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Scheme for realizing entanglement concentration via generalized measurements

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How to concentrate non-maximally entangled states for quantum communication is a fundamental problem in quantum information. In this paper, we will apply generalized measurements to entanglement concentration of known non-maximally entangled pure states in arbitrary dimensional system. How to design the generalized measurements for the unambiguous discrimination of linearly independent non-orthogonal states is crucial for the concentration of the known non-maximally entangled states. The result shows that, any known non-maximally entangled pure state (for arbitrary dimensional system) can be transformed to the maximally entangled state only by introducing a qubit as ancilla and a joint unitary transformation operation on one of the entangled particles and the ancilla. In addition, because the less entangled state of each fail round will be re-concentrated too, the entanglement waste during the concentration process will be greatly reduced.

1. Introduction. Quantum entangled states play a very important role in quantum communication. The fidelity and efficiency of quantum communication are both determined by the entanglement degree of quantum entangled state serving as quantum channel. Only when the quantum entangled state we use is a maximally entangled state, the fidelity and efficiency of quantum communication are both 1.0. However, there exists interactions between a quantum system and its surrounding environment inevitably. Meanwhile quantum operations will bring in some errors. So the quantum entangled states we achieve in experiments are all non-maximally entangled states. They will reduce the fidelity and efficiency of quantum communication and weaken quantum communication's capacity. To achieve quantum communication with high fidelity and efficiency, we must transform the non-maximally entangled states into maximally entangled ones. This transformation is called entanglement concentration for pure states [1, 2] or entanglement purification for mixed states [3].

Up to now, there exist two kinds of entanglement concentration methods: Schmidt projection scheme and Procrustean scheme [1]. Schmidt projection scheme requires operating on multiple pairs of particles in identical non-maximally entangled states in each round. Procrustean scheme only needs one pair of particles in non-maximally entangled state in each round, but, we must know the detailed form of the non-maximally entangled state. Moreover, Procrustean scheme is a probabilistic one. If we only operate on one pair of the particles in non-maximally entangled state, we can achieve the maximally entangled states with a finite probability. But, if we operate on multiple pairs of particles in identical non-maximally entangled state, a smaller (compared with the number of the pairs before concentration) number of the particle pairs can be kept in maximally entangled states, and the rest pairs will remain in non-maximally entangled states. In addition, entanglement swapping can be used to realize the entanglement concentration [2, 4]. In fact, entanglement swapping is a special kind of Schmidt projection process [2], and can be realized with the help of quantum state discrimination [5].

Many entanglement concentration schemes for two-state systems (qubits) have been presented. The Schmidt-projection-based concentration schemes have been proposed both theoretically [6–9] and experimentally [10, 11] in linear optics system. In cavity QED system, many entanglement concentration schemes have been presented too [12–15], and most of these entanglement concentration schemes are based on the Procrustean method. Paunković et al. proposed an entanglement scheme by using quantum statistics [16]. A recent review paper has reviewed most of the advances of the entanglement concentration and entanglement purification for the qubit system [17]. The result shows that there exists a special kind of entanglement, which can not be distilled. It is called bound entanglement [18]. For a pure nonmaximally entangled state, the bound entanglement is zero [18], which means the entanglement in a pure nonmaximally entangled state can be

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completely converted into the singlet form in principle. But, the current existing concentration schemes for pure nonmaximally entangled states can only extract the total entanglement of the initial state in an asymptotic way [1]. This asymptotic scheme is just an idea situation, and the implementation of it is a really tough goal. To enhance the efficiency of the concentration process, we have proposed a scheme for qubit system [12], where the entanglement in the failed state in each round of the concentration has been re-concentrated too. So, the waste in the original concentration process have been greatly reduced. Simple calculations show that the above-mentioned concentration schemes cannot concentrate the nonmaximally entangled states of high-dimensional system.

However, high-dimensional quantum systems have many merits with respect to the qubit systems. Maximally entangled states in high-dimensional quantum systems violate local realism stronger than the ones in two-state systems [19]. High-dimensional quantum system contains more information and enhances the noise limitation determined by quantum secret key distribution scheme [20, 21], and so on. Recently, high-dimensional entangled states have been generated experimentally in quantum optical systems by using the polarization freedom [22], time freedom [23], orbital angular momentum freedom [24], and space freedom [25], respectively.

