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Alloy-disorder scattering of the interacting electron gas in quantum wells and heterostructures of $Al_xGa_{1-x}As$

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The question whether alloy disorder is screened or unscreened is of fundamental importance. Therefore, we calculate the mobility of the interacting two-dimensional electron gas as realized in $Al_xGa_{1-x}As$ quantum wells and heterostructures in the presence of alloy-disorder scattering. For the screening we use the random-phase approximation and we include many-body effects due to exchange and correlation. We propose to determine the alloy disorder potential V_{AD} from mobility measurements. If we use $V_{AD} = 1.04\,eV$ we can explain recent experimental results obtained for quantum wells and heterostructures with ultrahigh mobility. From the anomalous linear temperature dependence of the mobility measured in heterostructures we conclude that the alloy disorder is screened. More experiments are needed to confirm the screening of the alloy disorder and we propose some measurements.

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Disorder and interaction effects (screening) are fundamental issues in the two-dimensional electron gas (2DEG) [1]. Alloy disorder (AD) scattering is an important scattering mechanism in some semiconductors and has been discussed for the 2DEG since a long time. Sometimes the alloy disorder was considered to be screened [2–4], sometimes screening effects were neglected and it was argued that AD as a short-range potential is unscreened [5–7]. In this paper we argue, using recent experimental results [8, 9], that AD is screened. The fractional quantum Hall effect in the 2DEG is due to Coulomb interaction effects and depends on disorder, see the discussion in Ref. [9]. Therefore, one may wonder why AD should be unscreened in the 2DEG, as supposed in Refs. [8, 9]. In a recent atomic approach in connection with the transport scattering time for AD in three-dimensional $Si_{1-x}Ge_x$ an unscreened electron gas was considered and the effects due to Coulomb interaction were neglected from the beginning [10]. All these arguments show that the screening issue for AD is an open issue. Is AD screened or unscreened, that is the question. This is certainly an important topic for all alloy systems in two and three dimensions.

Recently, AD scattering was studied by mobility measurements at low temperatures using ultra-high mobility samples made by $Al_{0.33}Ga_{0.67}As/Al_xGa_{1-x}As$ heterostructures (HS's) (mobility $\mu \leq 3.7 \cdot 10^6\,cm^2/Vs$ at electron density $N \approx 1.2 \cdot 10^{11}\,cm^{-2}$) [8] and $Al_{0.24}Ga_{0.76}As/Al_xGa_{1-x}As/Al_{0.24}Ga_{0.7}6As$ quantum wells (QW's) ($\mu \leq 1.6 \cdot 10^7\,cm^2/Vs$ at $N \approx 2.8 \cdot 10^{11}\,cm^{-2}$) [9]. In these structures the 2DEG is confined in the region of the $Al_xGa_{1-x}As$. In changing the Al content $0 < x < 0.01$ the typical $1/x(1-x)$ behavior of AD was found using low-temperature mobility measurements. Such samples are very useful to get information about AD. The HS samples have been used in studies of the quantum Hall effect [11]. The AD scattering in HS's and QW's was compared in Ref. [9] using the unscreened model, without taking into account that the form factor for the AD in QW's and HS's is quite different. We will find that for QW's the mobility is independent of the electron density for the unscreened 2DEG, but strongly dependent on the electron density for the screened 2DEG. Unfortunately, in the experiments the electron density was not modified. Therefore, some of our results are predictions to be tested in the future.

Let us stress that earlier calculations were made for AD in the barrier region, for instance for $Al_{1-x}Ga_xAs/GaAs$ HS's [2] or for $Al_xGa_{1-x}As/GaAs/Al_xGa_{1-x}As$ Qw's [3]. For the samples studied in Refs. [8, 9] the scattering in the barrier can be neglected. In the present paper we compare our theory for AD within the well for the interacting 2DEG [4] with the recent experimental results [8, 9]. We present some suggestions for future experiments in order to clarify the importance of interaction effects.

We consider a 2DEG with parabolic dispersion and an effective mass $m^* = 0.067m_e$ with m_e as the free electron mass. For the valley degeneracy we use $g_v = 1$.

