Pis'ma v ZhETF, vol. 97, iss. 10, pp. 654 – 656

Recovering partial conservation of axial current in diffractive neutrino scattering

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Submitted 10 April 2013

A model of diffractive neutrino scattering is formulated in terms of the chiral hadronic current which is conserved in the limit of vanishing pion mass. This current has the correct singularity structure and, naturally, does not lead to contradictions with a partial conservation of the axial current (PCAC). In that respect we differ from earlier work in the literature, where a breakdown of PCAC had been reported. We show that such a breakdown of PCAC is an artifact of the hadronic current non-conservation in the model developed there.

DOI: 10.7868/S0370274X13100020

1. Introduction. This communication is motivated by the publication [1] entitled "Breakdown of partial conservation of axial current in diffractive neutrino scattering". The analysis [1] is based on a specific model of diffractive neutrino scattering suggested earlier in Ref. [2]. Within this model interactions of high-energy neutrino with the nucleon or nuclear target

$$\nu + N \to l + X \tag{1}$$

in the axial channel are mediated by pions and $a_1(J^{PC} = 1^{++})$ mesons. Corresponding matrix elements of the axial hadronic current, A_μ, are expanded over π and a_1 components. For the specific final state $|X\rangle = |\pi N\rangle$ this expansion, with certain reservations, leads to the requirement[2)]

$$\sigma(\pi N \to \pi N) = \sigma(\pi N \to a_1 N). \tag{2}$$

The latter equality is considered in [1] as an indispensable property of a partial conservation of the axial current (PCAC). In [1] it was found that Eq. (2) cannot be reconciled with experimental data and the breakdown of PCAC was claimed.

Below we show that Eq. (2) does not follow from PCAC and cannot be a basis for radical questioning of PCAC.

2. The π-a_1-model and a_1-dominance. Below in Sects. 2 and 3 we briefly sketch the derivation of Eq. (2). For more details see [2, 3].

Within the $\pi - a_1$-model developed in [2] and exploited in [1, 3] the matrix element of the hadronic axial current

$$A_\mu = \langle X|a_\mu(0)|N\rangle \tag{3}$$

entering the amplitude

$$T(\nu N \to lX) = \frac{G_{\rm F}}{\sqrt{2}} L_\mu (V_\mu + A_\mu), \tag{4}$$

of the process (1) is saturated by the two lowest hadronic states, π and a_1 mesons,

$$A_\mu = f_\pi \frac{q_\mu}{q^2 - \mu^2} T(\pi N \to X) + \\ + f_a \frac{g_{\mu\nu} - q_\mu q_\nu / M^2}{q^2 - M^2} T_\nu(a_1 N \to X). \tag{5}$$

Hereafter, μ stands for the pion mass and M – for the a_1.

One comment on Eq. (5) is in order. This equation provides the off-mass-shell extrapolation of physical amplitudes $\pi N \to X$ and $a_1 N \to X$. Far from the π-, a_1-pole the representation (5) becomes rather uncertain at least for the a_1-exchange which is always very far the mass shell in the reaction (1). To minimize uncertainties in Sect. 5 we make use of the symmetry property of the problem. The latter turns out to be crucial for (in) validity of Eq. (2).

The leptonic current

$$L_\mu = \bar{u}(k')\gamma_\mu(1+\gamma_5)u(k) \tag{6}$$

is purely transversal (we neglected the lepton mass, $m_l = 0$, and introduced $q = k - k'$),

$$q_\mu L_\mu = 0. \tag{7}$$

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2) In addition to the longitudinal a_1, in Ref. [3], contributions of the ρ–π-state and higher axial excitations were also considered.

From Eq. (7) it follows that the pion pole in Eq. (5) does not contribute to the νp-scattering cross-section. Then, the longitudinal component of the differential cross section of the process (1) within the $\pi{-}a_1$-model of Ref. [2] (see also [3]) is dominated by the a_1 contribution,

$$\frac{d^2\sigma(\nu p \to lX)}{dQ^2 d\nu} \propto \frac{f_a^2 Q^2}{M^4}\sigma^L(a_1 p \to X;\, Q^2), \tag{8}$$

where we denoted $Q^2 = -q^2$ [3)].

