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The critical regime of elastic scattering of protons at the LHC

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It is shown that the darkness of the interaction region of protons is governed by the ratio of the slope of the diffraction cone to the total cross section. At LHC energies, it becomes completely absorptive at small impact parameters. The lower limit of the ratio is determined. That imposes some restrictions on its energy behavior. It is argued that the black disk terminology should be replaced by the black torus.

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The total cross section of colliding protons σ_t depends on their energy. Another important experimental characteristic is the slope B of the differential cross section of elastic scattering. Both of them increase with energy at high energies. Let us show that their ratio uniquely defines the darkness (opacity) at the very center of the interaction region.

The differential cross section of elastic scattering $d\sigma/dt$ is related to the scattering amplitude $f(s,t)$ in a following way

$$\frac{d\sigma}{dt} = |f(s,t)|^2. \tag{1}$$

Here $s = 4E^2$, where E is the energy in the center of mass system. The four-momentum transfer squared is

$$-t = 2p^2(1-\cos\theta) \tag{2}$$

with θ denoting the scattering angle in the center of mass system and p the momentum. The amplitude f is normalized at $t=0$ by the optical theorem such that

$$\mathrm{Im}f(s,0) = \sigma_t/\sqrt{16\pi}. \tag{3}$$

Note that the dimension of f is GeV^{-2}.

It is known from experiment that protons mostly scatter at rather small angles within the so-called diffraction cone. As a first approximation, it can be described by the exponential shape with the slope B such that

$$\frac{d\sigma}{dt} \propto e^{-B|t|}. \tag{4}$$

To define the geometry of the collision we must express these characteristics in terms of the transverse distance between the centers of the colliding protons called the impact parameter b. It is easily done by the Fourier–Bessel transform of the amplitude f written as

$$i\Gamma(s,b) = \frac{1}{2\sqrt{\pi}}\int_0^\infty d|t| f(s,t) J_0(b\sqrt{|t|}). \tag{5}$$

Using the above formulae, one can write the dimensionless Γ as

$$i\Gamma(s,b) = \frac{\sigma_t}{8\pi}\int_0^\infty d|t| e^{-B|t|/2}[i+\rho(s,t)] J_0(b\sqrt{|t|}). \tag{6}$$

Here $\rho(s,t) = \mathrm{Re}f(s,t)/\mathrm{Im}f(s,t)$ and the diffraction cone approximation (4) is inserted. Herefrom, one calculates

$$\mathrm{Re}\Gamma(s,b) = \frac{1}{Z}e^{-b^2/2B}, \tag{7}$$

where $Z = 4\pi B/\sigma_t$ is the variable used in the review paper [1]. This dependence on the impact parameter was used, in particular, in [2].

The elastic scattering amplitude must satisfy the most general principle of unitarity which states that the total probability of outcomes of any particle collision sums to 1 and reads

$$G(s,b) = 2\mathrm{Re}\Gamma(s,b) - |\Gamma(s,b)|^2. \tag{8}$$

The left-hand side called the overlap function describes the impact-parameter profile of inelastic collisions of protons. It satisfies the inequalities $0 \le G(s,b) \le 1$ and determines how absoptive is the interaction region depending on the impact parameter (with $G=1$ for full absorption).

It is known from experiment that the ratio $\rho(s,t)$ is very small at $t=0$ and, at the beginning, we neglect it and get

$$G(s,b) = \frac{2}{Z}e^{-b^2/2B} - \frac{1}{Z^2}e^{-b^2/B}. \tag{9}$$

For central collisions with $b=0$ it gives

$$G(s,b=0) = \frac{2Z-1}{Z^2}. \tag{10}$$

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The energy behavior of Z and $G(s,0)$

$\sqrt{s}$, GeV	2.70	4.11	4.74	7.62	13.8	62.5	546	1800	7000
Z	0.64	1.02	1.09	1.34	1.45	1.50	1.20	1.08	1.00
$G(s,0)$	0.68	1.00	0.993	0.94	0.904	0.89	0.97	0.995	1.00

Thus, the darkness of the central region is fully determined by the ratio Z. It becomes completely absorptive only at $Z=1$ and diminishes for other values of Z. The energy evolution of the parameter Z is shown in the Table 2 of [1]. Here, in the Table, we show the energy evolution of both Z and $G(s,0)$ for pp and $p\bar{p}$ scattering.

