Pis'ma v ZhETF, vol. 97, iss. 6, pp. 344 – 349

Nambu sum rule in the NJL models: from superfluidity to top quark condensation

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Submitted 11 February 2013

It may appear that the recently found resonance at 125 GeV is not the only Higgs boson. We point out the possibility that the Higgs bosons appear in models of top-quark condensation, where the masses of the bosonic excitations are related to the top quark mass by the sum rule similar to the Nambu sum rule of the NJL models [1]. This rule was originally considered by Nambu for superfluid ^{3}He-B and for the BCS model of superconductivity. It relates the two masses of bosonic excitations existing in each channel of Cooper pairing to the fermion mass. An example of the Nambu partners is provided by the amplitude and the phase modes in the BCS model describing Cooper pairing in the s-wave channel. This sum rule suggests the existence of the Nambu partners for the 125 GeV Higgs boson. Their masses can be predicted by the Nambu sum rule under certain circumstances. For example, if there are only two states in the given channel, the mass of the Nambu partner is $\sim$325 GeV. They together satisfy the Nambu sum rule $M_1^2 + M_2^2 = 4M_t^2$, where $M_t \sim 174$ GeV is the mass of the top quark. If there are two doubly degenerated states, then the second mass is $\sim$ 210 GeV. In this case the Nambu sum rule is $2M_1^2 + 2M_2^2 = 4M_t^2$. In addition, the properties of the Higgs modes in superfluid ^{3}He-A, where the symmetry breaking is similar to that of the Standard Model of particle physics, suggest the existence of two electrically charged Higgs particles with masses around 245 GeV, which together also obey the Nambu sum rule $M_+^2 + M_-^2 = 4M_t^2$.

DOI: 10.7868/S0370274X13060039

1. Introduction. In 1985 Nambu noticed the relation between the energy gaps of bosonic and fermionic excitations in a certain class of the effective NJL-like models (i.e. the models with the 4-fermion interaction) [1]. This class includes superfluid ^{3}He-B and s-wave superconductors. The collective bosonic modes emerging in the fermionic system (Goldstone and Higgs bosons) can be distributed into the pairs of Nambu partners. For each pair one has the relation, $M_1^2 + M_2^2 = 4M_f^2$, where M_1 and M_2 are gaps in the bosonic spectrum, and M_f is the gap in the femionic spectrum (in relativistic systems they correspond to the masses of particles). The similar relation was also discussed in the Nambu–Jona–Lasinio (NJL) approximation [2] of QCD, where it relates the σ-meson mass and the constituent quark mass $M_\sigma \approx 2M_{\rm quark}$.

Here we suggest that Higgs bosons in the Standard Model are composite objects, and they obey the same relation which we call the Nambu sum rule

$$\sum M_{\mathrm{H},i}^2 \approx 4M_f^2. \qquad (1)$$

Here $M_{\mathrm{H},i}$ are the masses of composite Higgs bosons within the given channel, and M_f is the mass of the heaviest fermion, which contributes to their formation. We assume that this is the top quark.

We suggest the hypothesis that Eq. (1) holds in the theories that admit the NJL approximation if there is the fermion whose mass M_f dominates the fermion spectrum. We apply this sum rule for the estimation of the masses of extra Higgs bosons, since the analogy with the superconductivity and superfluidity prompts that the Higgs boson may be composite. (See [3, 4, 5] for the foundation of the Higgs mechanism in quantum field theory.)

It is worth mentioning that the particles within the masses larger than 130 GeV are not excluded by present experiments if they have the cross-sections smaller than that of the standard Higgs boson of the Standard Model [6, 7]. For example, on Fig. 4 of [8] the solid black curve separates the region, where the scalar particles are excluded (above the curve) from the region, where they are not excluded. In particular, the particle with mass around 200 GeV and with the cross section about 1/3 of the Standard Model cross section is not excluded by

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these data. (The similar exclusion curve was announced by ATLAS (plenary talk [9] at ICHEP 2012, slide 34).)