The 2DEG is embedded into an insulating background with effective dielectric constant $\varepsilon_L = 12.8$. The interaction effects of the 2DEG are treated within the random-phase approximation (RPA) [12] and a finite local- field correction (LFC) [13] for many-body effects. The importance of many-body effects are described by the Wigner–Seitz parameter $r_s \equiv 1/\sqrt{\pi a^{*2}N}$, where $a^* \equiv 0.53\,\text{Å}\ \varepsilon_L m_e/m^* \approx 101\,\text{Å}$ is the effective Bohr radius. For $N = 2\cdot 10^{11}\,\text{cm}^{-2}$ the Wigner–Seitz parameter takes the value $r_s \cong 1.25$. The LFC $G(q)$ describes many-body effects (exchange and correlation) beyond the RPA. These effects can be neglected for very high electron density $r_s \ll 1$ and we apply in the present paper analytical expressions of the LFC according to the numerical results given in Ref. [14]. We study the transport properties of the 2DEG in the xy-plane in the presence of AD scattering at zero temperature. Extension effects of the electron gas perpendicular to the interface are described by an envelope wave function $\Psi(z)$. In the case of a QW of width L, with infinite barriers for $z < 0$ and $z > L$, we apply $\Psi(0 \le z \le L) \propto \sin(\pi z/L)$. For a HS, with an infinite barrier for $z < 0$ and a triangular potential well for $z > 0$, we use the Stern–Howard expression $\Psi(z > 0) \propto z\exp(-bz/2)$ with the extension parameter b [15]. We note that $b \propto (N_{\rm Depl} + 11N/32)^{1/3}$ depends on the electron density N and the depletion density $N_{\rm Depl}$. The AD random potential $\langle|U(\mathbf{q})|^2\rangle$ is characterized by the strength parameters $V_{\rm AD}$ and the lattice parameter $a = 5.65\,\text{Å}$ by [2–4]

$$\langle|U(\mathbf{q})|^2\rangle = x(1-x)\frac{a^3}{4}V_{\rm AD}^2 F_{\rm AD}. \qquad (1)$$

The AD form factor is defined as $F_{\rm AD} \equiv \int dz|\psi(z)|^4$. For a QW one gets $F_{\rm AD} = \frac{3}{2L}$ [4] and for a HS one finds easily $F_{\rm AD} = \frac{3b}{16}$ [7]. We stress that the form factor in the case of a HS is electron density and depletion density dependent. In Ref. [9] experimental results for the mobility due to AD in QW's and HS's were compared without taking into account the different form factors. Penetration effects into the barriers have been neglected in our calculation. Note that the AD random potential is proportional to $x(1-x)$ and this factor was studied in the experiments [8, 9] as the characteristic sign of AD. The transport scattering time τ_t, which defines the mobility $\mu = e\tau_t/m$, is expressed by [4]

$$\frac{\hbar}{\tau_t} = \frac{1}{2\pi\varepsilon_{\rm F}}\int_0^{2k_{\rm F}} dq \frac{q^2}{\sqrt{4k_{\rm F}^2 - q^2}}\frac{\langle|U(\mathbf{q})|^2\rangle}{\varepsilon(q)^2}, \qquad (2)$$

with $\varepsilon(q) = 1 + q_S[1 - G(q)]F_{\rm C}(q)/q$ as the dielectric function. $\varepsilon_{\rm F}$ represents the Fermi energy, $k_{\rm F}$ is the Fermi wave number, $q_S = 2g_V/a^*$ is the screening wave number, $\hbar$ is Planck's constant, and $F_{\rm C}(q)$ represents the form factor for the Coulomb interaction due to the finite width of the 2DEG. $F_{\rm C}(q)$ for QW's depends on L and can be found in Ref. [16]. For HS's $F_{\rm C}(q)$ depends on b and can be found in Ref. [15]. The limiting behaviour is $F_{\rm C}(q \to 0) = 1$. For the non-interacting 2DEG on can use Eq. (2) with $\varepsilon(q) = 1$. Note that for a QW the mobility is independent of electron density in case of a non-interacting 2DEG. In the case of a HS, due to the density dependent factor b, the mobility is density dependent even for the non-interacting 2DEG.

