Pis'ma v ZhETF, vol. 99, iss. 7, pp. 427 – 431

Color screening, absorption and σ_{tot}^{pp} at LHC

R. Fiore[+1)], *N. N. Nikolaev*[*], *V. R. Zoller*[×1)]

[+]*Dipartimento di Fisica, Università della Calabria*
and
Istituto Nazionale di Fisica Nucleare, Gruppo collegato di Cosenza,
I-87036 Rende, Cosenza, Italy

[*]*Landau Institute for Theoretical Physics of the RAS, 142432 Chernogolovka, Russia*

[×]*Alikhanov Institute for Theoretical and Experimental Physics, 117218 Moscow, Russia*

Submitted 4 March 2014

We show that a growth of the proton-proton total cross section with energy can be entirely attributed to the purely perturbative mechanism. The infrared regularization at rather short distances $R_c \simeq 0.3$ fm allows to extend the BFKL technique from deep inelastic to hadron-hadron scattering. With the account of the absorption corrections our results are in agreement with the LHC data on σ_{tot}^{pp}.

DOI: 10.7868/S0370274X14070017

1. Introduction. In deep inelastic scattering (DIS) of leptons on nucleons, the density of BFKL [1] gluons has been established to grow fast to smaller values of Bjorken x, $xg(x) \sim x^{-\Delta}$, where, phenomenologically, $\Delta \simeq 0.3$. In practice, the perturbative QCD base phenomenology of DIS structure functions is rather sensitive to the infrared regularization which defines a transition between the nonperturbative and perturbative domains. It is generally accepted, that in the QCD vacuum the non-perturbative fields form structures with sizes $\sim R_c$ significantly smaller than $\Lambda_{\text{QCD}}^{-1}$ and local field strengths much larger than Λ_{QCD}^2. Instantons are one of prominent candidates [2]. A direct confirmation of this picture comes from the lattice [3]. The nonperturbative fluctuations in the QCD vacuum restrict the phase space for the perturbative (real and virtual) gluons. The perturbative gluons with short propagation length, $R_c \sim 0.2{-}0.3$ Fm, as it follows from the fits to lattice data on field strength correlators [3], do not walk to large distances, $r > R_c$. This is the vacuum color screening effect.

Explicit IR regularization with such a small R_c, allows one to extend the BFKL technique from DIS to hadron-hadron scattering. Take for instance proton-proton scattering. There is always a contribution from small-size dipoles in the proton to the color dipole factorization formula $\sigma_{\text{tot}}^{pp} = \int d^2r |\Psi_p(r)|^2 \sigma(r)$. For example, in the symmetric oscillator approximation for the 3-quark proton, a probability $w_p(r < R_c)$ to find dipoles of size $r \lesssim R_c$ can be estimated as

$$w_p(r < R_c) \simeq \frac{R_c^2}{2\langle r_p^2 \rangle}. \tag{1}$$

In this approximation the proton looks as 3/2 color dipoles spanned between quark pairs and $\langle r_p^2 \rangle = 0.658\,\text{Fm}^2$ as suggested by the standard dipole form factor of the proton. The latter gives quite a substantial fraction of the proton,

$$w_p(r < R_c) \simeq 5 \cdot 10^{-2} \tag{2}$$

the interaction of which with the target nucleon proceeds in the hard regime typical of DIS. The corresponding contribution to σ_{tot}^{pp} must exhibit the same rapid rise with energy as the proton structure function. Furthermore, in the BFKL approach there is always a diffusion in the dipole size by which there is a feedback from hard region to interaction of large dipoles and vice versa. At large $r \gtrsim R_c$ a sort of the additive quark model is recovered: the quark of the dipole $\mathbf{r}$ develops its own perturbative gluonic cloud and gluonic clouds of different quarks do not overlap at $r \gg R_c$.

Below we discuss how substantial such a hard BFKL contribution to the proton-proton total cross section could be.

For high enough parton densities the phenomenon of parton fusion becomes important [4, 5]. Corresponding unitarity, e.g. absorption corrections to the BFKL evolution are described by the non-linear BK equation [6, 7]. The strength of non-linear effects depends crucially on

1)e-mail: roberto.fiore@cs.infn.it; zoller@itep.ru

the IR cutoff R_c [8–10]. Hence, one more issue we address in this communication is the role of the absorption corrections to $\sigma_{\rm tot}^{pp}$ and the non-linear dynamics of the perturbative component of pp-interactions at the LHC energies. For alternative approaches to the problem of $\sigma_{\rm tot}^{pp}$ at superhigh energies see [11, 12].

