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On the nonintegrability of the free surface hydrodynamics[1)]

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We study integrability of the compact 1-D Zakharov equation. The results of numerical experiments on multiple collisions of breathers (which correspond to envelope solitons in the NLSE approximation) show that collisions are not pure elastic. Also we analyse the amplitude of six-wave interactions for compact 1-D Zakharov equation. It was found that six-wave amplitude is not canceled for this equation. Thus, 1-D Zakharov equation is not integrable.

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1. Introduction. The work described here is motivated by remarkable fact regarding two-dimensional free surface hydrodynamics – four-wave interaction coefficient vanishes on the resonant manifold

$$k + k_1 = k_2 + k_3,$$
$$\omega_k + \omega_{k_1} = \omega_{k_2} + \omega_{k_3}.$$

This cancellation was derived in [1] and brought the hypothesis of integrability of 2-D free surface hydrodynamics. Also the cancellation allows to consider surface waves moving in the same direction only. Namely, if initial state consists of such waves, evolution equation keeps this property. In this article we study the problem of integrability in more details.

So, we consider two-dimensional potential flow of an ideal incompressible fluid with a free surface in a gravity field fluid which is described by the following set of equations:

$$\phi_{xx} + \phi_{zz} = 0 \qquad (\phi_z \to 0,\ z \to -\infty),$$
$$\eta_t + \eta_x\phi_x = \phi_z\Big|_{z=\eta},$$
$$\phi_t + \frac{1}{2}(\phi_x^2 + \phi_z^2) + g\eta = 0\Big|_{z=\eta};$$

here $\eta(x,t)$ is the shape of a surface, $\phi(x,z,t)$ is a potential function of the flow, and g is a gravitational acceleration. As was shown in [2] these equations are Hamiltonian with respect to variables $\eta(x,t)$ and $\psi(x,t) = \phi(x,z,t)\Big|_{z=\eta}$,

$$\frac{\partial\psi}{\partial t} = -\frac{\delta H}{\delta\eta}, \qquad \frac{\partial\eta}{\partial t} = \frac{\delta H}{\delta\psi}.$$

Here $H = K + U$ is the total energy of the fluid with the following kinetic and potential energy terms:

$$K = \frac{1}{2}\int dx\int_{-\infty}^{\eta} v^2 dz, \qquad U = \frac{g}{2}\int \eta^2 dx.$$

Hamiltonian can be expanded in infinite series of characteristic wave steepness $k\eta_k \ll 1$ and we consider this series up to the fourth order:

$$H = \frac{1}{2}\int g\eta^2 + \psi\hat{k}\psi dx - \frac{1}{2}\int\{(\hat{k}\psi)^2 - (\psi_x)^2\}\eta dx + \\ + \frac{1}{2}\int\{\psi_{xx}\eta^2\hat{k}\psi + \psi\hat{k}[\eta\hat{k}(\eta\hat{k}\psi)]\}dx + \ldots \tag{1}$$

Applying canonical transformation along with introducing normal complex variable $b(x,t)$ Hamiltonian (1) transforms to the equivalent compact form:

$$H = \int b^*\hat{\omega}_k b dx + \\ + \frac{1}{2}\int |b'|^2\left[\frac{i}{2}(bb'^* - b^*b') - \hat{k}|b|^2\right]dx. \tag{2}$$

Here $b' = \partial b/\partial x$, $\omega_k = \sqrt{gk}$, and $\hat{k}$ – modulus k operator. All the details of this transformation can be found in [3].

Corresponding equation of motion is the following:

[1)]See Supplemental material for this paper on JETPL Letters site: www.jetpletters.ac.ru, v. 98, iss. 1.