In Fig. 1 we show the mobility versus electron density for a QW of width $L = 300\,\text{Å}$ and $x = 0.01$ (1%)

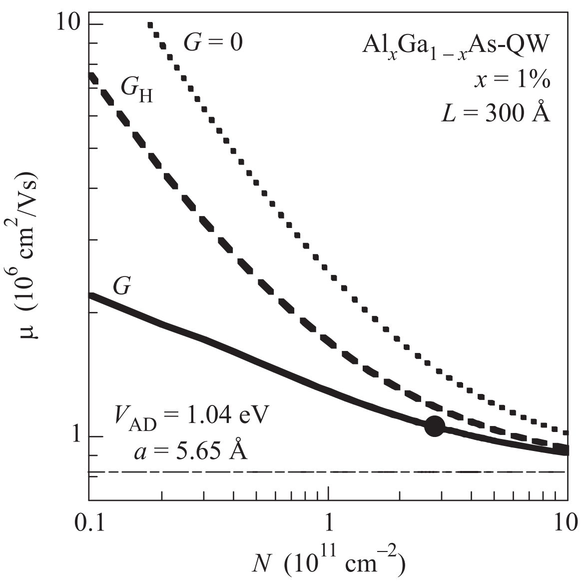

Fig. 1. Mobility μ versus electron density N of the 2DEG in the $Al_xGa_{1-x}As$ quantum well with $x = 0.01$ of width $L = 300\,\text{Å}$ for alloy disorder. For the solid line the full local-field correction is taken into account. The dashed line is for the Hubbard approximation and the dotted line represents the calculation where the local-field correction is neglected. The dashed-dotted line represents the unscreened 2DEG. The solid dot represents the sample of Ref. [9], however scattering for $x = 0$ is subtracted, see text, and scaling to $x = 0.01$ was performed

as the solid line with the complete LFC. The dotted line represents AD scattering with the LFC neglected [2] and the dashed line represents the Hubbard approximation of the LFC [4], where only exchange is taken into account and correlation is neglected. The AD potential $V_{\rm AD}$ was used as a fit parameter in order to explain the experimental result of Ref. [9] and we obtain $V_{\rm AD} = 1.04\,\text{eV}$. The value $V_{\rm AD} = 1.04\,\text{eV}$ in Fig. 1 was obtained by finding agreement between experiment and theory using screening within the full LFC for one x value where experimental results are available. Because in experiment [9] the characteristic $x(1-x)$ behavior for AD scattering was verified, there is agreement between experiment and theory for all other x values. Therefore, we only show one x value in Fig. 1 (and in Fig. 3). All other values of x can be obtained using the $x(1-x)$ behavior. If we would apply the RPA to fit the experimental result one would need a larger value for V_{AD} than $V_{\mathrm{AD}} = 1.04$ eV. Fig. 1 illustrates that it is very important to take into account the LFC, which should be tested in the future: for $N = 2 \cdot 10^{10}\,\mathrm{cm}^{-2}$ a factor 5 already exists between the RPA and the theory using the full LFC.

We stress that the mobility in a QW scales as $\mu \propto L/x(1-x)$ and Fig. 1 predicts the mobility of $\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$ QW's for different values of x and different values of L; just simple scaling should be applied. The value $V_{\mathrm{AD}} = 1.04$ eV is in good agreement with an earlier estimation in Ref. [17]. We stress that for the data point we have reduced the inverse scattering time by the $x = 0$ value $1/\tau_t = 2.5\,\mathrm{ns}^{-1}$ and we have scaled to $x = 0.01$ (1%). The dashed-dotted line in Fig. 1 is for the unscreened case and $V_{\mathrm{AD}} = 1.04$ eV. In order to explain the experimental results of Ref. [9] using an unscreened AD potential one would need $V_{\mathrm{AD}} = 0.9$ eV.

