The cosmological constant: a lesson from the effective gravity of topological Weyl media

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Topological matter with Weyl points, such as superfluid ^{3}He-A, provide an explicit example where there is a direct connection between the properly determined vacuum energy and the cosmological constant of the effective gravity emerging in condensed matter. This is in contrast to the acoustic gravity emerging in Bose–Einstein condensates (S. Finazzi, S. Liberati and L. Sindoni, Phys. Rev. Lett. **108**, 071101 (2012); arXiv:1103.4841). The advantage of topological matter is that the relativistic fermions and gauge bosons emerging near the Weyl point obey the same effective metric and thus the effective gravity is more closely related to real gravity. We study this connection in the bi-metric gravity emerging in ^{3}He-A, and its relation to the graviton masses, by comparison with a fully relativistic bi-metric theory of gravity. This shows that the parameter λ, which in ^{3}He-A is the bi-metric generalization of the cosmological constant, coincides with the difference in the proper energy of the vacuum in two states (the nonequilibrium state without gravity and the equilibrium state in which gravity emerges) and is on the order of the characteristic Planck energy scale of the system. Although the cosmological constant λ is huge, the cosmological term $T_{\mu\nu}^\Lambda$ itself is naturally non-constant and vanishes in the equilibrium vacuum, as dictated by thermodynamics. This suggests that the equilibrium state of any system including the final state of the Universe is not gravitating.

I. Introduction. It is well-known that in Einstein's theory of gravitation the most natural candidate for the cosmological constant is the vacuum energy, since this would lead to a non-zero value of the vacuum energy-momentum tensor which should be reflected in the Einstein equation. In condensed matter analogs of gravity, typically only the first part of general relativity (GR) is reproduced: quasiparticles move in the effective metric produced by an inhomogeneity of the vacuum state. The second part of GR – the Einstein equation describing the dynamics of the metric field – as a rule is not reproduced. For example, the acoustic metric obeys hydrodynamic equations, which are certainly not diffeomorphism invariant. As a result, the analog of the cosmological constant may essentially differ from the vacuum energy, as was found for acoustic gravity in BEC [1] (see also Ref. [2]). We discuss here the connection between the vacuum energy and the cosmological constant in the effective gravity which emerges in Weyl topological matter on example of the axial p-wave superfluid ^{3}He-A.

There are many condensed-matter systems which exhibit an effective relativistic metric for scalar excitations such as sound or surface waves [3]. Superfluid ^{3}He-A, Weyl semimetals and graphene provide the effective gravity for fermionic quasiparticles. Here due to the topologically protected Fermi points in momentum

space (3+1-Weyl points in ^{3}He-A [4] and Weyl semimetals [5–9], and 2+1 Dirac points in graphene [10–13]) the emergence of effective gravity is accompanied by the appearance of Weyl and Dirac fermions and effective gauge fields intimately related to those of the Standard Model of particle physics, which all couple to the same effective metric near the topological point. This is a consequence of the topological theorem – the Atiyah–Bott–Shapiro construction – applied to systems with Weyl, Dirac or Majorana points [14]. The emergent gravity has recently also been used to study the anomalies in quantum field theories of topological insulators, such as the gravitational anomaly and thermal Hall effect [15]. Hořava quantum gravity with anisotropic scaling (see [16] and references therein) can be simulated in bilayer graphene, and the corresponding quantum electrodynamics with anisotropic scaling emerging in multilayer graphene has been recently discussed in [17, 18].

II. Thermodynamics and the vacuum equation of state. It is well-known that Einstein himself considered thermodynamics to be "the only physical theory of universal content" [19]. There are strong indications that this universal scope of thermodynamics indeed includes gravity. A crucial step was the observation that the Einstein equations can be interpreted as the thermodynamic equation of state of spacetime [20], while reports of recent work on the relation between gravity and thermodynamics include [21, 22]. This suggests that

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thermodynamics could well play a crucial role for the cosmological constant problem and its relation to the vacuum energy as well [23].