The function $G(s,b)$ in Eq. (9) has the maximum at $b_m^2 = -2B\ln Z$ with full absorption $G(b_m)=1$. Its position depends both on B and Z. Note, that, for $Z>1$, one gets $G(s,b)<1$ at any physical b with the largest value reached at $b=0$ because the maximum appears at non-physical values of b. The disk is semi-transparent. At $Z=1$, the maximum is positioned exactly at $b=0$, and $G(s,0)=1$. The disk becomes black in the center. At $Z<1$, the maximum shifts to positive physical impact parameters. The dip is formed at the center. It becomes deeper at smaller Z. The limiting value $Z=0.5$ is considered in more details below.

The maximum absorption in central collisions $G(s,0)=1$ is reached at the critical point $Z=1$ which is the case at $\sqrt{s}=7$ TeV considered first. Moreover, the strongly absorptive core of the interaction region grows in size as we see from expansion of Eq. (9) at small impact parameters:

$$G(s,b)=\frac{1}{Z^2}\left[2Z-1-\frac{b^2}{B}(Z-1)-\frac{b^4}{4B^2}(2-Z)\right]. \tag{11}$$

The second term vanishes at $Z=1$, and $G(b)$ develops a plateau which extends to quite large values of b about 0.4–0.5 fm. Even larger values of b are necessary for the third term to play any role at 7 TeV where $B\approx 20\,\mathrm{GeV}^{-2}$. The structure of the interaction region with a central core is also supported by direct computation [3] using the experimental data of the TOTEM collaboration [4, 5] about the differential cross section in the region of $|t|\le 2.5\,\mathrm{GeV}^2$. The results of analytical calculations and the computation practically coincide (see Fig. 1 in [6]). It was also shown in [6] that this two-component structure is well fitted by the expression with the abrupt (Heaviside-like) change of the exponential. The diffraction cone contributes mostly to $G(s,b)$. Therefore, the large-$|t|$ elastic scattering cannot serve as an effective trigger of the black core even though some models were proposed (see, e.g., [7–10]) which try to elaborate some predictions.

Inelastic exclusive processes can be effectively used for this purpose. One needs such triggers which enhance the contribution due to the central black core. Following the suggestions of [2, 11], it becomes possible [6] to study the details of the central core using the experimental data of CMS collaboration at 7 TeV about inelastic collisions with high multiplicity triggered by the jet production [12] as well as some other related data. Separating the core contribution with the help of these triggers, one comes to the important conclusion that the simple increase of the geometrical overlap area of the colliding protons does not account for properties of jet production at very high multiplicities. It looks as if the parton (gluon) density must strongly increase in central collisions and rare configurations of the partonic structure of protons are involved.

It is interesting that the positivity of $G(s,b)$ imposes some limits on the relative role of B and σ_t. Namely, it follows from (10) that

$$2Z=\frac{8\pi B}{\sigma_t}\ge 1. \tag{12}$$

This relation implies that the slope B should increase asymptotically at least as strong as the total cross section σ_t. This inequality must be fulfilled even at intermediate energies.