For HS's the mobility depends on the density and the depletion density. In Fig. 2 we show the inverse transport scattering time versus depletion density for $N = 1.2 \cdot 10^{11}\,\mathrm{cm}^{-2}$ and $x = 0.0085$. The dotted line represents the experimental result of Ref. [8], reduced by the $x = 0$ scattering rate $1/\tau_t = 7.0\,\mathrm{ns}^{-1}$, probably due to interface-roughness scattering. We see from Fig. 2 that we get agreement with experiment for $N_{\mathrm{Depl}} \approx 2.3 \cdot 10^{11}\,\mathrm{cm}^{-2}$. This is a reasonable value for the depletion density.

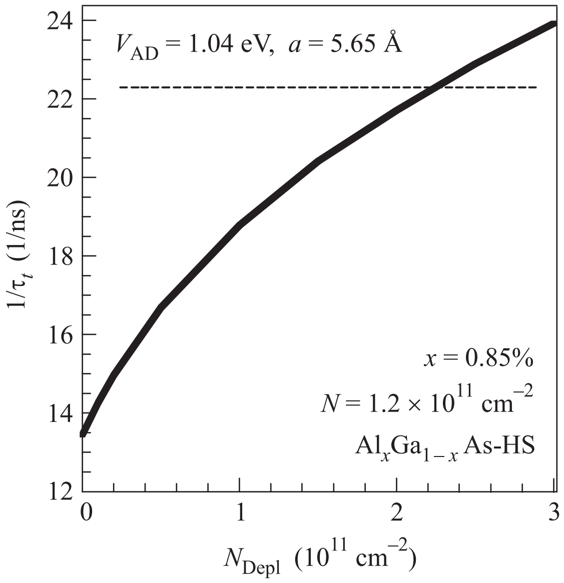

Fig. 2. Inverse transport scattering time $1/\tau_t$ versus depletion density N_{Depl} of the 2DEG in the $\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$ heterostructure with $x = 0.0085$ for alloy disorder and $N = 1.2 \cdot 10^{11}\,\mathrm{cm}^{-2}$. The dotted line represents the experimental result of Ref. [8], reduced by the $x = 0$ scattering rate

The mobility versus density for $\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$ HS's is shown in Fig. 3 for different depletion densities. The

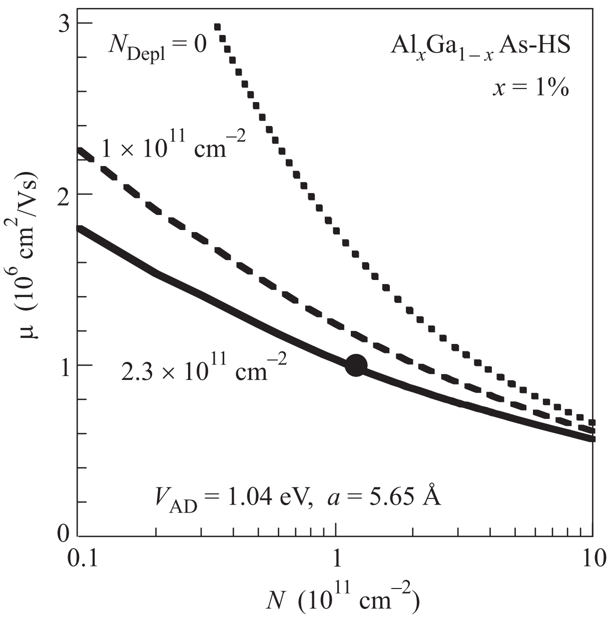

Fig. 3. Mobility μ versus electron density N of the 2DEG in the $\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$ heterostructure with $x = 0.01$ for alloy disorder for different depletion densities: $N_{\mathrm{Depl}} = 1 \cdot 10^{11}\,\mathrm{cm}^{-2}$ for the dashed line, $N_{\mathrm{Depl}} = 2.3 \cdot 10^{11}\,\mathrm{cm}^{-2}$ for the solid line, and $N_{\mathrm{Depl}} = 0$ for the dotted line. The solid dot represents the sample of Ref. [8], however the scattering rate for $x = 0$ was subtracted, and scaling to $x = 0.01$ was performed

experimental result of Ref. [8] is given as a solid dot. We stress that our theory has strong predictive power, especially for $N < 2 \cdot 10^{11}\,\mathrm{cm}^{-2}$, where the mobility strongly depends on the electron density and the depletion density. We propose that mobility measurements at low density could be used to get information about the depletion density in a given sample.