For a Lorentz invariant vacuum, $\rho_{\text{vac}} = -P_{\text{vac}}$ is the only possible equation of state as a perfect fluid, and so one can immediately see from the thermodynamic Gibbs–Duhem relation

$$P = -\epsilon + Ts + \sum_i \mu_i q_i, \quad q_i = Q_i/V \qquad (1)$$

(with s the specific entropy and the temperature $T = 0$ in the vacuum) that the relevant thermodynamic quantity, which plays the role of the vacuum energy is the analog of grand-canonical energy $\rho_{\text{vac}} = \epsilon(q_i) - \sum_i q_i d\epsilon/dq_i$. Any conserved quantity Q_i, which characterizes the quantum vacuum, should be explicitly taken into account together with its corresponding Lagrange multiplier μ_i. And indeed it is demonstrated that the quantity which enters the cosmological term in Einstein equations is the density of the grand-canonical energy, ρ_{vac}, rather than the energy density ϵ [22–24].

By reversing the above argument, one sees that the vacuum equation of state $\rho_{\text{vac}} = -P_{\text{vac}}$ is more generally valid [23]. The energy of the vacuum of quantum fields emerging in a many body condensed matter system is the grand canonical energy $\rho_{\text{vac}} = \epsilon(n) - nd\epsilon/dn$, where the particle density $N = nV$ is a conserved quantity and the corresponding Lagrange multiplier is the chemical potential $\mu = d\epsilon/dn$. The use of the grand canonical energy here corresponds to the fact that the many-body Hamiltonian in second quantization is $\hat{H}_{\text{QFT}} = \hat{H} - \mu\hat{N}$, where $\hat{H}$ is obtained from the Schrödinger many-body Hamiltonian and $\hat{N}$ is the number operator. The cosmological equation of state $w = P/\rho$ for the vacuum energy is then again $w = -1$ due to the Gibbs–Duhem relation, regardless of the Lorentz invariance or not of the vacuum.

Bi-metric gravity in systems with Weyl points. Gravity in superfluid ^{3}He-A is bi-metric, with $g_{\mu\nu}$ the effective dynamical gravitational field acting on the quasiparticles (the Weyl fermions) and $g_{\mu\nu}^{(0)}$ its value in the equilibrium vacuum state. The effective spacetime emerging for quasiparticles is anisotropic due to the uni-axial anisotropy of the broken symmetry superfluid state:

$$g^{(0)\mu\nu} = \text{diag}\left(-1, c_{\perp 0}^2, c_{\perp 0}^2, c_{\parallel}^2\right), \quad \sqrt{-g^{(0)}} = \frac{1}{c_{\perp 0}^2 c_{\parallel}}. \qquad (2)$$

Here, $c_{\perp 0}$ is the "speed of light" in the direction perpendicular to the anisotropy axis (the limiting speed for "relativistic" quasiparticles propagating in that direction),

and appears as the amplitude of the equilibrium order parameter in the symmetry breaking phase transition from the normal liquid ^{3}He to ^{3}He-A. $c_{\parallel}$ is the "speed of light" propagating along the anisotropy axis. One may say that in ^{3}He-A gravity appears as a result of the broken symmetry. Note that $c_{\parallel}$ is already present in the normal state of liquid ^{3}He. In the weak-coupling BCS theory one has $c_{\perp 0}/c_{\parallel} \ll 1$ with $c_{\perp 0}/c_{\parallel} \sim 10^{-3}$ in ^{3}He-A. Therefore $c_{\parallel}$ is completely fixed in the low-energy corner where the relativistic quasiparticles emerge. Also note that this anisotropy cannot be detected by internal observers who only dispose of such relativistic quasiparticles to construct rods and clocks with which to effect measurements [4, 27, 28].

