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Simply quantum information processing with rf superconducting qubit

F.-Y. Zhang[1)], *Z.-H. Chen, C. Li, H.-S. Song*

School of Physics and Optoelectronic Technology, Dalian University of Technology, 116024 Dalian, China

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Utilizing rf superconducting quantum interference devices (SQUIDs) coupled with transmission line resonator (TLR), we propose a scheme to implementing quantum information processing. In this system, the high fidelity two-qubit maximally entangled states and quantum logic gate are realized. Under the large detuning condition, the excited state of rf SQUID is adiabatically eliminated. So the excited state spontaneous emission of the superconducting qubit can be effectively avoided in this paper. At last, the experimental feasibility and the challenge of our schemes have been discussed.

I. Introduction. Entangled states and quantum logic gate are the main backbone in quantum information processing (QIP). The entangled states are not only helpful in QIP (quantum teleportation [1], quantum dense coding [2], quantum cryptography [3]), but also useful in quantum mechanics to prevail over local hidden theory [4]. On the other hand, quantum logic gates are the important information process unit in QIP. Until now, many theoretical and experimental schemes have been proposed to prepare entangled states, which include two-qubit entangled states [5, 6], Greenberger–Horne–Zeilinger (GHZ) states [7], W-type states [8], and cluster states [9], and to implement quantum logic gates, such as, Hadamard gate [10], $\sqrt{\texttt{SWAP}}$ gate [11], and Toffoli gate [12]. Hence, how to generate entangled states and implement quantum logic gates with the graceful way is still a hot topic.

Recently, the solid superconducting devices (Cooper pair boxes, Josephson junctions, and SQUID) were proposed as candidates to serve as the qubits for a superconducting quantum computer [13], due to its advantage in design flexibility, large-scale integration, and compatibility to conventional electronics. Therefore, there have been broad investigated in quantum informaton field. For instance, the coherent control of macroscopic quantum states in a single-Cooper-pair box has been realized [14]. The detection of geometric phases in superconducting qubit has been reported [15]. In particular, the current experiment has shown two nearby superconducting charge qubits can be readily coupled with a single high-Q TLR [16]. Then logical gates are realized by driving the resonator with microwave fields in this system [17]. However, one of the flux qubit, rf-SQUID has long coherence time than the charge qubit [18]. Yang et al. presented a scheme to achieve maximally entangled states and SWAP gate for two rf-SQUID qubits, which were placed in a microwave cavity [19]. In this scheme, there are some disadvantages. For instance, (i) the redundant operation: generating maximally entangled states requires three-step operations and achieving SWAP gate needs five-step operations, respectively. (ii) The short-lived excited state is acted as auxiliary level, which is the requisite in the all process. (iii) The influence of dephasing rate and relaxation rate of superconducting qubits on the results do not consider in the Yang's scheme.

In this paper, we propose a simply alternative scheme to generate entangled states and implement logic gates by using single-mode TLR which induces the interaction of two rf SQUIDs. Under the large detuning condition, the excited state of rf SQUID is adiabatically eliminated. The information is encoded in two ground states of superconducting qubit. Our schemes have the following advantages: (i) The excited level of rf SQUID, which is very robust to decoherence due to spontaneous emission, is only virtually coupling. (ii) The rf SQUIDs coupling with TLR has experimental feasibility with currently available technology. (iii) Comparing with Ref. [19], our scheme only needs one-step evolution for generating maximally entangled states and two-step operations to implementing SWAP gate, respectively.

The rf SQUID consisting of one Josephson junction enclosed by a superconducting loop. The Hamiltonian of rf SQUID can be written as [20, 21]

$$H_q = \frac{Q^2}{2C} + \frac{(\Phi - \Phi_x)^2}{2L} - E_{\rm J}\cos\left(2\pi\frac{\Phi}{\Phi_0}\right), \qquad (1)$$

where C expresses junction capacitance, L is loop inductance, Q describes the total charge on the capacitor, Φ depicts the magnetic flux threading the ring (Φ and Q are the conjugate variables of the system), Φ_x is the quasistatic external magnetic flux through the ring, and $E_{\rm J} = I_c\Phi_0/2\pi$ defines the Josephson coupling energy, where I_c and $\Phi_0 = h/2e$ are the critical current of the junction and the flux quantum, respectively.

