MAGNETOSPECTROSCOPY OF 2D ELECTRON GAS: CUSPS IN EMISSION SPECTRA AND COULOMB GAPS

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A theory of photoluminescence due to radiative trapping of 2D electrons from a confinement layer on neutral impurity centers is presented. In the dependence of the emission band position on the filling factor ν there are cusps at fractional values $\nu = p/q$, these cusps are closely connected to cusps in the ground state energy of interacting 2D electrons; the latter cusps arising due to formation of incompressible liquids at these points. The effect of the finite state interaction on the cusp shape is investigated, and the possibility for determining Coulomb gaps from cusp strengths is discussed.

The incompressible two-dimensional (2D) electron liquid [1] which manifests itself in the fractional quantum Hall effect [2] is one of the most remarkable objects of the modern solid state physics. Surveys may be found in [3–5]. However, the number of experimental methods permitting to measure parameters of the 2D liquid is highly restricted. Recently spectroscopic observations of the 2D liquid have been reported [6–9]. In particular, numerous features in the dependence of the position of photoluminescence band corresponding to radiative trapping of 2D carriers by neutral acceptors on the filling factor ν have been observed (see [6,7] and references therein). These features may be attributed to the electron-electron interaction and can be described in terms of steps. Theoretical papers on the photoluminescence in a strong magnetic field which are not based on the mean field approximation, and which accordingly may be applied for describing the quantum liquid are not numerous yet. The papers [10,11] deal with the effect of spin polarization on optical spectra, and the papers [10,12] with Auger processes. It has been shown [12] that under some conditions the energy Δ_r of the magneto-roton minimum [13] may be found from the shape of the impurity emission spectrum. We show in this paper that the gap Δ for quasielectron-hole pairs [1] to be created may be found from the dependence of the center-of-gravity $\overline{\omega}$ of the emission band corresponding to trapping of electrons from the 2D liquid by neutral impurity centers on ν. According to the theory for fractional $\nu = p/q \equiv \nu_{pq}$, p is an integer, q is an odd integer, in the dependence $\overline{\omega}(\nu)$ one can expect cusps and discontinuities in the derivative, $d\overline{\omega}/d\nu$, in these points are connected to the gaps, Δ, in them. The fact that the effect of the impurity centers on 2D electrons is weak in the initial state is a distinctive feature of this type of transitions, apparently it can be neglected. It is this assumption that is of crucial importance for the procedure of determining Δ from experimental data on cusps strengths discussed in what follows.

The model used below is as follows. In the initial state the electron density is homogeneous over the confinement layer and an impurity center by which an electron is trapped is neutral. The distance h of the impurity center from the confinement layer and the magnetic length $l = \sqrt{c/eH}$ ($\hbar$=1) exceed considerably both the layer width and the center radius r_{imp}. These both quantities are neglected as well as the initial state interaction of the impurity center with 2D electrons. The potential of the impurity center in the final state is a Coulomb one. The temperature $T = 0$, in the initial state the system of electrons is on its ground level i, if this level is degenerate all the states belonging to it are equally populated. A magnetic field is strong, $\omega_c \gg \epsilon_C = e^2/\kappa l$, here ω_c is the cyclotron frequency, ϵ_C is the Coulomb energy, κ is the dielectric constant. Mixing of different Landau levels is disregarded. The quantum transition occurs at the point $\vec{r}_0$ which is the point of the 2D layer closest to the impurity center. This recombination scheme is especially well suited for describing the trapping of electrons from a deep quantum well by neutral shallow acceptors residing near it. Numerical calculations are performed in the spherical geometry [14]. Therefore, all equations are written for a homogeneous system with a finite number of particles.

The normalized first moment of the emission spectrum (i.e., divided by its zero moment) determines the position of the center-of-gravity. It equals

$$\overline{\omega} = E_i - \langle H_f \rangle_{av} \quad , \tag{1}$$

where E_i is the energy of the i level, i.e., of the ground state of N interacting particles. H_f is the Hamiltonian of the system in its final state, i.e., the Hamiltonian of $(N-1)$ interacting particles $\vec{r}_1, \ldots, \vec{r}_{N-1}$ in the 2D layer subjected to repulsive Coulomb center at the point $\vec{r}_I$ where the impurity center resides, see [12]. The symbol $\langle \ldots \rangle_{av}$ stands for averaging over wave functions $\Psi_\alpha(\vec{r}_1 \ldots \vec{r}_N)$ of all the states α belonging to the i level under the condition $\vec{r}_N = \vec{r}_0$. For the energy of the "excess" electron existing in the initial state the difference of the ground Landau level in the conduction band and the impurity level is chosen as the reference point. Hence, $\overline{\omega}$ only includes the Coulomb interaction energy.