We stress that the same effective metric is relevant both for the relativistic fermions emerging in the vicinity of the Weyl points and for the gauge bosons, which also emerge in the vicinity of the Weyl points. Thus the effective gravity in ^{3}He-A couples universally and interacts with all matter in the same way. This is ultimately a consequence of the Attiyah–Bott–Shapiro topological theorem [29]. This construction guarantees that the low-energy fermionic modes close to the manifold of zeroes in their energy spectrum (the generalized Fermi surface) exhibit relativistic invariance in the directions transverse to the surface [14]. Lorentz invariance in fermionic systems is therefore obtained in all directions in case the spectrum is characterised by a topologically protected point node: the Fermi point (Weyl, Dirac or Majorana point) [14, 30, 31]. The effective spacetime emerges as a consequence of the topological universality class, not of its specific realisation, and it is therefore independent of the concrete background metric. Thus, in spite of the (non-relativistic) background structure (the Helium atoms), the effective theory for the quasi-matter obeys both background independence (at least, at the kinematical level[2]) and the equivalence principle. Both are broken at the trans-Planckian level, as occurs in many scenarios for quantum gravity.

The grand canonical energy which governs the transition from normal liquid ^{3}He to superfluid ^{3}He-A represents the analogue of the Ginzburg–Landau energy, and can be obtained from the p-wave pairing model in BCS theory. Let us first consider the perturbations of the metric which preserve the structure of the equilibrium metric and correspond to a variation of $c_\perp$ only, i.e. we consider a non-equilibrium metric of the type

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[2] Evidently, as remarked above, the dynamical equations of the effective metric are not covariant. However, the energy-momentum tensor in the effective theory nevertheless satisfies a generalized conservation law which accounts for the interaction between vacuum and quasiparticles [4].

$g^{\mu\nu} = \mathrm{diag}\left(-1, c_\perp^2, c_\perp^2, c_\parallel^2\right)$. Then according to Ref. [4] one obtains that the grand canonical energy of ^{3}He-A at zero external pressure is:

$$\rho_{\mathrm{vac}} = \lambda\sqrt{-g^{(0)}}\left(\frac{\sqrt{-g^{(0)}}}{\sqrt{-g}}\ln\frac{\sqrt{-g^{(0)}}}{\sqrt{-g}} - \frac{\sqrt{-g^{(0)}}}{\sqrt{-g}} + 1\right),$$
$$(3)$$

where $\lambda = \Delta_0^4/(6\pi^2\hbar^3)$ with $\Delta_0 = p_{\mathrm{F}}c_{\perp 0}$ the amplitude of the angular dependent superfluid gap in anisotropic ^{3}He-A, and p_{F} the Fermi momentum. Δ_0 plays the analogue role of a Planck energy E_{Planck}, i.e. the "UV cutoff" for the superfluid phase in which the effective gravity emerges. Close to equilibrium this vacuum energy density is quadratic in deviations from equilibrium:

$$\rho_{\mathrm{vac}} \approx \frac{\lambda}{2\sqrt{-g^{(0)}}}\left(\sqrt{-g} - \sqrt{-g^{(0)}}\right)^2. \qquad (4)$$

This demonstrates that in equilibrium, when $g^{\mu\nu} = g^{(0)\mu\nu}$, the proper vacuum energy ρ_{vac} is nullified. This is a consequence of the fact that ^{3}He-A belongs to the class of self-sustained systems such as liquids and solids, which may exist without external environment, and thus at zero pressure. The self-sustained systems adjust themselves to external conditions, so that their grand canonical energy is automatically nullified in the absence of environment [24–26]. Indeed, $\rho_{\mathrm{vac}} = -P_{\mathrm{vac}}$, so for the equilibrium vacuum ($T = 0$ and no quasiparticles), one obtains $\rho_{\mathrm{vac}} = P_{\mathrm{ext}} = 0$. This nullification is obtained from the macroscopic laws of thermodynamics. At the microscopic level, it entails a dynamic self-adjustment of the degrees of freedom around the Planck scale. This phenomenon is so well known in condensed-matter physics that the nullification of $\rho_{\mathrm{vac}} = \epsilon - \mu n$ in self-sustained systems in equilibrium is generally used as a sanity check for numerical calculations of ϵ and μ from the microscopic degrees of freedom.