[1)]e-mail: zhangfy@mail.dlut.edu.cn; zhangfy1986@gmail.com

II. Generation of entangled states and realizing logic gate. In general, the rf SQUID is defined a Λ-type three energy levels structure (the two lower flux states $|0\rangle$ and $|1\rangle$, and the excited state $|2\rangle$) [19]. The energy level configuration of the rf SQUID is shown in Fig. 1. We take the energy of level $|0\rangle$ to be zero as the reference point. The lower lying level $|1\rangle$ and the upper level $|2\rangle$ have the energy ω_1 and ω_2 ($\hbar = 1$), respectively.

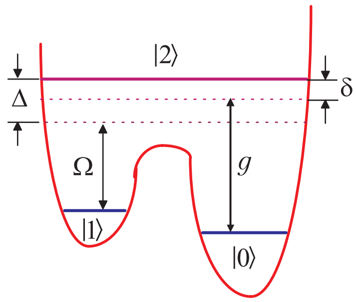

Fig. 1. (Color online) The energy level configuration of the rf-SQUID. The TLR mode interact with the transition $|0\rangle \leftrightarrow |2\rangle$. The transition $|1\rangle \leftrightarrow |2\rangle$ is driven by classical field

A rf SQUID was trapped in an antinode of the TLR, and driven by a classical microwave pulse. The resonator mode with frequency ω_r is coupled to the $|0\rangle \leftrightarrow |2\rangle$ transition with coupling constant g, but far-off resonant with the $|0\rangle \leftrightarrow |1\rangle$ and $|1\rangle \leftrightarrow |2\rangle$ transitions. The microwave pulse with the frequency ω and Rabi frequency Ω is coupled to the $|1\rangle \leftrightarrow |2\rangle$ transition while far-off resonant with the $|0\rangle \leftrightarrow |2\rangle$ and $|0\rangle \leftrightarrow |1\rangle$ transitions (see the Fig. 1). In the interaction picture, under the rotating-wave approximation (RWA), the Hamiltonian of the system can be written as

$$H = ga^\dagger|0\rangle\langle 2|e^{-i\delta t} + \Omega|1\rangle\langle 2|e^{-i\Delta t} + \text{h.c.}, \qquad (2)$$

where the detuning $\delta = \omega_2 - \omega_r$ and $\Delta = \omega_2 - \omega_1 - \omega$. If the detuning $\delta(\Delta)$ is sufficiently large, i.e. $|\delta|(|\Delta|) \gg \gg g, \Omega$, the upper state $|2\rangle$ is only virtually exciting and can be adiabatically eliminated, and the corresponding to effective Hamiltonian is [22]

$$H_{\text{eff}} = \lambda_0 a^\dagger a|0\rangle\langle 0| + \lambda_1|1\rangle\langle 1| + + (\lambda_2 a|1\rangle\langle 0|e^{i(\Delta-\delta)t} + \text{h.c.}), \qquad (3)$$

where the parameters $\lambda_0 = |g|^2/\delta$, $\lambda_1 = |\Omega|^2/\Delta$, $\lambda_2 = = g^*\Omega/\Delta'$, and $1/\Delta' = (1/\delta + 1/\Delta)/2$. The first two terms in H_{eff} represent photon-induced and laser-induced dynamic energy shifts, respectively. The photon induced level shifts can be eliminated when the TLR is initially prepared in the vacuum state. The laser induced level shifts can be compensated straightforwardly using additional lasers with appropriate frequencies. Therefore, the effective Hamiltonian can be further reduced to

$$H_I' = \lambda_2 a|1\rangle\langle 0|e^{i\eta t} + \text{h.c.}, \qquad (4)$$

with $\eta = \Delta - \delta$.

Next, we discuss how to generate two-qubit entangled states and achieve logic gate by our scheme. We consider two rf SQUIDs were trapped in an adjacent antinode of the single-mode TLR (see Fig. 2). In the interaction picture, the Hamiltonian can be written as

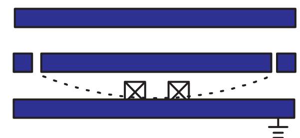

Fig. 2. Two superconducting qubits (crossed box) were trapped the antinodes of a quantum electromagnetic field in a TLR

$$H_I = \sum_{j=1,2} (\lambda_2^j a|1\rangle_{jj}\langle 0|e^{i\eta_j t} + \text{h.c.}). \qquad (5)$$

