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CONTINUOUS TOPOLOGICAL DEFECTS ON THE ^{3}He $A-B$ INTERFACE

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The microscopic structure of topological defects on the ^3He $A-B$ interface is considered. An explicit description of certain class of such defects is presented. Nonequivalence of positive and negative topological charges is demonstrated.

Recently a topological classification of the defects on the ^3He $A-B$ interface was proposed[1–3]. Here we consider possible microscopic structure of some of the defects. Characteristic for our solution is non vanishing and everywhere continuous distribution of the order parameter. Seemingly, these properties are in a contradiction with the topological character of the defects because, as it is well known, defects with nonzero topological charges have a singular "hard" core. Inside the hard core region the order parameter no longer belongs to the vacuum manifold of a given phase and may vanish. However, we show that in some cases this singularity can be escaped via certain changing of the shape of the interface involving creation of handles[1)].

The bulks of the $A-$ and $B-$phases are described by distributions of the order parameter which has a form $A^A_{\alpha i} = \Delta_A d_\alpha(e_{1i}+ie_{2i})$ in the $A-$phase and $A^B_{\alpha i} = \Delta_B \exp(i\Phi)R_{\alpha i}$ in the $B-$phase. As a boundary condition we require that the vector $\vec{l} = \vec{e}_1 \times \vec{e}_2$ in the $A-$phase near the interface is parallel to it[4,5]. Also other constraints[2,3] should be added in order to make the boundary condition complete. They specify for each value of the order parameter $A^A_{\alpha i}$ in the $A-$phase a set of permissible values of the order parameter $A^B_{\alpha i}$ in the $B-$phase on the opposite side of the interface and vice versa. In other words, a pair $(A^A_{\alpha i}, A^B_{\alpha i})$ satisfy the boundary condition if it can be obtained from the pair $(A^{0A}_{\alpha i}, A^{0B}_{\alpha i})$ where $A^{0A}_{\alpha i} = \Delta_A \hat{x}_\alpha(\hat{x}_i - i\hat{z}_i)$, $A^{0B}_{\alpha i} = \Delta_B \delta_{\alpha i}$ by the action of some element of the symmetry group $G = U(1) \times SO(2)^L \times SO(3)^S$. Here x is normal to the

[1)]As I was told by G.Volovik the first idea of such flaring-out of singularities into the shape of the $A-B$ interface belongs to E.Thuneberg.

interface; U(1) is the gauge group; $SO(2)^L$ denotes the group of space rotations around x; $SO(3)^S$ is the group of all spin rotations.

The result of the topological analysis is as follows [2,3]: a pointlike singularity of the interface is characterized by a triplet (m_Φ, m_l, m_R) where $m_\Phi, m_l \in Z$ are winding numbers for the phase Φ of the order parameter (both in the $A-$ and $B-$phases) and for the vector $\vec{l}$ (in the $A-$phase); the index $m_R \in Z_2$ stands for disclinations in the field of $R-$matrix in the $B-$phase. Here we study two types of defects (fig.1 a,b):

a) pointlike singularities localized on the interface (boojums) for which m_l is even; $m_\Phi = m_R = 0$.

b) singular lines (vorticies and disclinations) of the $B-$phase terminating in the pointlike defect of the interface; in this case $m_\Phi + m_l$ is even.

A possible microscopic picture of the defects a), b) is presented in fig.1 c,d. The $A - B$ interface is bent to form a connected surface C separating the bulks of the $A-$ and $B-$phases. This changing of the shape of the interface can be energetically preferable if there exists a continuous distribution of the order parameter in the bulks compatible with the boundary conditions on C. Then one gets the structure which macroscopically looks like the appropriate boojum or vortex but has no singularities in the microscopic order parameter distribution.

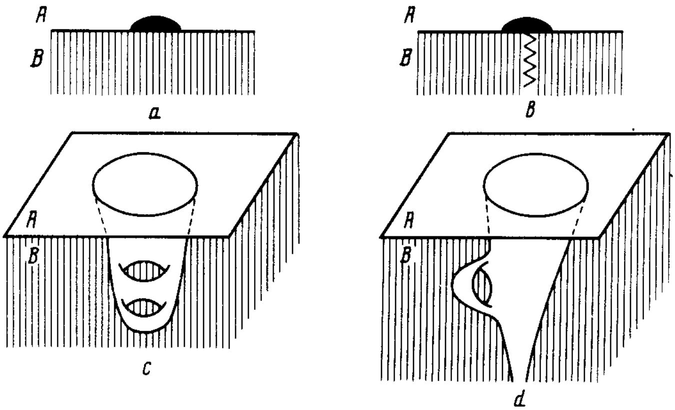

Fig. 1. Schematic illustration of (a) boojums and (b) singular lines in the $B-$phase terminating on the interface; (c) microscopic structure in the case with $g = 2$, (d) microscopic structure in the case with $g = 1$.