Due to anomalous screening [18] and the Kohn singularity [19, 20] in an interacting 2DEG on expects an anomalous linear temperature dependence of the transport scattering time as [19]

$$\tau_t(k_{\mathrm{B}}T \ll \varepsilon_{\mathrm{F}}) = \tau_t(0)\left[1 - C(\alpha, N)\frac{k_{\mathrm{B}}T}{\varepsilon_{\mathrm{F}}} + O(T^{3/2})\right]. \quad (3)$$

For a non-interacting 2DEG the linear temperature dependence is absent and $C(\alpha, N) \equiv 0$. The coefficient $C(\alpha, N)$ was predicted to be density dependent and also depends on the kind of disorder: $\alpha = 0$ for AD scattering and interface-roughness scattering and $\alpha = -1$ for charged-impurity scattering. Numerical results concerning $C(\alpha, N)$ have been published long ago [19]. We conclude that the presence of the linear temperature dependence of the mobility according to Eq. (3) is a strong hint for the existence of screening.

In the HS's studied in Ref. [8] a linear temperature dependence has been observed, with a rounding at very low temperatures. From the data we estimate that $C(\alpha, 1.25 \cdot 10^{11}\,\mathrm{cm}^{-2}) \approx 2.5$–$4.5$, which is in reasonable agreement with the prediction [19] for $\alpha = 0$: $C(0, 1.2 \cdot 10^{11}\,\mathrm{cm}^{-2}) \approx 3.3$. For $x = 0.85$, 0.33, 0% we find from the experimental data the values $C(\alpha, 1.25 \cdot 10^{11}\,\mathrm{cm}^{-2}) \approx 2.5$, 3.6, 4.5, respectively. In principle we expect a constant value for $C(\alpha, N)$ when one scattering mechanism (AD) is dominant, independent of x. On the other hand, it is known from the 2DEG in silicon that with increasing disorder the linear temperature dependence disappears. The fact that in Ref. [8] even for $x = 0$ a linear temperature dependence is found might have its origin in impurity scattering ($\alpha = -1$), or/and interface-roughness scattering (also $\alpha = 0$) or/and the presence of some Al even for $x = 0$, see the discussion in Ref. [9]. We believe that more systematic measurements are needed, especially for QW's. In general this linear temperature dependence is observed for a 2DEG as realized in silicon due to impurities at the Si/$\mathrm{SiO_2}$ interface or at the $\mathrm{Si_{1-x}Ge_x}$/Si interface. We stress that such a linear temperature dependence is not found when remote doping is dominant [21], which confirms that in the HS's used in Ref. [8] a short-range random potential ($\alpha = -1$ or 0) is present for $x = 0$. We claim that the observation of the linear temperature dependence in the mobility of (i) the *right sign* and (ii) the *right order of magnitude* is in favor for the presence of screening effects for AD.

Concerning another disorder of short-range, we mention that for interface-roughness (IR) scattering in thin GaAs QW's a nearly linear temperature dependence was measured [22], for the authors of that paper of unknown origin. This linear temperature dependence is of (i) the right sign and (ii) the right order of magnitude. Therefore, we believe that it is due to anomalous screening. From the experimental data we estimate $C(\alpha, N)$ for the two thinnest QW's as $C(\alpha, 5.5 \cdot 10^{10}\,\mathrm{cm}^{-2}) \approx 0.8$–$1.4$. In fact, due to the large value of $\Delta\Lambda = 600\,\text{Å}^2$ [22], the product of the two IR parameters, we believe that $k_\mathrm{F}\Lambda > 1$ applies and the value of $C(0, N)$ is strongly reduced by $\exp(-k_\mathrm{F}^2\Lambda^2) = 0.25$ compared to $k_\mathrm{F}\Lambda \ll 1$. For more details, see Ref. [16]. We note that it was argued that $k_\mathrm{F}\Lambda \ll 1$ [22], but this is impossible with $\Delta \approx 3\,\text{Å}$ for one monolayer and the density of the sample. We conclude from the temperature dependence of the scattering time that IR scattering in GaAs is screened, as in silicon [19].