Since the effective gravity in ^{3}He-A does not obey the Einstein equations, there is a priori no reason to define a cosmological term $T_{\mu\nu}^\Lambda = (2/\sqrt{-g})\partial\rho_{\mathrm{vac}}/\partial g^{\mu\nu}$. But in any case, whether the action is generally covariant or not, note that any (generalized) cosmological term $T_{\mu\nu}^\Lambda \propto \partial\rho_{\mathrm{vac}}/\partial g^{\mu\nu}$ will nullify in equilibrium. This is a general phenomenon, applicable not only to self-sustained systems but also to non-self-sustained systems such as weakly interacting Bose–Einstein condensates discussed in [1], and ^{4}He and ^{3}He solids. The nullification of the cosmological term is a consequence of the thermodynamic stability of the system, which implies an extremum of the function $\rho_{\mathrm{vac}}(c_\perp)$, and of $\rho_{\mathrm{vac}}(g^{\mu\nu})$ in general. A non-zero but constant pressure, such as

the one necessary to maintain a gas in equilibrium, does therefore not affect the nullity of the cosmological term.

Note that the nullification of the cosmological term corresponds to the experimental fact that quasiparticles in ^{3}He-A near the zero-T limit and phonons in a homogeneous sample of BEC move in straight trajectories, like free particles in a Minkowski spacetime. They certainly do not see a de Sitter or anti de Sitter spacetime with a huge cosmological constant, as could naively be expected based on the $(E_{\mathrm{Planck}})^4$-dependence of the parameter λ.

IV. Cosmological constant and vacuum readjustment. Comparison of Eq. (4) with the standard form $\Lambda\sqrt{-g}$ does not allow us to unambiguously define the cosmological constant Λ. Nevertheless, a natural choice would be to use for the cosmological constant the parameter λ in Eq. (3), $\Lambda = \lambda$. Incidentally, as follows from Eq. (3), this is the energy density of the original non-superfluid or false vacuum state. Since $c_{\perp 0} \propto \Delta_0$, the false vacuum has $c_\perp = 0$, $1/\sqrt{-g} = 0$, and thus

$$\rho_{\mathrm{vac}}(\text{false vacuum}) = \lambda\sqrt{-g^{(0)}}. \qquad (5)$$

The false vacuum is an unstable equilibrium state corresponding to a local energy maximum in the $T \to 0$ limit, which existed before the phase transition to the (stable) equilibrium state (the true vacuum) with emerging gravity. Eq. (5) for the energy of the false vacuum is written assuming that the true vacuum exists in the absence of external environment, and thus $\rho_{\mathrm{vac}}(\text{true vacuum}) = 0$. In the general case the cosmological constant λ determines the difference in the energies of the false and true vacuum,

$$\lambda\sqrt{-g^{(0)}} = \rho_{\mathrm{vac}}(\text{false vacuum}) - \rho_{\mathrm{vac}}(\text{true vacuum}).$$
$$(6)$$

For example, if the false vacuum is originally in its quasi-equilibrium state in the absence of external environment, then its vacuum energy density is zero, $\rho_{\mathrm{vac}}(\text{false vacuum}) = 0$, while the energy of the true vacuum is negative, $\rho_{\mathrm{vac}}(\text{true vacuum}) = -\lambda\sqrt{-g^{(0)}}$. After the phase transition to the superfluid state, the microscopic degrees of freedom are readjusted to the new equilibrium state, such that the energy of the true vacuum is nullified, while the energy of the false vacuum becomes positive [23].

The identification of the parameter λ with the cosmological constant becomes more explicit through its relation with the graviton masses in ^{3}He-A. It is instructive to first recall some general results from relativistic bi-metric theories of gravity.

V. Bi-metric relativistic theory of gravity. Let us compare the bi-metric theory emerging in ^{3}He-A with

the so-called relativistic theory of gravity (RTG) discussed in Ref. [32].