We consider first the case a) (boojums). The boundary surface can be compactified at infinity, the $A-$phase bulk being in the interior of the compactified surface. Then we get a compact 2-dimensional orientable manifold $\tilde{C}$ homeomorphic to a Riemannian surface of some genus g i. e. to the 2-dimensional sphere S^2 with g handles. According to the boundary condition vectors $\vec{l}$ form a tangent field on $\tilde{C}$ continuous everywhere except for "the infinitly distant" point N added to the surface C to make it compact: $\tilde{C} = C \bigcup \{N\}$. Regarding this remark one faces the problem of finding obstructions for the existence of such a field. The answer is known as Euler theorem: the sum of the indicies of all singular points of a tangent vector field equals $2-2g$ (Euler's characteristic of the Riemannian surface of genus g).

Since the index of the $\vec{l}-$field in N is equal to $2 - m_l$, one gets $2 - m_l = 2 - 2g$ or $m_l = 2g$. We conclude that for $m_l < 0$ such a structure cannot exist. We checked that for $m_l = 2$ (in this case $g = 1$ and the appropriate surface $\tilde{C}$ is a torus) there exists a continuous distribution of all other components of the order parameter including

$\vec{e}_1, \vec{e}_2, \vec{d}, R_{\alpha i}, \Phi$ and satisfying all the boundary condition. Schematically it is presented in fig.2.

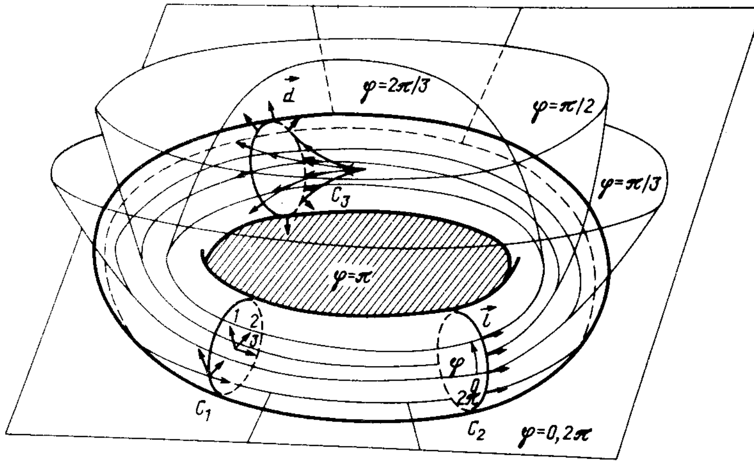

Fig. 2. Schematic illustration of the order parameter distribution for the case $m_l = 2, g = 1$. The A–phase fills the interior of the torus. Lines of the $\vec{l}$–vector coinside with the parallels of the torus (see section C_2). Triads $(\vec{e}_1, \vec{e}_2, \vec{l})$ are uniform throughout a given cross section (see e. g. triad (1, 2, 3) in the section C_1); $\vec{d}$–vectors on the surface of the torus are perpendicular to it and form a continuous funnel-like structure in the interior (section C_3). The matrix $R_{\alpha i}$ in the B–phase is $\delta_{\alpha i}$. Surfaces of a constant phase Φ of the B–phase look like closed domes leaning on the parallels of the torus. The disk bounded by the shortest parallel corresponds to $\Phi = \pi$. The horisontal plane surface corresponds to $\Phi = 0, 2\pi$.

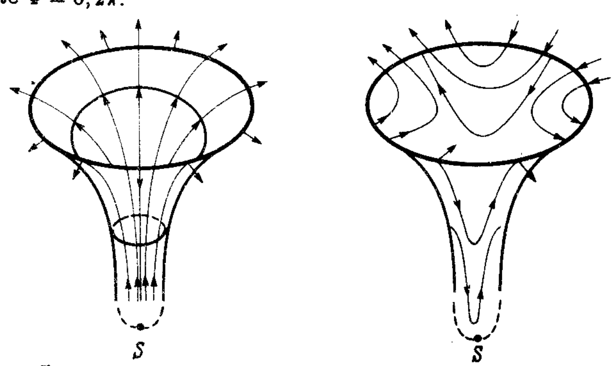

Fig. 3. Lines of $\vec{l}$–vector on the surface of the funnel in the cases $m_l = +1$ (left) and $m_l = -1$ (right). For the former we get $I_S = 1$ which allows for a uniform distribution of $\vec{l}$ near S while for the latter $I_S = -1$ and thus a texture of $\vec{l}$ near S arises inavoidably.

For larger m_l one can construct similar distributions. They contain pointlike singularities in the A–phase (hedgehogs of the $\vec{d}$–vector distribution). Their total topological charge is $1 - g$.

Let us consider now the case b) of vortex lines terminating on the interface. In order to compactify the surface C one has to add the point N and also to glue "the neck of the funnel" by a point S. Then the previous considerations of the $\vec{l}$–vector distribution applies and we find the index of the $\vec{l}$–field in S to be $I_S = m_l - 2g$. One can notice that only $I_S = 1$ allows for a space-uniform distribution of $\vec{l}$ near S. Any other I_S involves a texture with large $(\nabla\vec{l})^2$ in the vortex core. (Because for $m_l = +1$ one can take $g = 0$ and get $I_S = 1$ which is impossible for $m_l = -1$, see fig.3.) This observation implies that the vorticies with $m_l = +1$ and $m_l = -1$ are not equivalent with respect to their ability to penetrate the B−phase.

In conclusion I mention that as I was told by G.Volovik it is possible that just this inequivalence between different ends of the B−phase quantized vorticies is manifested in Helsinky NMR experiments on the phase boundary under rotation.

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