If the detuning η_j is much larger than the coefficient λ_2^j ($\eta_j \gg \lambda_2^j$), there is no energy exchange between the rf SQUIDs and the TLR. We assume two identical rf SQUIDs simultaneously interacting with the TLR, i.e. $\lambda_2^1 = \lambda_2^2$. Also, we assume the coupling strength g and Rabi frequency Ω are real numbers. And the TLR is initial assumed in the vacuum state. Then the effective Hamiltonian is given by [5]

$$H_e = \lambda_3(|1\rangle_{11}\langle 1| + |1\rangle_{22}\langle 1| + \sigma_1^+\sigma_2^- + \sigma_1^-\sigma_2^+), \qquad (6)$$

where $\lambda_3 = \lambda_2^2/\eta$ and $\sigma_j^+ = (\sigma_j^-)^\dagger = |1\rangle_{jj}\langle 0|$ ($j = 1, 2$). The first and second terms correspond to the dynamical energy shift regarding the level $|1\rangle_j$. The third and fourth terms describe the dipole coupling between the two rf SQUIDs induced by the TLR. Then the evolution operator of Eq. (6) is

$$U(t) = \begin{pmatrix} e^{-2i\lambda_3 t} & 0 & 0 & 0 \\ 0 & \frac{1+e^{-2i\lambda_3 t}}{2} & \frac{-1+e^{-2i\lambda_3 t}}{2} & 0 \\ 0 & \frac{-1+e^{-2i\lambda_3 t}}{2} & \frac{1+e^{-2i\lambda_3 t}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \qquad (7)$$

The two-qubit maximal entangled state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle_1|1\rangle_2 - i|1\rangle_1|0\rangle_2), \qquad (8)$$

can be obtained, when we choose the initial superconducting qubit in state $|0\rangle_1|1\rangle_2$ and evolution time $t = = \pi/4\lambda_3 + 2k\pi$ with the integer k, here the common phase factor $\pi/4$ has been omitted. Obviously, the time of generating two-qubit maximal entangled state is a periodic function. This is to say that the maximal entanglement of two rf SQUID can be deterministically generated at proper time. This result has been studied in the quantum dot system. The main advantages of our scheme are: (1) the superconducting qubit can be controlled by external flux and voltage; (2) the coupling between superconducting qubit and TLR has favourably experimental feasibility.

In order to realize quantum logic gate, we choose the basis $\{|0\rangle_1|0\rangle_2, |0\rangle_1|1\rangle_2, |1\rangle_1|0\rangle_2, |1\rangle_1|1\rangle_2\}$. When the evolution time $t = \pi/2\lambda_3$, a particular logic gate can be realized and written as

$$\begin{aligned} &|0\rangle_1|0\rangle_2 \to |0\rangle_1|0\rangle_2, \\ &|0\rangle_1|1\rangle_2 \to |1\rangle_1|0\rangle_2, \\ &|1\rangle_1|0\rangle_2 \to |0\rangle_1|1\rangle_2, \\ &|1\rangle_1|1\rangle_2 \to -|1\rangle_1|1\rangle_2, \end{aligned} \tag{9}$$

where an overall phase factor $\exp(-i\pi)$ is omitted. The Eq. (9) is a swap and π phase gate. To achieve a SWAP gate, a additional single-qubit operation is applied: $|1\rangle \to -|1\rangle$. Then the Eq. (9) becomes a normal SWAP gate.

For a real physical system, we should take account of decoherence effects. Following the standard quantum theory of damping, the master equation of two-qubit system is

$$\begin{aligned} \dot\rho = -i[H_e,\rho] + \sum_{j=1,2}\Big[\frac{\gamma_{\phi_j}}{2}\left(\sigma_z^j\rho\sigma_z^j - \rho\right) + \\ + \frac{\gamma_j}{4}\left(\sigma_j^-\rho\sigma_j^+ - \frac{1}{2}\sigma_j^+\sigma_j^-\rho - \frac{1}{2}\rho\sigma_j^+\sigma_j^-\right)\Big], \end{aligned} \tag{10}$$

where γ_{ϕ_j} and γ_j are the pure dephasing rate and relaxation rate, respectively, of individual qubits, and Pauli matrix $\sigma_z^j = |1\rangle_{jj}\langle 1| - |0\rangle_{jj}\langle 0|$. For simply, we choose the parameters $\gamma_{\phi_1} = \gamma_{\phi_2} = \gamma_\phi$ and $\gamma_1 = \gamma_2 = \gamma$. The Fig. 3 plots the relation of fidelity F of generating two-qubits maximally entangled states with the dephasing rate and relaxation rate. The high fidelity entangled states are generated at low dephasing rate and low relaxation rate. Also, the quantum logic gate's fidelity was plotted in the Fig. 4.