So the research on entanglement concentration in high-dimensional system has great physical significance. Yao et al. presented a scheme for realizing entanglement concentration of high-dimensional non-maximally entangled states via many ancillas which not only have the same dimension as the original system but also must be in the states depending on the original state [26]. These will increase the difficulty in experiment. Vaziri et al. made use of photon's orbital angular momentum to realize entanglement concentration of three-dimensional non-maximally polarized entangled states via the generalized Procrustean scheme [27].

As in the qubit system, the existing entanglement concentration schemes for high-dimensional system are asymptotic ones, and the efficiency is to be enhanced too. In this paper, we want to present a theoretical scheme to realize entanglement concentration of pure known nonmaximally entangled states for high-dimensional system via generalized measurements. The advantage of this scheme is twofold. Firstly, this scheme only needs one qubit as ancilla, so it can be easily carried out. On the other hand, the left less entangled state in each fail round will be re-concentrated, which greatly reduces the entanglement waste in the concentration process. Although, here, the scheme is mainly based on the quantum state discrimination, we only need one pair of the non-maximally entangled particles in each round, which makes the current scheme simpler and easier to implement in experiments.

2. POVM-based concentration of bipartite non-maximally entangled pure states. According to Neumark theorem, we can realize any given POVM through expanding the state space to a larger one and carrying out proper unitary operations and orthogonal measurements in the larger space. Now, the realization of entanglement concentration of known states is cast into the problem of how to design the corresponding unitary transformation of the POVM in the larger space.

If we have a known pure non-maximally entangled state, it can be rewritten in the Schmidt form:

$$|\Psi\rangle = \sum_{j=0}^{n-1} c_j |\alpha_j\rangle|\beta_j\rangle, \qquad (1)$$

where $\{|\alpha_j\rangle\}$ and $\{|\beta_j\rangle\}$ are orthonormal bases for particle 1 and particle 2, respectively. The letter n is the dimension of the systems involved and the superposition coefficients c_j satisfy the normalization relation $\sum_{j=0}^{n-1}|c_j|^2 = 1$. Through the discrete Fourier transformation, the basis of particle 1 can be transformed into Fourier-transformed basis:

$$|\gamma_k\rangle = \frac{1}{\sqrt{n}} \sum_{j=0}^{n-1} \exp\left(-\frac{2\pi ijk}{n}\right)|\alpha_j\rangle, \qquad (2)$$

where k and j are both integers and run from 0 to $n-1$. Because of

$$\frac{1}{n}\sum_{j=0}^{n-1} \exp\left[-\frac{2\pi ij(k-k')}{n}\right] = \delta_{k,k'}, \qquad (3)$$

we can prove that $|\gamma_k\rangle$ is still a set of orthonormal basis. Now $|\Psi\rangle$ can be rewritten in the following form:

$$|\Psi\rangle = \frac{1}{\sqrt{n}}\sum_k |\gamma_k\rangle|\psi_k\rangle, \qquad (4)$$

where $|\psi_k\rangle = \sum_{j=0}^{n-1} c_j \exp(\frac{2\pi ijk}{n})|\beta_j\rangle$ ($k = 0, 1, 2, \ldots, n-1$) are the corresponding states for particle 2. We find that all of the probability amplitudes are equal in Eq. (4). So long as the states $\{|\psi_k\rangle\}$ form a set of orthonormal basis, $|\Psi\rangle$ will be a maximally entangled state. Unfortunately, $\{|\psi_k\rangle\}$ cannot form a set of orthonormal basis, however, we can prove that they are linear independent [28]. According to the discussion in Ref. [28], the POVM operators A_k distinguishing non-orthogonal states for particle 2 can transform $\{|\psi_k\rangle\}$ into a set of orthogonal states. Because $\{|\psi_k\rangle\}$ is not a set of orthogonal states, we can't distinguish them unambiguously.