In [32], the gravitational field is regarded as a physical field $\Phi^{\mu\nu}$ on top of a fundamental Minkowski background spacetime, which creates a curvature of the secondary, effective Riemannian spacetime. Regardless of the observational and conceptual issues of this theory[3], we are interested in the formal treatment of any free parameters in the theory which extend the concept of Einstein's cosmological constant. Due to the bi-metricity, one can in principle add two "cosmological constants" $\lambda_0 \sqrt{-g^{(0)}}$ and $\Lambda\sqrt{-g}$, as well as two metric-coupling terms $\frac{1}{2}m^2\sqrt{-g}g_{\mu\nu}^{(0)}g^{\mu\nu}$ and $\frac{1}{2}M^2\sqrt{-g^{(0)}}g_{\mu\nu}g^{\mu\nu(0)}$ to the overall action without affecting the interplay between matter and the gravitational field. The parameters λ_0, Λ, m and M are constrained by the conservation equation $\nabla_\mu^{(0)}T^{\mu\nu}=0$ with respect to the Minkowski background metric $g_{\mu\nu}^{(0)}$ and by the consistency of the Minkowski limit $L_{\mathrm{matter}}=0; \Phi^{\mu\nu}=0$. This turns out to give $M=0$ and $\Lambda=\lambda_0$ as well as $m^2=16\pi G_N\Lambda$. Furthermore, linearizing the global equations of motion gives $(\Box-m^2)\Phi_{\mu\nu}=0$, hence m corresponds to the graviton mass, which is equal for the usual spin $S=2$ graviton modes and the additional $S=0$ graviton[4], and is thus directly related to the cosmological constant Λ.[5]

The bi-metric generalization of the "vacuum energy" term in the Einstein equations can then be written in the following way:

$$\rho_\lambda = \Lambda\left(\frac{1}{2}\sqrt{-g}g_{\mu\nu}^{(0)}g^{\mu\nu}-\sqrt{-g}-\sqrt{-g^{(0)}}\right)= \quad (7)$$

$$= \Lambda\left[\sqrt{-g}-\sqrt{-g^{(0)}}+\frac{1}{2}\sqrt{-g}(g^{\mu\nu}-g^{(0)\,\mu\nu})g_{\mu\nu}^{(0)}\right]. \quad (8)$$

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[3] One generic problem with modifications of GR that are bi-metric and/or include massive gravitons is the possible presence of ghosts [33]. In [34] it is argued that such negative-energy contributions do not appear in RTG because the causality condition must necessarily be imposed in the (physical) background spacetime; see also [35] for several useful comments. As has been shown recently, it is possible to explicitly construct ghost-free models of massive gravity [36] and extend these to bi-metric gravity [37]. A rigorous study of the subtle relation between (ghost-free) bi-metric and massive models of gravity is given in [38], which also emphasises that there is no formal reason to consider only flat background metrics. We stress that we mention here Logunov's RTG because it provides the most direct route to the case relevant for our discussion of the effective gravity in ^{3}He-A, namely that of a flat fundamental background metric, without committing ourselves to the underlying philosophy of RTG–or bi-metric theories in general–as opposed to GR.

[4] The bi-metric gravity with arbitrary mass ratio ζ can be found in [39].

[5] The relation between the introduction of a graviton mass and a cosmological constant is actually a rather generic feature of many models of massive and bi-metric theories of gravity, although not necessarily in the simplest form that we discuss here, see [38].

In the Minkowski vacuum, i.e. for $g_{\mu\nu}=g_{\mu\nu}^{(0)}$, which is an analogue of the equilibrium vacuum in ^{3}He-A, ρ_λ is zero. The cosmological term

$$T_{\mu\nu}^\Lambda \equiv \frac{2}{\sqrt{-g}}\frac{\partial\rho_\lambda}{\partial g^{\mu\nu}}=$$

$$= -\Lambda\left[\left(g_{\mu\nu}-g_{\mu\nu}^{(0)}\right)+\frac{1}{2}g_{\mu\nu}g_{\alpha\beta}^{(0)}\left(g^{\alpha\beta}-g^{(0)\alpha\beta}\right)\right], \quad (9)$$

is also zero in the Minkowski vacuum, even though the cosmological constant $\Lambda\propto m^2 G_N^{-1}$ itself is nonzero and is on the order of the Planck scale.