The mobility data [8, 9] shown in Figs.1–3 are taken at the temperature of 0.3 K. Note that the Fermi temperature for the HS is about 50 K which results in $C(\alpha, N)\frac{k_\mathrm{B}T}{\varepsilon_\mathrm{F}} \approx 0.02$. Therefore we can directly compare experimental values with our theoretical results obtained at zero temperature, as done in Figs. 1–3.

There exists another scattering time in the 2DEG, the single-particle relaxation time τ_s, which is measured by SdH oscillations in a weak perpendicular magnetic field [23]. This time is related to the Dingle temperature T_D via $k_\mathrm{B}T_\mathrm{D} = \hbar/2\pi\tau_s$. For the transport scattering time backscattering is most important, while small angle scattering is less important. All scattering events contribute for the single-particle relaxation time. For AD scattering, and neglecting many-body effects, it was shown that

$$2/3 < \tau_t/\tau_s < 1, \tag{4}$$

where $\tau_t/\tau_s \to 1$ is reached at high density [4]. Using existing samples [8, 9] the theory can be tested in measuring the single-particle relaxation time. For instance, this was done for the 2DEG as realized in $\mathrm{Ga_xIn_{1-x}As}$ [24].

Numerical results for the ratio τ_t/τ_s versus density for the HS are shown in Fig. 4. We see that for different depletion densities nearly the same value for the ratio is found. The small deviations for different depletion densities are due to small differences in the form factor $F_\mathrm{C}(q)$ of the Coulomb interaction. For a non-interacting 2DEG one gets $\tau_t/\tau_s \equiv 1$, independent of the electron density, the dotted line in Fig. 4. We conclude that the measurement of both scattering times allow to test the presence of interaction (screening) effects. For a QW, not shown in Fig. 4, we have checked that one finds practical identical values as for HS's, because the form factor of the AD cancel out in the ratio and the Coulomb interaction only weakly depends on extension effects.

In conclusion we have shown that the mobility data in specially designed $\mathrm{Al_xGa_{1-x}As}$ QW's and HS's with ultrahigh mobility can be interpreted by an interacting 2DEG with an alloy disorder potential $V_\mathrm{AD} = 1.04\,\mathrm{eV}$. The electron density dependence of the mobility is predicted and can be tested in experiment. Mobility measurements might be used to determine the depletion density in heterostructures. The linear temperature dependence of the mobility found in experiment is in favor

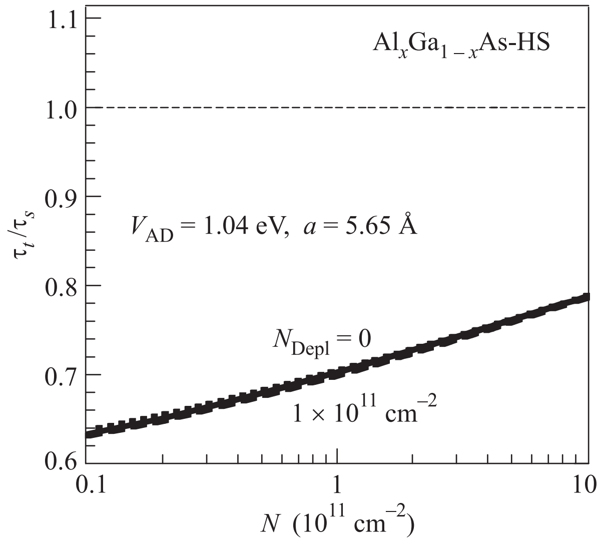

Fig. 4. Ratio τ_t/τ_s of the transport scattering time and the single-particle relaxation time versus electron density N for alloy disorder in the interacting 2DEG as realized in $Al_xGa_{1-x}As$ heterostructures for different depletion densities. The dotted line represents the non-interacting 2DEG

of interaction effects, indicating that the alloy disorder is screened. We propose to study the single-particle relaxation time in order to get more insight.

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