2. Vacuum color screening and CD BFKL. A distribution of perturbative gluons around the quark source is described by light cone radial wave function $\psi(\boldsymbol{\rho})$

$$\psi(\boldsymbol{\rho}) = \frac{\sqrt{C_F \alpha_S(R_i)}}{\pi} \frac{\boldsymbol{\rho}}{\rho R_c} K_1(\rho/R_c), \tag{3}$$

where the modified Bessel function, $K_1(t)$, parameterizes the exponential decay of the perturbative gluon fields by vacuum screening at large distances, $r > R_c$ [13, 14].

The effects of finite R_c are consistently incorporated by the generalized color dipole (CD) BFKL equation (hereafter CD BFKL) [13, 14]:

$$\partial_\xi \sigma(\xi, r) = \int d^2\boldsymbol{\rho}_1 \ |\psi(\boldsymbol{\rho}_1) - \psi(\boldsymbol{\rho}_2)|^2 \times \\ \times \left[\sigma_3(\xi, \mathbf{r}, \boldsymbol{\rho}_1, \boldsymbol{\rho}_2) - \sigma(\xi, r)\right], \tag{4}$$

where the 3-parton ($q\bar{q}g$-nucleon) cross section is

$$\sigma_3(\xi, \mathbf{r}, \boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \\ = \frac{C_A}{2C_F}\left[\sigma(\xi, \rho_1) + \sigma(\xi, \rho_2) - \sigma(\xi, r)\right] + \sigma(\xi, r), \tag{5}$$

where $C_A = N_c$ and $C_F = (N_c^2 - 1)/2N_c$. Denoted by $\boldsymbol{\rho}_{1,2}$ are the q–g and $\bar{q}$–g separations in the two-dimensional impact parameter plane for dipoles generated by the $\bar{q}$–q color dipole source. The one-loop QCD coupling

$$\alpha_S(R_i) = 4\pi/\beta_0 \ln(C^2/\Lambda_{\rm QCD}^2 R_i^2) \tag{6}$$

is taken at the shortest relevant distance $R_i = \min\{r, \rho_i\}$. In the numerical analysis $C = 1.5$, $\Lambda_{\rm QCD} = 0.3\,\text{GeV}$, $\beta_0 = (11N_c - 2N_f)/3$ and infrared freezing $\alpha_S(r > r_f) = \alpha_f = 0.8$ has been imposed.

The BFKL dipole cross section $\sigma(\xi, r)$, where $\xi = \ln(x_0/x)$ and r is the $q\bar{q}$-separation, sums the Leading-Log$(1/x)$ multi-gluon production cross sections within the QCD perturbation theory (PT). As a realistic boundary condition for the BFKL dynamics we take the lowest PT order $q\bar{q}$-nucleon cross section at some $x = x_0$. It is described by the Yukawa screened two-gluon exchange and is basically parameter-free one.

3. Non-perturbative component of the dipole cross section. The perturbative gluons are confined and do not propagate to large distances. Available fits [3] to the lattice QCD data suggest Yukawa screening of perturbative color fields with propagation/screening radius $R_c \approx 0.2$–$0.3\,\text{Fm}$. The value $R_c = 0.275\,\text{Fm}$ has been used since 1994 in the very successful color dipole phenomenology of small-x DIS [15–19]. Because the propagation radius is short compared to the typical range of strong interactions the dipole cross section obtained as a solution of the CD BFKL equation (4) would miss the interaction strength for large color dipoles. In [15, 16] this missing strength was modeled by the x-independent dipole cross section, so that our heterotic solution is that the perturbative, $\sigma(\xi, r)$, and non-perturbative, $\sigma_{\rm npt}(r)$, cross sections are additive,

$$\sigma_{\rm tot}(\xi, r) = \sigma(\xi, r) + \sigma_{\rm npt}(r). \tag{7}$$

The principal point about the non-perturbative component of $\sigma_{\rm tot}(\xi, r)$ is that it must not be subjected to the perturbative BFKL evolution. Thus, the arguments about the rise of $\sigma(\xi, r)$ due to the hard-to-soft diffusion do not apply to $\sigma_{\rm npt}(r)$. We reiterate, finite R_c means that gluons with the wave length $\lambda \gtrsim R_c$ are beyond the realm of perturbative QCD. Therefore, the intrusion of hard regime into soft pp-scattering is the sole source of the rise of total cross sections. Specific form of $\sigma_{\rm npt}(r)$ used in the present paper is found in [9].