VI. Cosmological constant and massive gravitons. Eq. (3) is the vacuum energy in ^{3}He-A as a function of $c_\perp$ only. Let us consider the more general energy as a function of the metric elements g^{11}, g^{22}, and g^{12}, which are induced in ^{3}He-A, and compare with the RTG of the previous Section. Expressing the perturbations of the metric in terms of the variables η_i corresponding to propagating gravitons

$$\delta g^{11}=c_{\perp 0}^2\frac{\eta_0+\eta_1}{\sqrt{2}}, \quad \delta g^{22}=c_{\perp 0}^2\frac{\eta_0-\eta_1}{\sqrt{2}}, \quad (10)$$

$$\delta g^{12}=\delta g^{21}=c_{\perp 0}^2\frac{\eta_2}{\sqrt{2}}, \quad (11)$$

one obtains [40] the non-equilibrium vacuum density

$$\rho_{\mathrm{vac}}^{\mathrm{pert}}=\frac{1}{4}\lambda\sqrt{-g^{(0)}}\left[\eta_0^2+\frac{1}{2}(\eta_1^2+\eta_2^2)\right]. \quad (12)$$

For $g^{11}=g^{22}=c_\perp^2$ and $g^{12}=0$, this transforms to Eq. (4).

The so-called clapping modes η_1 and η_2 are equivalent to gravitons propagating along the anisotropy axis with spin (helicity) $S=2$. The symmetry between these two $S=2$ graviton modes means that their coefficients in Eq. (12) must necessarily be equal. In addition there is a mode η_0 which is an analogue of the propagating graviton with spin $S=0$ (for this mode the analogy with relativistic theories is not perfect: in ^{3}He-A the perturbation of the metric is not traceless in the $S=0$ mode since the perturbation of the metric element g^{33} is missing).

One can write the ^{3}He-A energy in Eq. (12) in a manner similar to ρ_Λ in Eq. (8) for the RTG, as an expansion of the following expression:

$$\rho_{\mathrm{vac}}=\frac{\lambda}{2}\left[\sqrt{-g}-\sqrt{-g^{(0)}}+\frac{1}{2}\sqrt{-g^{(0)}}(g^{\mu\nu}-g^{(0)\,\mu\nu})g_{\mu\nu}^{(0)}\right]. \quad (13)$$

The energy (12) determines the masses m_i of the gravitons, which are seen to obey the ratio

$$\zeta=\frac{m_{S=0}^2}{m_{S=2}^2}=2. \quad (14)$$

In relativistic bi-metric theories, such as the RTG discussed previously, the mass of the gravitons is determined by the product of the cosmological constant and the Newton constant, $m^2 \sim G_N \lambda$. In ^{3}He-A, the Newton constant is not well defined, because due to the lack of general covariance the terms in the action which describe gravity are not combined into a single Einstein action, and so they have different "Newton constants" [4, 41]. This can clearly be seen in the example of the gravitons propagating along the anisotropy axis. The effective action for these gravitons explicitly contains the mass term (12) (or (13)) [40], and it is an easy exercise to derive their equations of propagation. These contain two "Newton constants", G_N and $\tilde{G}_N$, which enter correspondingly to the space derivative and time derivative sectors of the Einstein tensor:

$$\frac{1}{16\pi\tilde{G}_N}\partial_t^2\eta_{1,2} - \frac{c_\parallel^2}{16\pi G_N}\partial_z^2\eta_{1,2} + \lambda\eta_{1,2} = 0, \qquad (15)$$

$$\frac{1}{16\pi\tilde{G}_N}\partial_t^2\eta_0 - \frac{c_\parallel^2}{16\pi G_N}\partial_z^2\eta_0 + \zeta\lambda\eta_0 = 0. \qquad (16)$$