The Nambu sum rule Eq. (1) gives an important constraint on the bosonic spectrum. For example, if there are only two states in the channel that contains the discovered 125 GeV Higgs boson, then the partner of this boson should have the mass around 325 GeV. Surprisingly, in 2011 the CDF collaboration [10] has announced the preliminary results on the excess of events in $ZZ \to ll\bar{l}\bar{l}$ channel at the invariant mass ≈ 325 GeV. CMS collaboration also reported a small excess in this region [11]. In [12, 13] it was argued that this may point out to the possible existence of a new scalar particle with mass $M_{\rm H2} \approx 325$ GeV. If there are two (doubly degenerated) Higgs bosons in the channel that contains the 125 GeV Higgs, then the partner of the 125 GeV boson should have mass around 210 GeV. (This possibility is realized in the model of Section 5 of the present paper.) In the channel with two states of equal masses the 245 GeV Higgs bosons should appear in analogy with ^{3}He-A considered in Section 3. A certain excess of events in this region has been observed by ATLAS in 2011 (see, for example, [14]).

We review several models, where the Nambu sum rule holds. The paper is organized as follows. In Section 2 we consider the appearance of the Nambu Sum rule in ^{3}He-B mentioned originally by Nambu. In Section 3 we consider the $3D$ A-phase of the superfluid ^{3}He. In this case the fermions are gapless. However, the Nambu sum rule Eq. (1) works if in its r.h.s. the average of the angle dependent energy gap is substituted. In Section 4 we review bosonic excitations and Nambu sum rules in the $2D$ thin films of He-3. There are two main phases, where the Nambu sum rule works within the effective four-fermion model similar to that of ^{3}He-B. In Section 5 we consider how the sum rule Eq. (1) appears in its nontrivial form in the relativistic NJL model. Namely, we deal with the particular case considered in [15] of the model of the top-quark condensation suggested in [16]. (This model is the direct generalization of the original model of [17] to the case, when all 6 quarks are included.) In Section 6 we review results on the bosonic excitations in dense quark matter. We consider diquarks in Hadronic phase and the color superconductor in the so-called Color-Flavor Locking (CFL) phase. In Section 7 we compare Veltman relation for the vanishing of quadratic divergences to the scalar boson mass with the Nambu sum rule. In Section 8 we end with the conclusions.

2. Nambu sum rules in ^{3}He-B. In the B-phase of ^{3}He the condensate is formed in the spin-triplet p-wave state, which is characterized by the quantum numbers of spin, orbital momentum and total angular momentum correspondingly $S = 1$, $L = 1$, $J = 0$ [18]. This corresponds to the symmetry breaking scheme $G \to H$ with the symmetry of physical laws $G = SO_S(3) \times SO_L(3) \times U(1)$ and the symmetry of the degenerate vacuum states $H = SO_J(3)$. The order parameter is 3×3 complex matrix

$$A_{\alpha i} = \Delta\delta_{\alpha i} + u_{\alpha i} + iv_{\alpha i}\,. \qquad (2)$$

Here the first term corresponds to the equilibrium state, with Δ being the gap in the fermionic spectrum. The other two terms are the deviations from the equilibrium. They represent 18 collective bosonic modes, which are classified by the total angular momentum quantum number $J = 0, 1, 2$. At each value of $J = 0, 1, 2$ the modes u and v are orthogonal to each other and correspond to different values of the bosonic energy gaps. Four modes are gapless. They represent Goldstone bosons, which result from the symmetry breaking. The other 14 modes are Higgs bosons with non-zero gaps. (Higgs bosons in other condensed matter systems have been discussed in recent papers [19–21] and in references therein.)

The energy gaps of bosons in ^{3}He-B are given by:

$$E^{(J)}_{u,v} = \sqrt{2\Delta^2[1 \pm \eta^{(J)}]}, \qquad (3)$$

where parameters $\eta^{(J)}$ are determined by the symmetry of the system, $\eta^{J=0} = \eta^{J=1} = 1$, and $\eta^{J=2} = 1/5$ [15]. Eq. (3) proves the sum rule for ^{3}He-B found by Nambu for ^{3}He-B:

$$[E^{(J)}_u]^2 + [E^{(J)}_v]^2 = 4\Delta^2\,. \qquad (4)$$

For $J = 0$ there is one pair of the Nambu partners (the gapless Goldstone sound mode and the Higgs mode, which is called the pair-breaking mode):

$$E^{(0)}_1 = 0, \quad E^{(0)}_2 = 2\Delta. \qquad (5)$$

For $J = 1$ there are three pairs of Nambu partners (three gapless Goldstone modes – spin waves, and three Higgs pair-breaking modes):

$$E^{(1)}_1 = 0, \quad E^{(1)}_2 = 2\Delta. \qquad (6)$$

For $J = 2$ there exist five pairs of Higgs partners – five so-called real squashing modes with the energy gap $E^{(2)}_1$, and, correspondingly, five imaginary squashing modes with the energy gap $E^{(2)}_2$:

$$E^{(2)}_1 = \sqrt{2/5}\,(2\Delta), \quad E^{(2)}_2 = \sqrt{3/5}\,(2\Delta). \qquad (7)$$

Zeeman splitting of imaginary squashing mode in magnetic field has been observed in [22], for the latest experiments see [23].

