**, 5192 (2000).
11. Ph. Brune, C. Bruder, and H. Schoeller, Phys. Rev. B **56**, 4730 (1997).
12. C. A. Stafford and N. Wingreen, Phys. Rev. Lett. **76**, 1916 (1996).
13. B. Hazelzet, M. Wagewijs, H. Stoof, and Yu. Nazarov, Phys. Rev. B **63**, 165313 (2001).
14. E. Cota, R. Aguadado, and G. Platero, Phys. Rev. Lett. **94**, 107202 (2005).
15. J. Splettstoesser, M. Governale, J. Konig, and R. Fazio, Phys. Rev. Lett. **95**, 246803 (2005).
16. J. P. Pekola, J. J. Vartiainen, M. Mopttopnen et al., Nature Phys. **4**, 120 (2008).
17. D. Averin and J. P. Pekola, Phys. Rev. Lett. **101**, 066801-1 (2008).
18. V. F. Maisi, Y. A. Pashkin, S. Kafanov et al, New Journal of Phys. **11**, 113057 (2009).
19. F. Rezoni and T. Brandes, Phys. Rev. Lett. **64**, 245301 (2001).
20. M. Moskalets and M. Buttiker, Phys. Rev. B **66**, 205320 (2002).
21. L. V. Keldysh, Sov. Phys. JETP **20** , 1018 (1964).