It is convenient to introduce the pair correlation function

$$g(|\vec{\rho}_N - \vec{\rho}_{N-1}|) = (N-1)Al^2 \sum_\alpha \int |\Psi_\alpha(\vec{r}_1 \ldots \vec{r}_N)|^2 d\vec{r}_1 \ldots d\vec{r}_{N-2}/g_i \quad ,$$

$$\int g(\rho) d\vec{\rho} = N - 1 \quad , \tag{2}$$

here g_i is the multiplicity of the i level, A is the area of the 2D layer, $\vec{\rho} = \vec{r}/l$. In the thermodynamic limit $g(r) \to \nu/2\pi$ at $\rho \to \infty$. The function g depends only on the difference $\rho = |\vec{\rho}_N - \vec{\rho}_{N-1}|$ since due to summation over α in (2) it possesses the full symmetry of the system prior to the optical transition when the impurity center is neutral and does not perturb 2D electrons. For the same reason all integrals entering $\langle H_f \rangle_{av}$ may be expressed through the function g. The Hamiltonian H_f equals

$$H_f = \sum_{jk=1}^{N-1} V(|\vec{\rho}_j - \vec{\rho}_k|) - \sum_{j=1}^{N-1} V(|\vec{\rho}_j - \vec{\rho}_I|) \quad , \; j < k \quad , \tag{3}$$

here $V(\rho)$ is the electron-electron interaction potential. The potential of the positive background ensuring the electric neutrality of the system is omitted since it makes no contribution to energy differences which are our final results. As

$$\frac{N}{2}\int V(\rho)g(\rho)d\vec{\rho} = E_i \quad , \tag{4}$$

the eq.(1) may be rewritten as

$$\begin{aligned}\overline{\omega} &= \int\{V(|\vec{\rho}_0-\vec{\rho}\,|)-V(|\vec{\rho}_I-\vec{\rho}\,|)\}g(|\vec{\rho}_0-\vec{\rho}\,|)d\vec{\rho} \\ &= 2E_i/N - \int V(|\vec{\rho}_I-\vec{\rho}\,|)g(|\vec{\rho}_0-\vec{\rho}\,|)d\vec{\rho} \quad .\end{aligned} \tag{5}$$

The integration in (4) and (5) is performed over 2D layer.

Let us consider first short range potential $V(r)$. In this case the second term in (5) may be omitted when the distance $h = |\vec{r}_I - \vec{r}_0|$ of the point $\vec{r}_I$ from the confinement layer is larger than the potential radius. When one takes into account that $\partial E_i/\partial N = \mu$, where μ is the chemical potential, and $\mu(\nu)$ is a discontinuous function of ν at $\nu = \nu_{pq}$ [15]

$$\delta\mu \equiv \mu(\nu_{pq}+0) - \mu(\nu_{pq}-0) = q\Delta \quad , \tag{6}$$

it is easy to obtain from (5)

$$\Delta = (\nu/2q)\,\delta\{\partial\overline{\omega}/\partial\nu\} \quad . \tag{7}$$

Therefore, the function $\overline{\omega}$ must show cusps in the points ω_{pq}, the discontinuity in the derivative permits to find a gap. It is clear from (5) that these cusps are closely connected to cusps of the function $E_i(\nu)$ [15]. A magnitude of $\delta\{\partial\overline{\omega}/\partial N\}$ will be termed below a cusp strength.

The long range Coulomb behavior of $V(r)$ does not permit to disregard the second term in (5), the both terms contribute to the cusp strength. In fact, the singularities of $\overline{\omega}(\nu)$ appear due to a nonanalytical behavior of g treated as a function of ν. The previous computations imply [16,17] that the behavior $g(\rho) = g_1(\rho) + |\nu - \nu_{pq}|g_2(\rho)$ near the points $\nu = \nu_{pq}$, here g_1 and g_2 are smooth functions of ν. Such a behavior results in cusps in the both integrals of eq.(5). If $h \to 0$, then $\overline{\omega} = 0$. This means that the intrinsic and extrinsic terms cancel one another and the interaction does not affect the position of the emission band. For intermediate values of h the extrinsic contribution to the cusp strength may have a considerable magnitude, but is less than the intrinsic contribution. The general expression for the cusp strength is

$$\delta\{\partial\overline{\omega}/\partial\nu\} = 2q\Delta/\nu - \int V(|\vec{\rho}_I-\vec{\rho}|)\,\delta\{\partial g(|\vec{\rho}_0-\vec{\rho}|)/\partial\nu\}d\vec{\rho} \quad . \tag{8}$$

In the expansion of eq.(8) at $h \gg l$ the term $\sim h^{-1}$ is absent due to normalization condition (2), at $h \ll l$ the expansion starts with the term $\sim h^2$.