3. Nambu sum rules in bulk ^{3}He-A. In the A-phase of ^{3}He the condensate is formed in the state with $S_z = 0$ and $L_z = 1$ [18]. In the orbital sector the symmetry breaking in ^{3}He-A is similar to that of the electroweak theory: $U(1) \otimes SO_L(3) \to U_Q(1)$, where the quantum number Q plays the role of the electric charge (see e.g. Ref. [24]), while in the spin sector one has $SO_S(3) \to SO_S(2)$. The order parameter matrix has the form

$$A_{\alpha i} = \Delta_0 \hat{z}_\alpha (\hat{x}_i + i\hat{y}_i) + u_{\alpha i} + iv_{\alpha i}. \quad (8)$$

The A-phase is anisotropic. The special direction in the orbital space appears that is identified with the direction of the spontaneous orbital angular momentum of Cooper pairs, which is here chosen along the axis z. In this phase fermions are gapless: the gap in the fermionic spectrum depends on the angle between the momentum $\mathbf{k}$ and the anisotropy axis, $\Delta(\theta) = \Delta_0 \sin\theta$, and nullifies at $\sin\theta = 0$. The spectrum of the collective modes has been considered in [25], see also [26] for extra Goldstone modes related to hidden symmetry of the A-phase. The energy spectrum of the Higgs modes has, in general, the imaginary part due to the radiation of gapless fermions. However, if the radiation processes are neglected, one obtains that there are the Nambu partners that satisfy a version of the Nambu sum rule, written in the form

$$E_1^2 + E_2^2 = 4\bar{\Delta}^2. \quad (9)$$

The role of the square of the fermion mass is played by the angle average of the square of the anisotropic gap in the fermionic spectrum:

$$\bar{\Delta}^2 \equiv \left\langle \Delta^2(\theta) \right\rangle = \frac{2}{3}\Delta_0^2. \quad (10)$$

One (triply degenerated) pair of bosons (the phase and amplitude collective modes in Nambu terminology) is formed by the “electrically neutral” ($Q = 0$) massless Goldstone mode and the “Higgs boson” with $Q = 0$:

$$E_1^{(Q=0)} = 0, \quad E_2^{(Q=0)} = 2\bar{\Delta} = \sqrt{8/3}\Delta_0. \quad (11)$$

The other (triply degenerated) pair represents the analog of the charged Higgs bosons in ^{3}He-A with $Q = \pm 2$. These are the so-called clapping modes whose energies are

$$E_1^{(Q=2)} = E_2^{(Q=-2)} = \sqrt{2}\bar{\Delta} = \sqrt{4/3}\Delta_0. \quad (12)$$

4. Superfluid phases in 2+1 films. The same relations (11) and (12) take place for the bosonic collective modes in the quasi two-dimensional superfluid ^{3}He films. There are two possible phases in thin films, the A-phase and the planar phase. Both phases have isotropic gap Δ in the $2D$ case, as distinct from the $3D$ case where such phases are anisotropic with zeroes in the gap, and both have similar spectrum of 12 collective modes: 3 Goldstone bosons and 9 Higgs modes. The energy gaps of bosons are given by Eq. (3), where instead of J there is the corresponding quantum number. This proves that the collective modes obey the Nambu sum rule. The parameters η are determined by the symmetry of the system, but in both cases they get three possible values $\eta = 1$, $\eta = -1$, and $\eta = 0$.

Let us enumerate the modes in the thin film of A-phase, where the symmetry breaking is $SO(2) \otimes SO(3) \otimes \otimes U(1) \to U(1)_Q \otimes SO(2)$, and the bosonic modes are classified in terms of the $U(1)$ charge Q, which is similar to the electric charge in Standard Model. These modes form two pairs of Nambu partners (triply degenerated), with $Q = 0$ and $|Q| = 2$:

$$E_1^{(Q=0)} = 0, \quad E_2^{(Q=0)} = 2\Delta, \quad (13)$$

$$E^{(Q=+2)} = \sqrt{2}\Delta, \ E^{(Q=-2)} = \sqrt{2}\Delta. \quad (14)$$

This spectrum was originally obtained in Ref. [27].