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$$i\frac{\partial b}{\partial t} = \hat{\omega}_k b + \frac{i}{4}\hat{P}^+ \left[b^*\frac{\partial}{\partial x}(b'^2) - \frac{\partial}{\partial x}\left(b^{*\prime}\frac{\partial}{\partial x}b^2\right)\right] - \\ - \frac{1}{2}\hat{P}^+\left[b\cdot\hat{k}(|b'|^2) - \frac{\partial}{\partial x}[b'\hat{k}(|b|^2)]\right]. \quad (3)$$

For Fourier harmonics Hamiltonian can be written as following

$$H = \int \omega_k |b_k|^2 + \\ + \frac{1}{2}\int T^{k_3k_4}_{k_1k_2} b^*_{k_1} b^*_{k_2} b_{k_3} b_{k_4} \delta_{k_1+k_2-k_3-k_4} dk_1 dk_2 dk_3 dk_4.$$

Here

$$T^{kk_1}_{k_2k_3} = \frac{\theta(k)\theta(k_1)\theta(k_2)\theta(k_3)}{8\pi}\times \\ \times \{[kk_1(k+k_1) + k_2k_3(k_2+k_3)](kk_2|k-k_2| + \\ + kk_3|k-k_3| + k_1k_2|k_1-k_2| + k_1k_3|k_1-k_3)\}, \quad (4)$$

where θ – functions in (4) is the manifestation of waves moving in the same direction. Corresponding evolution equation is the following:

$$i\frac{\partial b_k}{\partial t} = \omega_k b_k + \\ + \int T^{k_2k_3}_{kk_1} b^*_{k_1} b_{k_2} b_{k_3} \delta_{k+k_1-k_2-k_3} dk_1 dk_2 dk_3. \quad (5)$$

Below we will analyze this equation from the point of view of its integrability.

2. Breathers and numerical simulation of its collisions. Breather is the localized solution of (3) of the following type:

$$b(x,t) = B(x - Vt)e^{i(k_0x-\omega_0 t)}, \quad (6)$$

where k_0 is the wavenumber of the carrier wave, V is the group velocity and ω_0 is the frequency close to ω_{k_0}. In the Fourier space breather can be written as follow:

$$b_k(t) = e^{-i(\Omega+Vk)t}\phi_k, \quad (7)$$

where Ω is close to $\omega_{k_0}/2$.

For ϕ_k the following equation is valid:

$$(\Omega + Vk - \omega_k)\phi_k = \\ = \int \tilde{T}^{k_2k_3}_{kk_1}\phi^*_{k_1}\phi_{k_2}\phi_{k_3}\delta_{k+k_1-k_2-k_3} dk_1 dk_2 dk_3. \quad (8)$$

One can treat ϕ_k as pure real function of k.

To solve Eq. (8) one can use Petviashvili iteration method (n is the number of iteration):

$$(\Omega + Vk - \omega_k)\phi^{n+1}_k = \\ = M^n \int \tilde{T}^{k_2k_3}_{kk_1}\phi^{*\ n}_{k_1}\phi^n_{k_2}\phi^n_{k_3}\delta_{k+k_1-k_2-k_3} dk_1 dk_2 dk_3.$$

Petviashvili coefficient M^n is the following:

$$M^n = \left[\frac{\langle\phi^n_k(\Omega + Vk - \omega_k)\phi^n_k\rangle}{\langle\phi^n_k\int\tilde{T}^{k_2k_3}_{kk_1}\phi^{*n}_{k_1}\phi^n_{k_2}\phi^n_{k_3}\delta_{k+k_1-k_2-k_3}dk_1dk_2dk_3\rangle}\right]^{\frac{3}{2}}.$$

In the limit of NLSE breather solution (6) is nothing but well-known NLSE soliton. Space profile of typical breather is shown in Fig. 1.