The Newton constants are on the order of the Planck scale, $G_N \sim \Delta_0^{-2}$. In the BCS limit they obey the relation $G_N = 3\tilde{G}_N$, while the masses of the gravitons are $m_{S=0}^2 = (8/3)\Delta_0^2$ and $m_{S=2}^2 = (4/3)\Delta_0^2$. This difference in mass ratio ($\zeta = 2$ in Eq. (14) in ^{3}He-A, as compared with $\zeta = 1$ in the RTG) is caused by the minimal difference between RTG and the bi-metric gravity in ^{3}He-A ($\sqrt{-g^{(0)}}$ in the third term of Eq. (13) instead of $\sqrt{-g}$ in Eq. (8)), as well as the different composition of the $S = 0$ graviton.[6] But crucially, Eqs. (15), (16) show that the link between the graviton masses and the generalized bi-metric cosmological constant λ in RTG survives in ^{3}He-A. Moreover, in ^{3}He-A, this term is directly determined by the vacuum energy through Eqs. (3) and (6).

Finally, comparison of Eqs. (13) and (8) shows that, even though the action in ^{3}He-A is not generally covariant, it makes good sense to define a cosmological term in analogy with (9):

$$T_{\mu\nu}^\Lambda = \frac{2}{\sqrt{-g}}\frac{\partial\rho_{\text{vac}}}{\partial g^{\mu\nu}} = -\frac{\lambda}{\sqrt{-g}}\left(\sqrt{-g}g_{\mu\nu} - \sqrt{-g^{(0)}}g_{\mu\nu}^{(0)}\right),$$
$$(17)$$

which indeed vanishes in equilibrium, as required by the thermodynamic arguments discussed above.

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[6] In the future it would be interesting to discuss whether the problem of the van Dam–Veltman–Zakharov discontinuity [42, 43] can be resolved in the effective gravity emerging in the systems with Weyl points.

VII. Grand canonical vacuum energy and cosmological constant – Using their example of effective gravity in BEC, the authors of Ref. [1] state that "it is conceivable that the very notion of cosmological constant as a form of energy intrinsic to the vacuum is ultimately misleading". From our point of view this conclusion is based on an example where the effective gravity is very far from GR or from a relativistic bi-metric theory of gravity. In our example, gravity in ^{3}He-A is a bi-metric gravity. Though this gravity emerges on a particular non-relativistic background, its characteristics are universally determined by the topological class (the Weyl point). The effective theory thus obeys both Lorentz invariance and the equivalence principle, and some terms in the effective action for gravity are similar to that in relativistic bi-metric gravity, which in particular describe the propagation of gravitons. The latter allows us to identify the analogue of the cosmological constant. In ^{3}He-A, this cosmological constant is directly related to the properly defined vacuum energy. The cosmological constant turns out to be equal to the difference in energy densities of the vacuum states with and without gravity. In other words, the cosmological constant coincides with that part of the vacuum energy which comes from the degrees of freedom responsible for the emergence of the effective low-energy space-time and thus for the effective gravity. Moreover, the close relationship with a relativistic bi-metric theory suggests to maintain the definition of the cosmological term $T_{\mu\nu}^\Lambda$, which vanishes in equilibrium. Contrarily to GR, for which the high-energy theory is unknown, in the effective gravity of ^{3}He-A, the microscopic physics can be traced explicitly. ^{3}He-A thus shows a possible scenario for the decoupling of the high-energy degrees of freedom, which could be relevant for GR as well.

A related example is provided by the q-theory – the phenomenological theory of the quantum vacuum [22–24]. In this theory the quantity q is the Lorentz invariant variable, which characterizes the vacuum. At first glance, the connection between the cosmological constant Λ and the vacuum energy is lost in the q theory: the cosmological constant is not equal to the vacuum energy $\epsilon(q)$ of the field q. But it coincides with the analog of the grand canonical energy of the field, $\Lambda = \rho_{\text{vac}}(q) = \epsilon(q) - qd\epsilon/dq$: from a thermodynamic analysis and from the dynamic equations of the q-theory it follows that it is this energy $\rho_{\text{vac}}(q)$ which is gravitating, rather than $\epsilon(q)$. In this q-theory, cosmology is the process of relaxation of the Universe towards the equilibrium vacuum state. The vacuum energy density ρ_{vac} has initially a Planck-scale value, which corresponds to the ^{3}He false vacuum state without gravity. In the process

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