Note that since masses of $Q = +2$ and $Q = -2$ modes are equal, the Nambu sum rule necessarily leads to the definite value of the masses of the “charged” Higgs bosons. Because of the common symmetry breaking scheme in the electroweak theory and in ^{3}He-A we consider the listed above energy gaps as an indication of the existence of the Higgs boson with mass

$$M_{\rm H} = \sqrt{2} M_t. \quad (15)$$

This mass is about 245 GeV.

5. Nambu sum rules in the relativistic models of top-quark Condensation. In this section we consider the Nambu sum rule in the context of the extended NJL model of top-quark condensation. The simplest models of this kind were considered in a number of papers (see, for example, [17, 28, 29]). Here we consider the particular case of the model suggested by Miransky and coauthors in [16]. It involves 6 quarks and has the action of the form

$$S = \int d^4x \Big[\bar{\chi}[i\nabla\gamma]\chi + + \frac{8\pi^2}{N_C \Lambda^2} (\bar{\chi}_{\alpha A,L} \chi_R^{\beta B})(\bar{\chi}_{\bar{\beta}\bar{B},R} \chi_L^{\bar{\alpha}A}) L_{\bar{\alpha}}^{\alpha} R_{\beta}^{\bar{\beta}} I_B^{\bar{B}} \Big]. \quad (16)$$

Here $\chi_{\alpha A}^T = (u, d); (c, s); (t, b)$ is the set of the doublets with the generation index α, Λ is the dimensional parameter, $N_C = 3$. Hermitian matrices L, R, I contain dimensionless coupling constants. It is implied that all eigenvalues of matrices L, R, I are close to each other.

This means that the unknown microscopic theory should have the approximate symmetry, which provides that these values are equal. Small corrections to this equality gives the eigenvalues of L, R, I that only slightly deviate from each other. (After the suitable rescaling Λ plays the role of the cutoff, while the eigenvalues of L, R, I are all close to 1.) The possible origin of this pattern was discussed in [30], where it is suggested that the given NJL model originates from the gauge theory of Lorentz group coupled in an equal way to all existing fermions. The basis of observed quarks corresponds to the diagonal form of L, R, I. We denote $L = \mathrm{diag}(1 + L_{ud}, 1+L_{cs}, 1+L_{tb})$, $R = \mathrm{diag}(1+R_{ud}, 1+R_{cs}, 1+R_{tb})$, $I = \mathrm{diag}(1 + I_{\rm up}, 1 + I_{\rm down})$, and

$$\begin{aligned}
&y_u = L_{ud} + R_{ud} + I_{\rm up}, \quad y_d = L_{ud} + R_{ud} + I_{\rm down},\\
&y_c = L_{cs} + R_{cs} + I_{\rm up}, \quad y_s = L_{cs} + R_{cs} + I_{\rm down},\\
&y_t = L_{tb} + R_{tb} + I_{\rm up}, \quad y_b = L_{tb} + R_{tb} + I_{\rm down},\\
&y_{ud} = L_{ud} + R_{ud} + I_{\rm down}, \quad y_{du} = L_{ud} + R_{ud} + I_{\rm up},\\
&y_{uc} = L_{ud} + R_{cs} + I_{\rm up}, \quad y_{cu} = L_{cs} + R_{ud} + I_{\rm up},\\
&y_{us} = L_{ud} + R_{cs} + I_{\rm down}, \quad y_{su} = L_{cs} + R_{ud} + I_{\rm up},\\
&\dots
\end{aligned} \tag{17}$$

These coupling constants satisfy the relation $y_{q_1q_2} + y_{q_1q_2} = y_{q_1} + y_{q_2}$. As it was mentioned above, it is implied that $|y_q|, |y_{q_1q_2}| \ll 1$. Bosonic spectrum of this model was calculated in one-loop approximation in [15]. It is implied that in vacuum the composite scalar fields $h_q = \bar{q}q$ are condensed for all quarks $q = u, d, c, s, t, b$. The induced quark masses M_q are related to the coupling constants y_q, Λ as $\frac{M_q^2}{\Lambda^2}\log\frac{\Lambda^2}{M_q^2} = y_q$.