Adler's observation is that the above differential cross section (8) at $Q^2 \to 0$ is expressible also in terms of the on-shell pion-nucleon cross section $\sigma(\pi N \to X)$ [4].

3. Pions and Adler's theorem. In Ref. [4] it was noticed that at $Q^2 = 0$

$$L_\mu \propto q_\mu. \tag{9}$$

Consequently,

$$T(\nu p \to lX) \propto q_\mu A_\mu. \tag{10}$$

The constraint of PCAC implies [5]

$$q_\mu a_\mu = f_\pi \mu^2 \varphi, \tag{11}$$

where f_π is the pion decay constant, φ is the pion field operator, and a_μ is the axial current operator (see Eq. (3)). Therefore,

$$|q_\mu A_\mu|^2 = \frac{f_\pi^2}{\sqrt{\nu^2 + Q^2}}\sigma(\pi N \to X) \tag{12}$$

and at $Q^2 \to 0$

$$\frac{d^2\sigma(\nu p \to lX)}{dQ^2 d\nu} \propto f_\pi^2 \sigma(\pi N \to X). \tag{13}$$

In Ref. [2] (see also [3]) from comparison of Eqs. (13) and (8) supplemented with Weinberg sum rules [6] and certain assumptions on the off-shell properties of hadronic cross sections Eq. (2) was obtained.

4. The $\boldsymbol{\pi{-}a_1}$-model – the model with built-in current non-conservation. In [1] it was noticed that the cross sections $\sigma(\pi N \to \pi N)$ and $\sigma(\pi N \to a_1 N)$, where N represents the target nucleon/nucleus, have different dependence on the collision energy as well as very different dependence on the nuclear opacity. The principal conclusion of Ref. [1] is that the PCAC hypothesis is in conflict with well established properties of high-energy hadronic amplitudes.

However, it is quite clear that the basic expansion (5) has at least one serious flaw. The current (5) is not conserved. It is not conserved even "partially". Consequences are obvious. The requirement of PCAC (11) supplemented with the equation of motion of the pseudoscalar field φ applied to the matrix element (3) implies

$$q_\mu A_\mu = f_\pi \frac{\mu^2}{q^2 - \mu^2} T(\pi N \to X) \tag{14}$$

and for A_μ defined by Eq. (5) results in

$$f_\pi T(\pi N \to X) = f_a M^{-2} q_\nu T_\nu(a_1 N \to X). \tag{15}$$

Eq. (15) like its counterpart (2) can hardly be reconciled with the experimental data.

5. Introducing the chiral hadronic current $\boldsymbol{A_\mu^\chi}$. To meet the requirement of chiral symmetry the matrix element of the axial hadronic current in the basis of π, a_1-states should be constructed as follows

$$A_\mu^\chi = g_A \frac{M^2}{q^2 - M^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2 - \mu^2}\right)\mathcal{T}_\nu(q^2, ...). \tag{16}$$

Below we keep in $\mathcal{T}_\nu(q^2, ...)$ only one argument. The dependence on additional variables arises in specific problems for particular final states.

Two poles in (16) correspond to both the pion and the a_1-meson. At $q^2 \approx \mu^2$

$$A_\mu^\chi = g_A\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2 - \mu^2}\right)\mathcal{T}_\nu(q^2) \tag{17}$$

and for $q^2 \approx M^2$

$$A_\mu^\chi = g_A \frac{M^2}{q^2 - M^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M^2}\right)\mathcal{T}_\nu(q^2), \tag{18}$$

as it should be. In the limit $\mu \to 0$ the current (16) is conserved,

$$q_\mu A_\mu^\chi = 0. \tag{19}$$

The current conservation is, thus, a purely kinematical effect. The dynamics is concentrated in the invariant amplitude $\mathcal{T}_\nu(q^2)$, which is controlled by the QCD. The latter implies that in the particular two-channel model one and the same function $\mathcal{T}_\nu(q^2)$ describes neutrino scattering in a wide range of virtualities q^2 including both π and a_1 poles, where $\mathcal{T}_\nu(q^2)$ satisfies the following on-shell conditions