This shows that there must be an inconclusive operator A_I, and it can't give any information of the states, which means entanglement concentration fails. Obviously, there is a completeness relation

$$A_O^\dagger A_O + A_I^\dagger A_I = I, \tag{5}$$

where $A_O^\dagger A_O = \sum_{k=0}^{n-1} A_k^\dagger \sum_{k=0}^{n-1} A_k$. So long as we can realize all of the POVM operators $A_k, k = 0, 1, 2, \ldots, n-1$, simultaneously, we can distinguish the non-orthogonal states $\{|\psi_k\rangle\}$ with some probability, and now the non-orthogonal set $\{|\psi_k\rangle\}$ have been transformed into a set of orthogonal states. When POVM measurement acts on particle 2, there will be two outcomes: $A_O^\dagger A_O$ and $A_I^\dagger A_I$. The outcome $A_O^\dagger A_O$ shows that we have transformed a set of non-orthogonal quantum states $\{|\psi_k\rangle\}$ into a set of orthogonal quantum states $\{|\phi_k\rangle\}$. But the outcome $A_I^\dagger A_I$ shows that the concentration process fails. According to the discussion in Ref. [28], we can get

$$A_k = \frac{P^{1/2}}{\langle\psi_k^\perp|\psi_k\rangle}|\phi_k\rangle\langle\psi_k^\perp|, \tag{6}$$

where $P = n \times \min(|c_j|^2)$ is the successful probability of distinguishing the non-orthogonal quantum states. Here,

$$|\psi_k^\perp\rangle = N^{-1/2}\sum_{j=0}^{n-1} c_j^{*\,-1} \exp\left(\frac{2\pi ijk}{n}\right)|\beta_j\rangle \tag{7}$$

are orthogonal complement states of $|\psi_k\rangle$, and it means $|\psi_k^\perp\rangle$ and $|\psi_k\rangle$ satisfy the following relation

$$\langle\psi_k^\perp|\psi_{k'}\rangle = \frac{n}{\sqrt{N}}\delta_{k,k'}. \tag{8}$$

Here, $N = \sum_{j=0}^{n-1}|c_j|^{-2}$ and $\{|\phi_k\rangle\}$ is another set of orthonormal basis. It is not difficult to see that

$$A_O^\dagger A_O = \sum_{k=0}^{n-1} A_k^\dagger \sum_{k=0}^{n-1} A_k = \sum_{k=0}^{n-1} A_k^\dagger A_k. \tag{9}$$

He and Bergou proposed a scheme to realize POVM measurement in Ref. [29]. We will adopt this scheme to realize the above mentioned POVM measurement and design the unitary transformation U we need to realize entanglement concentration of the non-maximally entangled states. First, let's introduce another particle (labeled by 3) as an ancilla qubit. The ancilla particle 3 is prepared in state $|0_3\rangle$. Now, the n-dimension Hilbert space expanded by the state $|\psi_k\rangle$ becomes a subspace of $2n$-dimension Hilbert space of particles 2 and 3. According to the above discussion and with the help of Eq. (3), we can get $A_O^\dagger A_O$:

$$A_O^\dagger A_O = \sum_k \frac{P}{|\langle\psi_k^\perp|\psi_k\rangle|^2}|\psi_k^\perp\rangle\langle\psi_k^\perp| = \\ = \sum_j |c_j|^{-2}\min(|c_j|^2)|\beta_j\rangle\langle\beta_j|. \tag{10}$$

According to the completeness relation in Eq. (5), there must exist another operator labeled by $A_I^\dagger A_I$:

$$A_I^\dagger A_I = I - A_O^\dagger A_O = \\ = \sum_j [1 - |c_j|^{-2}\min(|c_j|^2)]|\beta_j\rangle\langle\beta_j|, \tag{11}$$

and it is diagonalized already. Define two Hermitian operators:

$$A_O^\dagger = A_O = \sum_{j=0}^{n-1}|c_j|^{-1}\min(|c_j|)|\beta_j\rangle\langle\beta_j|, \tag{12}$$

$$A_I^\dagger = A_I = \sum_{j=0}^{n-1}\sqrt{1 - |c_j|^{-2}\min(|c_j|^2)}|\beta_j\rangle\langle\beta_j|. \tag{13}$$