As a result we have two excitations in each $q\bar{q}$ channel:

$$M^P_{q\bar{q}} = 0; \quad M^S_{q\bar{q}} = 2M_q \tag{18}$$

and four excitations (i.e. two doubly degenerated excitations) in each $q_1\bar{q}_2$ channel. We denote the masses $M^{\pm}_{q_1q_2}, M^{\pm}_{q_2q_1}$ for $q_1, q_2 = u, d, c, s, t, b$. They are given by

$$\begin{aligned}
M^2_{q_1q_2} = M^2_{q_1} + M^2_{q_2} \pm\\
\pm \sqrt{(M^2_{q_2} - M^2_{q_1})^2\zeta^2_{q_1q_2} + 4M^2_{q_1}M^2_{q_2}}
\end{aligned} \tag{19}$$

with

$$\zeta_{q_1q_2} = \frac{2y_{q_1q_2} - y_{q_2} - y_{q_1}}{y_{q_2} - y_{q_1}} = \zeta_{q_2q_1} \tag{20}$$

(parameters $y_q, y_{q_1q_2}$ are listed in Eq. (17)).

One can see, that the Nambu sum rule holds in the form

$$\begin{aligned}
&[M^+_{q_1\bar{q}_2}]^2 + [M^-_{q_1\bar{q}_2}]^2 + [M^+_{q_2\bar{q}_1}]^2 + [M^-_{q_2\bar{q}_1}]^2 \approx\\
&\approx 4[M^2_{q_1} + M^2_{q_2}], \quad (q_1 \neq q_2);\\
&[M^P_{q\bar{q}}]^2 + [M^S_{q\bar{q}}]^2 \approx 4M_q^2.
\end{aligned} \tag{21}$$

In the case when the t-quark contributes to the formation of the given scalar excitation, its mass dominates, and in each channel ($t\bar{t}, t\bar{c}$, ...) we come to the relation

$$\sum M^2_{{\rm H},i} \approx 4M_t^2\,, \tag{22}$$

where the sum is over the scalar excitations in the given channel.

The symmetry breaking pattern of the considered model is $U_{L,1}(2) \otimes U(2)_{L,2} \otimes U(2)_{L,3} \otimes U(1)_u \otimes \dots \otimes U(1)_b \to U(1)_u \otimes \dots \otimes U(1)_t \otimes U(1)_b$. Among the mentioned Higgs bosons there are 12 Goldstone bosons that are exactly massless (in the channels $t(1 \pm \gamma^5)\bar{b}, t\gamma^5\bar{t}, c(1 \pm \gamma^5)\bar{s}, c\gamma^5\bar{c}, u(1 \pm \gamma^5)\bar{d}, u\gamma^5\bar{u}, b\gamma^5\bar{b}, s\gamma^5\bar{s}, d\gamma^5\bar{d}$). There are Higgs bosons with the masses of the order of the t-quark mass ($t(1 \pm \gamma^5)\bar{b}, t\bar{t}, t(1 \pm \gamma^5)\bar{s}, t\gamma^5\bar{c}, t(1 \pm \gamma^5)\bar{d}, t\gamma^5\bar{u}$). The other Higgs bosons have masses much smaller than the t-quark mass. A lot of physics is to be added in order to make this model realistic. In particular, extra light Higgs bosons should be provided with the masses of the order of M_t.

6. Nambu sum rules in dense quark matter. In dense quark matter with $\mu > \Lambda_{\rm QCD}$ there may appear several phases with different diquark condensates. For example, in the color-flavor locking phase (CFL) in the framework of the phenomenological model with three massless quarks u, d, s the condensate has the form [31, 32]

$$\langle[\psi^i_\alpha]^t i\gamma^2\gamma^0\gamma^5\psi^j_\beta\rangle \sim \Phi^I_J\,\epsilon_{\alpha\beta J}\epsilon^{ijI} \sim (\beta V)^{1/2}C\ \epsilon_{\alpha\beta I}\epsilon^{ijI}. \tag{23}$$

The symmetry breaking pattern is $SU(3)_L \otimes SU(3)_R \otimes SU(3)_F \otimes U(1)_A \otimes U(1)_B \to SU(3)_{CF}$. There are 36 scalar and pseudoscalar fluctuations of Φ around this condensate [33]). Among them there are $9 + 9$ massless Goldstone modes. The remaining $9 + 9$ Higgs modes contain two octets of the traceless modes and two singlet trace modes. The quark excitations also form singlets and octets. There are two fermionic gaps (for the octet and for the singlet) $\Delta_1 = 2\Delta_8$ (Sect. 5.1.2. of [32]). The scalar singlet and octet masses are $M_1 = 2\Delta_1, M_8 = 2\Delta_8$. This may be derived from the results presented in [34, 35].