It is usually stated that the equality $2Z=8\pi B/\sigma_t = 1$ corresponds to the black disk limit with equal elastic and inelastic cross sections $\sigma_{el}=\sigma_{in}=0.5\sigma_t$. However, one sees that $G(s,b=0)=0$, i.e. the interaction region is completely transparent in central collisions. This paradox is resolved if we write the inelastic profile of the interaction region using Eq. (9). At $Z=0.5$ it looks like

$$G(s,b)=4[e^{-b^2/2B}-e^{-b^2/B}]. \tag{13}$$

Recalling that $B=R^2/4$, we see that one should rename the black disk as a black torus (or a black ring) with full absorption $G(s,b_m)=1$ at the impact parameter $b_m=R\sqrt{0.5\ln 2}\approx 0.59R$, complete transparency at $b=0$ and rather large half-width about $0.7R$. Thus, the evolution to values of Z smaller than 1 at higher energies (if this happens in view of energy tendency of Z shown in the Table) would imply quite special transition from the two-scale features at the LHC to torus-like configurations of the interaction region. Its implications for inelastic processes are to be guessed and studied.

In principle, the positivity of the inelastic cross section

$$\sigma_{in} = \frac{\pi B}{Z^2}(4Z-1) \geq 0 \tag{14}$$

admits the value of Z as small as 0.25 which corresponds to $\sigma_{el} = \sigma_t$ and $\sigma_{in} = 0$. However, this possibility looks unphysical and has no interpretation in terms of eikonal (blackness).

Another consequence of Eq. (10) follows from study of energy evolution of $G(s,0)$ shown in the Table. In connection with torus-like structure, it is interesting to point out the value of $Z = 0.64$ or $G(s,0) = 0.68$ at $\sqrt{s} = 2.70$ GeV and maximum 1 at $b_m^2 = 4B\ln 2$. One also notices that, in the energy interval $4\,\text{GeV} < \sqrt{s} < 8\,\text{GeV}$, the values of Z are slightly larger than 1 so that the values of $G(s,0)$ are smaller but very close to 1. It looks as if the interaction region becomes black at the center $b = 0$ but at higher energies up to ISR loses this property trying to restore it at the LHC. This fact asks for further studies in the energy interval 4 GeV $< \sqrt{s} <$ 8 GeV especially in view of proposed experiments in Protvino. The dark core must be smaller there than at LHC because of smaller values of B. Moreover, the contribution due to the real part of the amplitude is larger at these energies as well as larger $|t|$ beyond the diffraction cone can be important. One should also notice that Z becomes less than 1 at even smaller energies. As is easily shown, that does not pose any problem with the requirement $G(s,b) \leq 1$ even though, at first sight, some problems could arise because the linear in b^2 term in Eq. (11) becomes positive.

Now, we come to assumptions used in getting our conclusions. First, the real part of the amplitude f (or the ratio ρ) has been neglected. At LHC, it is small at $t = 0$ and there are theoretical arguments that it is even smaller within the diffraction cone. Thus, it looks safely to say that its contribution to $G(s,b)$ is less than 10^{-2}–10^{-3} there. Surely, these values are within the accuracy of estimates of Z from experimental data. At lower energies it can become larger (of the order of 0.1) and change the conclusions. Second, the differential cross section was approximated by its diffraction cone expression (4) and no Orear region was attributed beyond it. Its comparison with fit of TOTEM experimental data done in [6] shows that it also works quite well there with accuracy about 10^{-3}. Nevertheless, at lower energies new analysis should be done.

In this connection, we should mention that the same parameter Z in combination with $\rho(s,t)$ determines the slope of the differential cross section in the Orear region as was shown a long ago [13, 14]. When $Z = 1$, the slope depends only on ρ. That allowed to estimate its value in the Orear region at 7 TeV [15] which happened to be surprisingly large in modulus and negative. No models have yet explained this finding.

In conclusion, it is shown that the absorption at the center of the interaction region of protons is determined by a single energy-dependent parameter Z. The region of full absorption extends to quite large impact parameters if Z tends to 1. This happens at $\sqrt{s} = 7$ TeV where the two-scale structure of the interaction region of protons becomes well pronounced. That leads to special consequences both for elastic and inelastic processes. Energy behavior of Z at higher energies is especially important in view of possible evolution of the geometry of the interaction region.

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