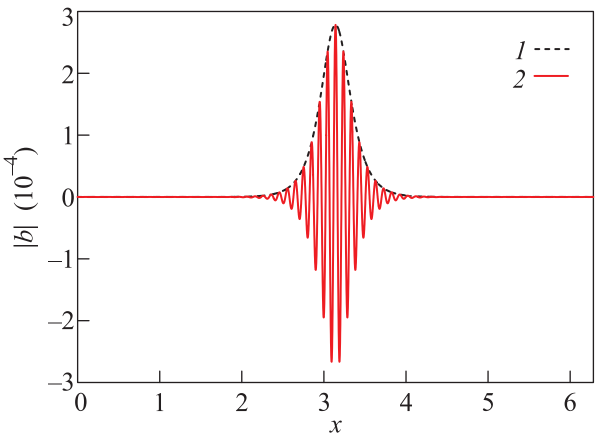

Fig. 1. Modulus of $b(x)$ and real part of $b(x)$ with $V = 1/16$ and $\Omega = 4.01$, carrier wave number appears to be ~ 64. Dashed line corresponds to modulus of $b(x)$, solid line corresponds to the real part of $b(x)$

If the Eq. (3) is integrable collisions of breathers would be pure elastic. To study breathers collision we performed the following numerical simulation.

- As initial condition we have used two breathers separated in space (distance was equal to π.) Number of space points was 4096. Length of the periodic domain was 2π.
- First breather has the following parameters: $\Omega_1 = 4.01$, $V_1 = 1/16$. Carrier wave number appears to be ~ 64.
- For the second breather – $\Omega_2 = 4.51$, $V_2 = 1/18$. Carrier wave number appears to be ~ 81.

The initial condition and state after 100 breather collisions are shown in Fig. 2. One can see radiation after collisions in Fig. that shows the zoomed profile of $|b(x)|$. During numerical simulation the total energy was conserved up to ninth digit after decimal point. Thus numerical simulation shows the collisions are not elastic.

It should be mentioned that in the papers [4, 5] similar collisions simulations seem to be elastic. Thus it was made suggestion about integrability of compact 1-D Zakharov equation. However, in these papers very few

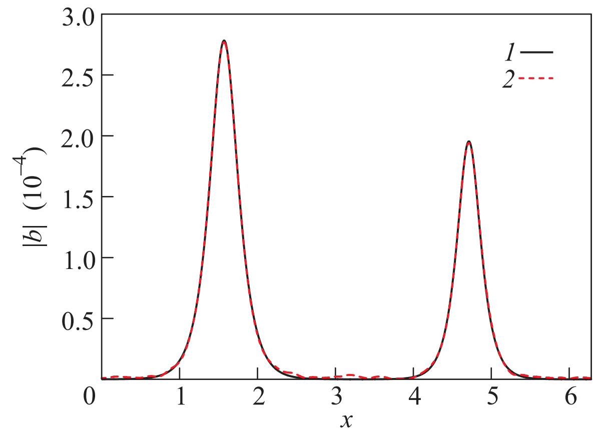

Fig. 2. Modulus of $b(x)$ for two points of time. Solid line corresponds the initial statement ($t = 0$), dashed line corresponds to the state after 100 breather collisions ($t \sim \sim 88000$)

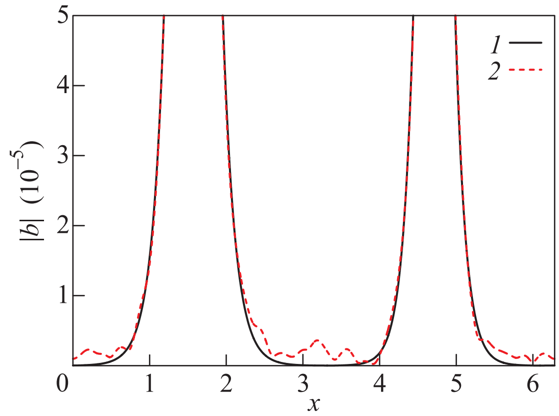

Fig. 3. Modulus of $b(x)$ for two points of time. Solid line corresponds the initial statement ($t = 0$), dashed line corresponds to the state after 100 breather collisions ($t \sim \sim 88000$)

collisions were simulated and radiation was hardly observed.

To prove nonintegrability rigorously, we analyse analytically amplitudes of six-wave interactions for this equation.