With the help of these POVM elements and their corresponding forms, we can design a unitary transformation $U = \begin{pmatrix} A_O & -A_I \\ A_I & A_O \end{pmatrix}$, acting in the expanded $2n$-dimension Hilbert space. It can be proven that U is unitary. Apply the unitary transformation $U = \begin{pmatrix} A_O & -A_I \\ A_I & A_O \end{pmatrix}$ on the $2n$-dimension Hilbert space expanded by particles 2 and 3, and carry out an Von Neumann projection measurement on the particle 3. According to the outcomes, we can judge whether the states discrimination or the entanglement concentration is successful or not: the outcome $|0_3\rangle$ shows entanglement concentration is successful, and the outcome $|1_3\rangle$ shows entanglement concentration fails. The probability of successful entanglement concentration is equal to the probability of distinguishing the non-orthogonal quantum states $|\psi_k\rangle$ successfully, $P = n \times \min(|c_j|^2)$ [28]. If we get $|1_3\rangle$, the state of particles 1 and 2 is still a non-maximally entangled state, we can design another unitary transformation U' to realize entanglement concentration further. That is to say, by using the current scheme, we can even make use of the garbage state (fail state in the concentration process), and the garbage state can be concentrated too. This point can be clearly seen in the following example. In this sense, the current concentration scheme can concentrate the entanglement of non-maximally entangled state as much as possible.

3. An example. In this section, we will give an example to examine the scheme we presented in Section 2. Suppose there is a non-maximally entangled state:

$$|\Pi\rangle = \frac{1}{\sqrt{3}}|0_A\rangle|0_B\rangle + \frac{1}{\sqrt{6}}|1_A\rangle|1_B\rangle + \frac{1}{\sqrt{2}}|2_A\rangle|2_B\rangle, \quad (14)$$

where $\{|0_A\rangle, |1_A\rangle, |2_A\rangle\}$ and $\{|0_B\rangle, |1_B\rangle, |2_B\rangle\}$ are orthonormal bases for particle A and particle B, respectively. Express the state in the Fourier-transformed basis on particle A, we can get:

$$\begin{aligned}|\Pi\rangle = {} & \frac{1}{\sqrt{3}}|\gamma_0\rangle\left(\frac{1}{\sqrt{3}}|0_B\rangle + \frac{1}{\sqrt{6}}|1_B\rangle + \frac{1}{\sqrt{2}}|2_B\rangle\right) + \\ & + \frac{1}{\sqrt{3}}|\gamma_1\rangle\left[\frac{1}{\sqrt{3}}|0_B\rangle + \frac{1}{\sqrt{6}}\exp\left(\frac{2\pi i}{3}\right)|1_B\rangle + \right. \\ & \left. + \frac{1}{\sqrt{2}}\exp\left(\frac{4\pi i}{3}\right)|2_B\rangle\right] + \\ & + \frac{1}{\sqrt{3}}|\gamma_2\rangle\left[\frac{1}{\sqrt{3}}|0_B\rangle + \frac{1}{\sqrt{6}}\exp\left(\frac{4\pi i}{3}\right)|1_B\rangle + \right. \\ & \left. + \frac{1}{\sqrt{2}}\exp\left(\frac{2\pi i}{3}\right)|2_B\rangle\right], \end{aligned} \quad (15)$$

where

$$\begin{aligned}|\gamma_0\rangle &= \frac{1}{\sqrt{3}}(|0_A\rangle + |1_A\rangle + |2_A\rangle), \\ |\gamma_1\rangle &= \frac{1}{\sqrt{3}}\left[|0_A\rangle + \exp\left(-\frac{2\pi i}{3}\right)|1_A\rangle + \right. \\ & \left. + \exp\left(-\frac{4\pi i}{3}\right)|2_A\rangle\right], \\ |\gamma_2\rangle &= \frac{1}{\sqrt{3}}\left[|0_A\rangle + \exp\left(-\frac{4\pi i}{3}\right)|1_B\rangle + \right. \\ & \left. + \exp\left(-\frac{2\pi i}{3}\right)|2_A\rangle\right], \end{aligned} \quad (16)$$

are a set of fourier basis of particle A. In the new form, the coefficients of $|\Pi\rangle$ are equal. If we can transform the set of linear independent state vectors in the parentheses of Eq. (15) into a set of orthogonal states, $|\Pi\rangle$ will become a maximally entangled state.