$$g_A q_\nu \mathcal{T}_\nu(\mu^2) = f_\pi T(\pi N \to X), \tag{20}$$

$$g_A M^2 \mathcal{T}_\nu(M^2) = f_a T_\nu(a_1 N \to X). \tag{21}$$

The product of leptonic and hadronic currents – recall that $q_\mu L_\mu = 0$, – is as follows

$$T(\nu N \to lX) \sim L_\mu A_\mu^\chi = g_A L_\mu \mathcal{T}_\mu(q^2). \tag{22}$$

[3)] We are interested in the limit $Q^2 \to 0$, where $\sigma^L(Q^2)$ is singular. Recall that in the axial channel $\sigma^L = |\epsilon_\mu^L A_\mu|^2/\sqrt{Q^2+\nu^2}$ and the longitudinal polarization vector is defined as $\epsilon^L = (\sqrt{\nu^2+Q^2}, 0, 0, \nu)/\sqrt{Q^2}$ with $q = (\nu, 0, 0, \sqrt{\nu^2+Q^2})$.

Here, $\mathcal{T}_\mu(q^2)$ is specified by Eqs. (20, 21). Comparison of Eq. (22) at $q^2 \to 0$ with Adler's amplitude dictated by PCAC leads simply to the identity (20) and does not yield any new relation with $T(a_1 N \to X)$ because now the off-shell extrapolations of amplitudes $T(\pi N \to X)$ and $T(a_1 N \to X)$ are interrelated by the current conservation condition. Indeed, the chiral current A_μ^χ can be represented as a superposition of π and a_1 poles. In the limit of $\mu^2 \to 0$

$$A_\mu^\chi = g_A \frac{q_\mu q_\nu}{q^2 - \mu^2} \mathcal{T}_\nu(q^2) + + g_A \frac{M^2}{q^2 - M^2} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M^2} \right) \mathcal{T}_\nu(q^2). \tag{23}$$

Eq. (5) follows from Eq. (23) if $\mathcal{T}_\nu(q^2)$ is substituted with two different on-shell amplitudes. Evidently, this operation breaks down PCAC.

6. Summary and Conclusions. The breakdown of PCAC claimed in [1] has been derived from the $\pi - a_1$-model of Ref. [2]. An important ingredient of the model is the matrix element of the axial current (5) which is saturated by the lowest axial hadronic states, π and a_1. In the kinematical domain of the reaction (1) the exchanged a_1-meson is always very far from the mass shell. Postulated in [2, 3] the off-shell extrapolation of (5) has serious flaw, the current (5) is not conserved. The model [2] simply does not respect the chiral symmetry. No wonder, the PCAC in the model is badly broken.

We introduced the axial hadronic current A_μ^χ (see Eq. (16)). This current is conserved in the chiral limit. Also it has a correct $\pi - a_1$-pole structure. Naturally, this current does not lead to any troubles with PCAC.

Thanks are due to N.N. Nikolaev for careful reading the manuscript. The work was supported in part by the RFBR grants # 11-02-00441 and 12-02-00193.

1. B. Z. Kopeliovich, I. K. Potashnikova, I. Schmidt, and M. Siddikov, Phys. Rev. C **84**, 024608 (2011).
2. C. A. Piketty and L. Stodolsky, Nucl. Phys. B **15**, 571 (1970).
3. B. Z. Kopeliovich and P. Marage, Int. J. Mod. Phys. A **8**, 1513 (1993); A. A. Belkov and B. Z. Kopeliovich, Sov. J. Nucl. Phys. **46**, 499 (1987).
4. S. Adler, Phys. Rev. B **135**, 963 (1964).
5. Y. Nambu, Phys. Rev. Lett. **4**, 380 (1960); M. Gell-Mann and M. Levy, Nuovo Cimento **17**, 705 (1960).
6. S. Weinberg, Phys. Rev. Lett. **18**, 507 (1967).