The function $g(\rho)$ has been found by us in the spherical geometry [14,18] for three values of ν: $\nu = 1/3$ and in the two neighbour points. The function $\delta\{\partial g(\rho)/\partial\nu\}$ at $\nu = 1/3$ found by means of these data is shown in Fig.1a. It exhibits the ρ^2 behavior at $\rho \ll 1$ and one strong oscillation. Oscillations at $\rho > 5$ decrease with increasing N. This function has been used to calculate in the plane geometry the last term in (8) as a function of h, it is shown in Fig.1b. In the limit cases

$$\delta\{\partial\overline{\omega}/\partial\nu\} \approx \begin{cases} 18\Delta - 37\epsilon_c(l/h)^3 & \text{for} \quad h \gg l \ , \\ 0.77(h/l)^2 & \text{for} \quad h \ll l \ , \end{cases} \tag{9}$$

the coefficients in (9) have been found by extrapolation $1/N \to 0$. A rapid decrease of the extrinsic term with increasing h may considerably facilitate determining the gaps from experimental data on cusp strengths.

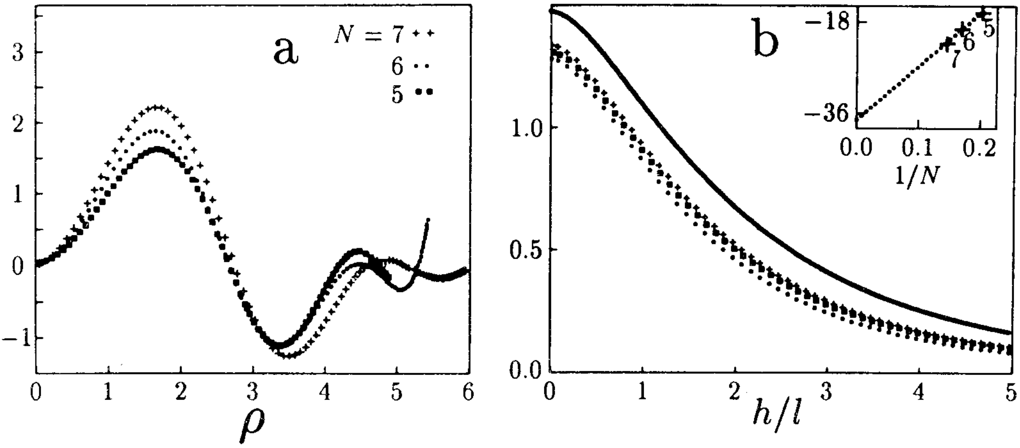

Fig.1. a). Function $\delta\{\partial g(\rho)/\partial\nu\}$ found in the spherical geometry for different number N of particles; $\nu = 1/3$.

b). Extrinsic contribution to the cusp strength (second term in eq.(8)) vs. distance h, $\nu = 1/3$. The heavy line is obtained by extrapolation $1/N \to 0$. Energy is in ϵ_C units. Inset: magnitude of the coefficient at $(l/h)^3$ in (9) found for different number of particles.

In Fig.2 and 3 are shown the first moment of the spectrum and the width γ of the emission band versus the integer parameter $2S$, $S = (R/l)^2$, R is a radius of a sphere, $N = 6$. The width γ is defined as $\gamma^2 = \langle\omega^2 - \overline{\omega}^2\rangle$. The impurity center is inside the sphere at a distance h^* from its north pole [12]. The dependence $\overline{\omega}(2S)$ is shown in Fig.2 for three values of h^*/R, it is natural to put the value $h^* = R$ into correspondence with $h = \infty$ in the plane geometry. Three cusps are seen on a smooth background, all of them correspond to nondegenerate i states. The cusp $2S = 15$ corresponds to $\nu = 1/3$ (according to equation $2S = (N-1)/\nu$ [18]), the cusp $2S = 9$ to $\nu = 2/3$ (according to charge symmetry relation: $\nu \to (1-\nu)$, $N \to (2S+1) - N$), the cusp $2S = 11$ has no definite assignment. The cusps $\nu = 1/3$ for $N = 7$ are shown for comparison. The cusp strength increases with h^* in accordance with (9). In Fig.3 $\gamma(2S)$ is shown for three values of h^*/R. It is seen that γ strongly depends on S, to all the cusps minima of γ correspond.