Introduce a 2-dimensional ancilla particle C prepared in state $|0_C\rangle$. According to the discussion in Section 2 we can design the corresponding unitary transformation:

$$U_{BC} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{3}} & 0 & 0 & -\frac{\sqrt{2}}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{\sqrt{2}}{\sqrt{3}} & 0 & 0 & \frac{1}{\sqrt{3}} \end{pmatrix}, \quad (17)$$

which will be applied on particles B and C. Then we get

$$\begin{aligned} & I_A \otimes U_{BC}|\Pi\rangle|0_C\rangle = \\ & = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{3}}|0_A\rangle|0_B\rangle + \frac{1}{\sqrt{3}}|1_A\rangle|1_B\rangle + \right. \\ & \left. + \frac{1}{\sqrt{3}}|2_A\rangle|2_B\rangle\right)|0_C\rangle + \\ & + \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{3}}|0_A\rangle|0_B\rangle + \frac{\sqrt{2}}{\sqrt{3}}|2_A\rangle|2_B\rangle\right)|1_C\rangle. \end{aligned} \quad (18)$$

Then the Von Neumann projection measurement on the ancilla particle C will tell us whether the concentration process succeed or not. The outcome $|0_C\rangle$ shows entanglement concentration succeeds, and the outcome $|1_C\rangle$ shows entanglement concentration fails. The success probability is $P_1 = 3 \times \frac{1}{6} = \frac{1}{2}$. In addition, we find the fail state $\frac{1}{\sqrt{3}}|0_A\rangle|0_B\rangle + \frac{\sqrt{2}}{\sqrt{3}}|2_A\rangle|2_B\rangle$ is still a bipartite non-maximally entangled state. We can design another unitary transformation to realize entanglement concentration of it with probability $P_2 = (1 - P_1) \times 2 \times \frac{1}{3} = \frac{1}{3}$. So the total success probability of the concentration process is $P = P_1 + P_2 = 0.8333$. E_{F}, entanglement of formation [30], is the maximum of the distillable entanglement [31]. Because the initial state is a pure state, there is no bound entanglement inside it [18]. So, we can use E_{F} to describe the distillable entanglement. E_{F} of the quantum state $|\Pi\rangle$ is 0.9206 [32]. Obviously, the distillable entanglement is a little bit larger than the success probability (0.8333) of the concentration process. Although our scheme is not the optimal one, the entanglement waste during the concentration process has been greatly reduced.

4. Summary. In this paper, we present a scheme to realize entanglement concentration of non-maximally entangled pure states in arbitrary dimensional systems via generalized measurements (POVM). How to design the joint unitary transformation on one of the entangled particles and the ancilla is crucial to the scheme. In fact, the unitary transformation realizes the entanglement concentration and a quantum state discrimination simultaneously. A qubit ancilla and a joint unitary transformation on the ancilla and one of the entangled particles will be sufficient for the concentration. Surprisingly, the previous works on the entanglement concentration are only the applications of the current universal scheme [12–14]. The advantage of this scheme is twofold. On one hand, to concentrate the known non-maximally entangled pure states in arbitrary dimensional systems only needs one qubit as ancilla particle, which greatly reduces the complexity of experiments. On the other hand, the left less entangled state in each fail round will be reconcentrated, which greatly reduces the entanglement waste in the concentration process. But we must know the Schmidt form of the non-maximally entangled pure states to be concentrated. That is to say this scheme can only be applied to the **known** non-maximally entangled **pure** states. Moreover, this scheme only can be applied to the entanglement concentration of pure non-maximally entangled states, rather than the mixed entangled states. This is because, for some entangled states (bound entangled states) no maximally entangled pairs can be distilled [17, 18]. So, seeking a more universal scheme of realizing entanglement purification of mixed entangled states needs further study.

Although the style and the notation of the paper seems rather "abstract", it contains the detailed scheme for entanglement concentration for pure known states of arbitrary dimensional systems, which can be easily followed by the experimentalists to design the corresponding implementation in different systems.

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