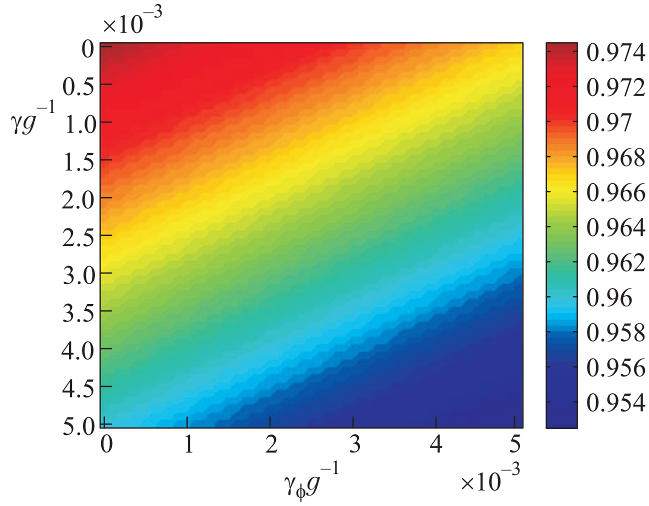

Fig. 3. The relationship of fidelity of generation two-qubit entangled state with dephasing rate γ_ϕ/g and relaxation rate γ/g of qubit

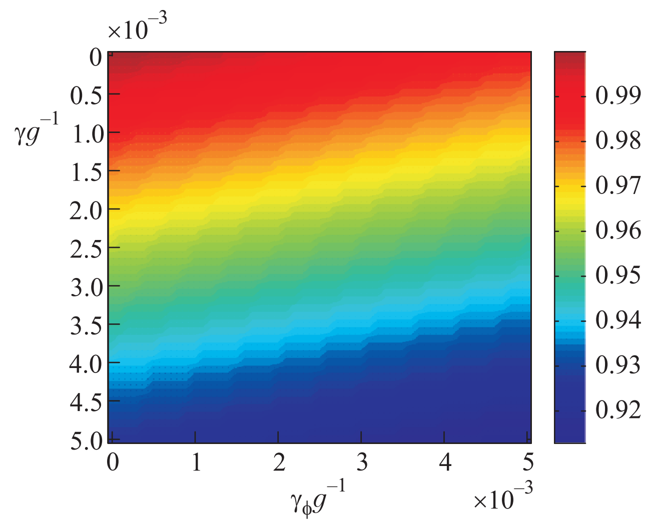

Fig. 4. The fidelity of quantum logic gate Eq. (9) was plotted with dephasing rate γ_ϕ/g and relaxation rate γ/g of qubit

III. Discussion and conclusion. We briefly address the experimental feasibility of the proposed scheme with the parameters already available in current experimental setups. The coupling technology between the TLR and superconducting qubit is rather mature in current experiment [23]. The Ref. [24] has reported the parameters of the TLR: eigenfrequency $\omega_r/2\pi = 10$ GHz, quality factor $Q = 1\cdot 10^5$, and decay rate $\kappa = 0.1$ MHz. The TLR is always empty and only virtually excited. The influence of the cavity loss is negligible. The length of the resonator's central conductor is 23 mm [25], the size of superconducting qubit is μm order [26]. The coupling of two superconducting qubits via data bus has been reported [16, 27]. For the superconducting qubit: the energy level frequencies are GHz order [28, 29]. The coupling strength $g/2\pi \sim 19$ MHz between superconducting qubit and TLR has been reported [23]. And the energy relaxation time $T_1 = 70\,\mu$s and quantum coherence time $T_2 = 95\,\mu$s have been observed [30]. Based on above experimental parameters, we get the following conclusions: if we choose the classical field's Rabi frequency $\Omega/2\pi = 0.9\,\text{MHz}$, the large detuning conditions were satisfied. In general, thus we argue that our proposal might be experimentally realized with current technology.

In summary, we have proposed schemes to generate two-qubit entangled states and realize quantum logic gate by the rf SQUID coupling with TLR. The information is encoded in the ground states $|0\rangle$ and $|1\rangle$ of rf SQUID. The excited state of rf SQUID was adiabatically eliminated. Therefore, our schemes are insensitive to the spontaneous emission of the excited state of rf SQUID. Finally, the experimental feasibility of our scheme is discussed.

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