We already mentioned in the introduction, that in the Hadronic phase the NJL approximation leads to the Nambu sum rule in the trivial form $M_\sigma = 2M_{\rm quark}$. However, at nonzero $\mu \ll M_{\rm quark}$ the Nambu sum rule in the nontrivial form appears for the diquark states. Namely, the following values of the masses of the diquarks are presented in Eq. (46) of [33]:

$$M^2_\Delta = (2M_{\rm quark} - 2\mu)^2; \quad M^2_{\Delta^*} = (2M_{\rm quark} + 2\mu)^2. \tag{24}$$

So that

$$M_{\Delta}^2 + M_{\Delta^*}^2 \approx 2 \cdot 4M_{\text{quark}}^2 \,\text{at}\, \mu \ll M_{\text{quark}}. \quad (25)$$

(Here Δ is the diquark while Δ^* is the antidiquark.)

7. Veltman identity. In the case of the single Higgs boson and in the absence of the gauge fields the quadratic divergences in the mass of the Higgs boson vanish if $3M_{\rm H}^2 = 4\sum_f M_f^2$ (see [36–40]). Here $M_{\rm H}$ is the scalar boson mass, while the sum is over the fermions of the model. For the model with triply degenerated quarks, this relation is reduced to $M_{\rm H}^2 = 4\sum_f M_f^2$. It looks similar to Eq. (1). Nevertheless, their origins are different. This follows from the fact that the cancellation of quadratic divergences relies on the identity $N_C = 3$ while the Nambu sum rule Eq. (1) in the models considered above works for any number of fermion colors. Besides, the number of the components of the scalar is relevant for Veltman relation. Therefore, its nature differs from the nature of the Nambu sum rule.

8. Conclusions and discussion. In this paper we consider the bosonic spectrum of various NJL models: from the condensed matter models of superfluidity to the relativistic models of top quark condensation. In each case the Nambu sum rule takes place that relates the masses (or, energy gaps) of the bosonic excitations with the mass (energy gap) of the heaviest fermion that contributes to the formation of the given composite scalar boson. (It is implied that its mass is essentially larger than the masses of the other fermions that contribute to the given composite boson.)

We suppose that the top quark contributes to the formation of the composite Higgs bosons. There may also appear the other composite Higgs bosons, whose formation is not related to the top quark. These Higgs bosons would be light. Since such states are not observed, their formation is to be suppressed. Some physics is to be added in order to provide this. For example, these bosons may be eaten by some extra gauge fields that acquire masses due to the Higgs mechanism.

The results presented in this paper belong to the NJL-like models considered in weak coupling. In any realistic models this is only an approximation. In QCD the use of the NJL approximation is limited at low energies, in particular, because confinement is not taken into account. However, the unknown theory, whose low energy approximation may have the form of the NJL model, should provide chiral symmetry breaking but cannot be confining. (Otherwise all quarks would be confined to the regions of space smaller than TeV^{-1}.) This justifies the use of this technique. Besides, Eq. (1) being derived using the NJL approximation does not contain the parameters of the NJL model: neither the coupling entering the four-fermion terms nor the cutoff. As Nambu noticed in [1], his sum rule may work better than the NJL approximation itself.

The authors kindly acknowledge useful remarks by J.D. Bjorken, D.I. Diakonov, T.W.B. Kibble, F.R. Klinkhamer, S. Nussinov, M.I. Polikarpov, A.M. Polyakov, M. Shaposhnikov, V.I. Shevchenko, T. Vachaspati, M.I. Vysotsky, V.I.Żakharov. The authors are very much obliged to V.B. Gavrilov and M.V. Danilov for the explanation of the experimental situation with the search of new particles at the LHC. This work was partly supported by RFBR grant # 11-02-01227, by the Federal Special-Purpose Programme "Human Capital" of the Russian Ministry of Science and Education, by Federal Special-Purpose Programme 07.514.12.4028. GEV acknowledges a financial support of the Academy of Finland and its COE program, and the EU FP7 program (# 228464 Microkelvin).

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