3. Scattering matrix for compact equation. For further consideration we introduce $c_k(t)$ in a following way:

$$b_k(t) = c_k(t)e^{-i\omega_k t}.$$

Then Eq. (5) can be rewritten as

$$i\dot{c}_k(t) = \int T^{kk_1}_{k_2k_3} c^*_{k_1}(t)c_{k_2}(t)c_{k_3}(t)e^{i(\omega_k+\omega_{k_1}-\omega_{k_2}-\omega_{k_3})t} \times \times \delta(k + k_1 - k_2 - k_3)dk_1dk_2dk_3 \tag{9}$$

and rewrite it in the Picard form ($\epsilon > 0$)

$$c_k(t) = c^-_k - i\lim_{\epsilon\to 0}\int\limits_{-\infty}^{t} dt \int T^{kk_1}_{k_2k_3} c^*_{k_1}(t)c_{k_2}(t)c_{k_3}(t) \times \times e^{i(\omega_k+\omega_{k_1}-\omega_{k_2}-\omega_{k_3})t+\epsilon t}\delta(k+k_1-k_2-k_3)dk_1dk_2dk_3. \tag{10}$$

This equation can be solved by iterations:

$$c_k = c^{(0)}_k + c^{(1)}_k(t) + c^{(2)}_k(t) + ..., \quad c^{(0)}_k = c_k(-\infty).$$

Following [6] we introduce a so-called formal scattering matrix for the Eq. (10) for

$$c^-_k = \lim_{t\to-\infty} c_k(t), \quad c^+_k = \lim_{t\to=+\infty} c_k(t), \quad c^+_k = S[c^-_k].$$

So far as (9) has only four-wave vortex, scattering matrix has the form:

$$S[c^-_k] = c^-_k + S_{22}[c^-_k] + S_{33}[c^-_k + \dots$$

Element S_{22} has already been calculated in [7]. In spite of it has logarithmic divergence (it is why scattering matrix is formal), it does not produce "new" wave vectors as should be in the integrable systems. Below we will calculate S_{33}. Performing two iteration of (10) one can get for $c^{(1)}_k(t)$:

$$c^{(1)}_k(t) = -i\lim_{\epsilon\to 0}\int\limits_{-\infty}^{t} dt \int T^{kk_1}_{k_2k_3} c^{*(0)}_{k_1}c^{(0)}_{k_2}c^{(0)}_{k_3} \times \times e^{i(\omega_k+\omega_{k_1}-\omega_{k_2}-\omega_{k_3})t+\epsilon t}\delta(k + k_1 - k_2 - k_3)dk_1dk_2dk_3$$

or

$$c^{(1)}_k(t) = -\int \frac{T^{kk_1}_{k_2k_3} c^{*(0)}_{k_1}c^{(0)}_{k_2}c^{(0)}_{k_3}}{\omega_k + \omega_{k_1} - \omega_{k_2} - \omega_{k_3}} \times \times e^{i(\omega_k+\omega_{k_1}-\omega_{k_2}-\omega_{k_3})t}\delta(k + k_1 - k_2 - k_3)dk_1dk_2dk_3.$$

Performing second iteration one can get:

$$c^{(2)}_{p_1}(+\infty) = -i\int\limits_{-\infty}^{+\infty} dt \int drdp_2dp_3dq_1dq_2dq_3\times \times c^{*(0)}_{p_2}c^{*(0)}_{p_3}c^{(0)}_{q_1}c^{(0)}_{q_2}c^{(0)}_{q_3}e^{i(\omega_{p_1}+\omega_{p_2}+\omega_{p_3}-\omega_{q_1}-\omega_{q_2}-\omega_{q_3})t}\times \times \left[\frac{T^{p_1r}_{q_1q_2}T^{p_2p_3}_{rq_3}\delta(r+p_1-q_1-q_2)\delta(r+q_3-p_2-p_3)}{\omega_r + \omega_{q_3} - \omega_{p_2} - \omega_{p_3}} + \right.$$