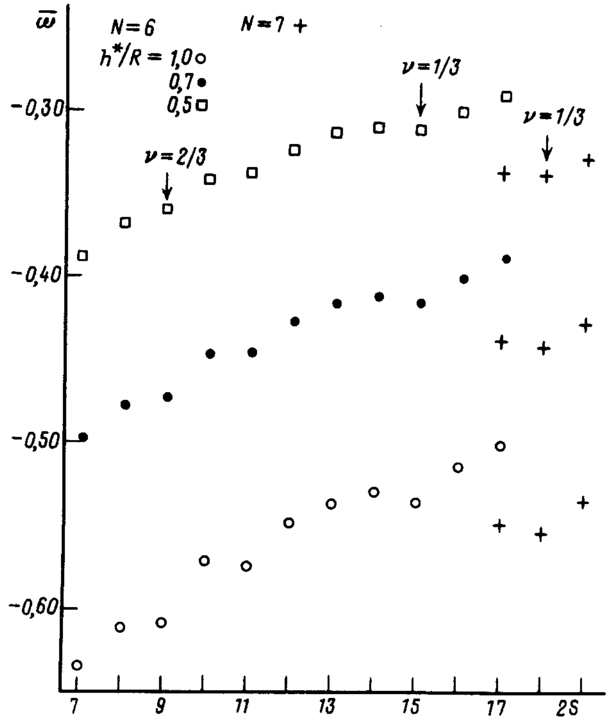

Fig.2. Dependence of the center-of-gravity $\overline{\omega}$ of the emission spectrum on $2S$. Cusps correspond to nondegenerate states. Calculations were performed at $N=6$ for three values of the parameter h^*/R. The $\nu=1/3$ cusp for $N=7$ is also shown. Energy $\overline{\omega}$ is in ϵ_C units.

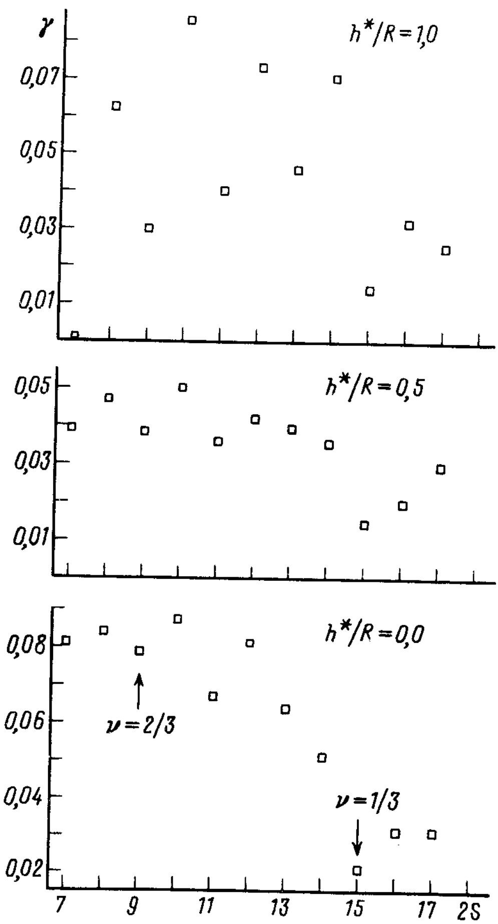

Fig.3. Width γ of the emission spectrum vs. $2S$ at $N=6$ for three values of h^*/R. γ is in ϵ_C units.

The larger the distance of an impurity center from the 2D layer is, the deeper these minima are.

The basic assumption used above is that an impurity center does not affect 2D electrons in the initial state. This interaction is small as compared to ϵ_C in the parameter r_{imp}/h. Nevertheless, it may prove to be important since it reduces the symmetry and lifts up the degeneracy of the ground level. This may result in discontinuities of the function $\overline{\omega}(\nu)$ (and not only of its derivative) having the scale of Δ_r, for examples see [12]. The actual effect of the initial state interaction on the optical spectrum depends on the magnitudes of T, h, r_{imp}, the discontinuities must be smeared by disorder and also by the nonequilibrium distribution of photoproduced charge carriers. The recombination scheme used above imposes some restrictions on the magnitude of h. For a shallow acceptor near a deep quantum well the appropriate condition is $h \ll l^2/r_{imp}$. The dependence of the shape of $\overline{\omega}(\nu)$ curves on the recombination scheme must be also investigated.

For the discussion of experimental data from the standpoint of this paper see the next paper in this issue [19].

In conclusion, gaps in the energy spectrum of the incompressible 2D liquid may be found from cusps of the function $\overline{\omega}(\nu)$ for radiative trapping of 2D electrons on neutral impurities. Due to a rapid decrease of the extrinsic contribution to the cusp strength with h this method may be hopefully efficient for actual distances of impurity centers from the confinement layer.

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