$$
\begin{aligned}
&+\frac{T^{p_1 r}_{q_3 q_2} T^{p_2 p_3}_{r q_1}\delta(r+p_1-q_2-q_3)\delta(r+q_1-p_2-p_3)}{\omega_r+\omega_{q_1}-\omega_{p_2}-\omega_{p_3}}+\\
&+\frac{T^{p_1 r}_{q_1 q_3} T^{p_2 p_3}_{r q_2}\delta(r+p_1-q_1-q_3)\delta(r+q_2-p_2-p_3)}{\omega_r+\omega_{q_2}-\omega_{p_2}-\omega_{p_3}}+\\
&+\frac{T^{p_1 p_2}_{r q_1} T^{r p_3}_{q_2 q_3}\delta(r+q_1-p_1-p_2)\delta(r+p_3-q_2-q_3)}{\omega_r+\omega_{p_3}-\omega_{q_2}-\omega_{q_3}}+\\
&+\frac{T^{p_1 p_2}_{r q_2} T^{r p_3}_{q_1 q_3}\delta(r+q_2-p_1-p_2)\delta(r+p_3-q_1-q_3)}{\omega_r+\omega_{p_3}-\omega_{q_1}-\omega_{q_3}}+\\
&+\frac{T^{p_1 p_2}_{r q_3} T^{r p_3}_{q_2 q_1}\delta(r+q_3-p_1-p_2)\delta(r+p_3-q_1-q_2)}{\omega_r+\omega_{p_3}-\omega_{q_1}-\omega_{q_2}}+\\
&+\frac{T^{p_1 p_3}_{r q_1} T^{r p_2}_{q_2 q_3}\delta(r+q_1-p_1-p_3)\delta(r+p_2-q_2-q_3)}{\omega_r+\omega_{p_2}-\omega_{q_2}-\omega_{q_3}}+\\
&+\frac{T^{p_1 p_3}_{r q_2} T^{r p_2}_{q_1 q_3}\delta(r+q_2-p_1-p_3)\delta(r+p_2-q_1-q_3)}{\omega_r+\omega_{p_2}-\omega_{q_1}-\omega_{q_3}}+\\
&+\frac{T^{p_1 p_3}_{r q_3} T^{r p_2}_{q_2 q_1}\delta(r+q_3-p_1-p_3)\delta(r+p_2-q_1-q_2)}{\omega_r+\omega_{p_2}-\omega_{q_1}-\omega_{q_2}}\Bigg]
\end{aligned}
$$

or

$$
\begin{aligned}
c^{(2)}_{p_1}(+\infty) = -2\pi i\int T^{p_1p_2p_3}_{q_1q_2q_3} c^{*(0)}_{p_2} c^{*(0)}_{p_3} c^{(0)}_{q_1} c^{(0)}_{q_2} c^{(0)}_{q_3}\times\\
\times\,\delta(\omega_{p_1}+\omega_{p_2}+\omega_{p_3}-\omega_{q_1}-\omega_{q_2}-\omega_{q_3})\times\\
\times\,\delta(p_1+p_2+p_3-q_1-q_2-q_3)dp_2dp_3dq_1dq_2dq_3.
\end{aligned}
$$

Here $T^{p_1p_2p_3}_{q_1q_2q_3}$ is equal to:

$$
\begin{aligned}
T^{p_1p_2p_3}_{q_1q_2q_3} = \Bigg[&\frac{T^{p_2p_3}_{p_2+p_3-q_3q_3} T^{q_1+q_2-p_1p_1}_{q_1q_2}}{\omega_{q_1+q_2-p_1}+\omega_{p_1}-\omega_{q_1}-\omega_{q_2}}+\\
&+\frac{T^{p_2p_3}_{p_2+p_3-q_1q_1} T^{q_2+q_3-p_1p_1}_{q_2q_3}}{\omega_{q_2+q_3-p_1}+\omega_{p_1}-\omega_{q_2}-\omega_{q_3}}+\\
&+\frac{T^{p_2p_3}_{p_2+p_3-q_2q_2} T^{q_1+q_3-p_1p_1}_{q_1q_3}}{\omega_{q_1+q_3-p_1}+\omega_{p_1}-\omega_{q_1}-\omega_{q_3}}+\\
&+\frac{T^{p_1p_2}_{p_1+p_2-q_1q_1} T^{q_2+q_3-p_3p_3}_{q_2q_3}}{\omega_{q_2+q_3-p_3}+\omega_{p_3}-\omega_{q_2}-\omega_{q_3}}+\\
&+\frac{T^{p_1p_2}_{p_1+p_2-q_2q_2} T^{q_1+q_3-p_3p_3}_{q_1q_3}}{\omega_{q_1+q_3-p_3}+\omega_{p_3}-\omega_{q_1}-\omega_{q_3}}+\\
&+\frac{T^{p_1p_2}_{p_1+p_2-q_3q_3} T^{q_1+q_2-p_3p_3}_{q_1q_2}}{\omega_{q_1+q_2-p_3}+\omega_{p_3}-\omega_{q_1}-\omega_{q_2}}+\\
&+\frac{T^{p_1p_3}_{p_1+p_3-q_1q_1} T^{q_2+q_3-p_2p_2}_{q_2q_3}}{\omega_{q_2+q_3-p_2}+\omega_{p_2}-\omega_{q_2}-\omega_{q_3}}+\\
&+\frac{T^{p_1p_3}_{p_1+p_3-q_2q_2} T^{q_1+q_3-p_2p_2}_{q_1q_3}}{\omega_{q_1+q_3-p_2}+\omega_{p_2}-\omega_{q_1}-\omega_{q_3}}+\\
&+\frac{T^{p_1p_3}_{p_1+p_3-q_3q_3} T^{q_1+q_2-p_2p_2}_{q_1q_2}}{\omega_{q_1+q_2-p_2}+\omega_{p_2}-\omega_{q_1}-\omega_{q_2}}\Bigg].
\end{aligned}
$$

Element $T^{p_1p_2p_3}_{q_1q_2q_3}$ is the kernel of six waves element of scattering matrix S_{33}. Symbolically it can be represented as 9 diagrams, see for details [8]. Now we calculate 6-waves element $T^{p_1p_2p_3}_{q_1q_2q_3}$ on the resonant manifold

$$
\begin{cases}
p_1+p_2+p_3 = q_1+q_2+q_3,\\
\omega_{p_1}+\omega_{p_2}+\omega_{p_3} = \omega_{q_1}+\omega_{q_1}+\omega_{q_3}.
\end{cases}
$$

On this manifold expression for $T^{p_1p_2p_3}_{q_1q_2q_3}$ can be simplified:

$$
\begin{aligned}
T^{p_1p_2p_3}_{q_1q_2q_3} = &\frac{1}{2\left(\omega_{p_1}\omega_{p_2}\omega_{p_3}-\omega_{q_1}\omega_{q_2}\omega_{q_3}\right)}\times\\
&\times[T^{p_2p_3}_{p_2+p_3-q_3q_3} T^{q_1+q_2-p_1p_1}_{q_1q_2}\times\\
&\times(\omega_{p_1}-\omega_{q_3})(\omega_{p_1}-\omega_{q_1}-\omega_{q_2}-\omega_{q_1+q_2-p_1})+\\
&+T^{p_2p_3}_{p_2+p_3-q_1q_1} T^{q_2+q_3-p_1p_1}_{q_2q_3}\times\\
&\times(\omega_{p_1}-\omega_{q_1})t(\omega_{p_1}-\omega_{q_2}-\omega_{q_3}-\omega_{q_2+q_3-p_1})+\\
&+T^{p_2p_3}_{p_2+p_3-q_2q_2} T^{q_1+q_3-p_1p_1}_{q_1q_3}\times\\
&\times(\omega_{p_1}-\omega_{q_2})(\omega_{p_1}-\omega_{q_1}-\omega_{q_3}-\omega_{q_1+q_3-p_1})+\\
&+T^{p_1p_2}_{p_1+p_2-q_1q_1} T^{q_2+q_3-p_3p_3}_{q_2q_3}\times\\
&\times(\omega_{p_3}-\omega_{q_1})(\omega_{p_3}-\omega_{q_2}-\omega_{q_3}-\omega_{q_2+q_3-p_3})+\\
&+T^{p_1p_2}_{p_1+p_2-q_2q_2} T^{q_1+q_3-p_3p_3}_{q_1q_3}\times\\
&\times(\omega_{p_3}-\omega_{q_2})(\omega_{p_3}-\omega_{q_1}-\omega_{q_3}-\omega_{q_1+q_3-p_3})+\\
&+T^{p_1p_2}_{p_1+p_2-q_3q_3} T^{q_1+q_2-p_3p_3}_{q_1q_2}\times\\
&\times(\omega_{p_3}-\omega_{q_3})(\omega_{p_3}-\omega_{q_1}-\omega_{q_2}-\omega_{q_1+q_2-p_3})+\\
&+T^{p_1p_3}_{p_1+p_3-q_1q_1} T^{q_2+q_3-p_2p_2}_{q_2q_3}\times\\
&\times(\omega_{p_2}-\omega_{q_1})(\omega_{p_2}-\omega_{q_2}-\omega_{q_3}-\omega_{q_2+q_3-p_2})+\\
&+T^{p_1p_3}_{p_1+p_3-q_2q_2} T^{q_1+q_3-p_2p_2}_{q_1q_3}\times\\
&\times(\omega_{p_2}-\omega_{q_2})(\omega_{p_2}-\omega_{q_1}-\omega_{q_3}-\omega_{q_1+q_3-p_2})+\\
&+T^{p_1p_3}_{p_1+p_3-q_3q_3} T^{q_1+q_2-p_2p_2}_{q_1q_2}\times\\
&\times(\omega_{p_2}-\omega_{q_3})(\omega_{p_2}-\omega_{q_1}-\omega_{q_2}-\omega_{q_1+q_2-p_2})].
\end{aligned}
$$

Manifold can be parametrized using three parameters λ, α, and β as following

$$
\begin{cases}
p_1 = \dfrac{\omega^2_{p_1}}{g} = \left(1+\lambda\dfrac{1+\alpha}{1+\alpha+\alpha^2}\right)^2,\\
p_2 = \dfrac{\omega^2_{p_2}}{g} = \left[1+\lambda\dfrac{\alpha(1+\alpha)}{1+\alpha+\alpha^2}\right]^2,\\
p_3 = \dfrac{\omega^2_{p_3}}{g} = \left(1-\lambda\dfrac{\alpha}{1+\alpha+\alpha^2}\right)^2,\\
q_1 = \dfrac{\omega^2_{q_1}}{g} = \left(1+\lambda\dfrac{1+\beta}{1+\beta+\beta^2}\right)^2,\\
q_2 = \dfrac{\omega^2_{q_2}}{g} = \left[1+\lambda\dfrac{\beta(1+\beta)}{1+\beta+\beta^2}\right]^2,\\
q_3 = \dfrac{\omega^2_{q_3}}{g} = \left(1-\lambda\dfrac{\beta}{1+\beta+\beta^2}\right)^2.
\end{cases}
$$

Plugging ω_i, p_i, and q_i for different values of λ, α, and β one can check that

$$
T^{p_1p_2p_3}_{q_1q_2q_3} \neq 0.
$$

This is the proof of nonintegrability of compact 1-D Zakharov equation.

Note, that if consider above theory for Nonlinear Schrodinger Equation for which

$$T^{k_1 k_2}_{k_3 k_4} = 1, \qquad \omega_k = k^2$$

simple calculations end up with $T^{p_1 p_2 p_3}_{q_1 q_2 q_3} = 0$, as it must be for integrable system.

4. Conclusion. Compact 1-D Zakharov equation (5), or equivalent system with Hamiltonian (1) is non-integrable system. We have proved it both numerically and analytically. However the question about integrability of fully nonlinear system (1) is still unclear. Exact equation has his own six wave term which could make total six wave coefficient changed.

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