4. Non-linear regime. Absorption effects. We considered above the non-unitarized running CD BFKL amplitudes too rapid a rise of which must be tamed by the unitarity absorption corrections. The simplest way to take them into account was suggested first in [6, 7]. In [9] the BK equation was rederived in terms of the $q\bar{q}$-nucleon partial-wave amplitudes (profile functions) and for the predominantly imaginary elastic dipole-nucleon amplitude $f(\xi, r, \mathbf{k}) = i\sigma(\xi, r)\exp(-Bk^2/2)$ upon integrating over the impact parameters it was reduced to the following form [9]

$$\partial_\xi \sigma(\xi, r) = \int d^2\boldsymbol{\rho}_1 \ |\psi(\boldsymbol{\rho}_1) - \psi(\boldsymbol{\rho}_2)|^2 \times \\ \times \Bigg\{\sigma(\xi, \rho_1) + \sigma(\xi, \rho_2) - \sigma(\xi, r) - \\ - \frac{\sigma(\xi, \rho_1)\sigma(\xi, \rho_2)}{4\pi(B_1 + B_2)} \exp\left[-\frac{r^2}{8(B_1 + B_2)}\right]\Bigg\}, \tag{8}$$

where $B_i = B(\xi, \rho_i)$. This form of equation with the above definition of the elastic amplitude f removes uncertainties with the radius R of the area within which interacting gluons are expected to be distributed, thus removing the frequently used in the literature parameter $S_\perp = \pi R^2$. The diffraction slope for the forward cone in the dipole-nucleon scattering is [20, 21]

$$B(\xi, r) = \frac{1}{2}\langle \mathbf{b}^2 \rangle = \frac{1}{8} r^2 + \frac{1}{3} R_N^2 + 2\alpha'_{\rm I\!P}\xi, \tag{9}$$

where $r^2/8$ is the purely geometrical term related to the elastic form factor of the color dipole of the size r, R_N represents the gluon-probed radius of the proton, the dynamical component of B is given by the last term in Eq. (9) where $\alpha'_{\mathbb{P}}$ is the Pomeron trajectory slope evaluated first in [20] (see also [21]). Here we only cite the order of magnitude estimate [21]

$$\alpha'_{\mathbb{P}} \sim \frac{3}{16\pi^2}\int d^2\mathbf{r}\ \alpha_S(r) R_c^{-2} r^2 K_1^2(r/R_c) \sim \sim \frac{3}{16\pi}\alpha_S(R_c)R_c^2, \qquad (10)$$

which clearly shows the connection between the dimensionful $\alpha'_{\mathbb{P}}$ and the non-perturbative infrared parameter R_c.

In Eq. (9) the gluon-probed radius of the proton is a phenomenological parameter to be determined from the experiment. The analysis of Ref. [22] gives $R_N^2 \approx \approx 12\,\mathrm{GeV}^{-2}$.

5. Absorption and large dipoles, $r \gtrsim R_c$. The proton size r_p is much larger than the correlation radius R_c. In high-energy scattering of large dipoles, $r \gg R_c$, a sort of the additive quark model is recovered: the quark of the dipole r develops its own perturbative gluonic cloud and the pattern of diffusion changes dramatically. Indeed, in this region the term proportional to $K_1(\rho_1/R_c)K_1(\rho_2/R_c)$ in the kernel of Eq. (4) is exponentially small, what is related to the exponential decay of the correlation function (the propagator) of perturbative gluons. Then, at large r the kernel will be dominated by the contributions from $\rho_1 \lesssim R_c \ll \rho_2 \simeq r$ and from $\rho_2 \lesssim R_c \ll \rho_1 \simeq r$. It does not depend on r and for large N_c the equation for the dipole cross section reads

$$\partial_\xi \sigma(\xi, r) = \frac{\alpha_S C_F}{\pi^2}\int d^2\rho_1 R_c^{-2} K_1^2(\rho_1/R_c) \times \times \left\{\sigma(\xi,\rho_1) + \sigma(\xi,\rho_2) - \sigma(\xi, r) - - \frac{\sigma(\xi,\rho_1)\sigma(\xi,\rho_2)}{4\pi(B_1+B_2)}\exp\left[-\frac{r^2}{8(B_1+B_2)}\right]\right\}, \qquad (11)$$

where $B_i = B(\xi, \rho_i)$.

From Ref. [9] it follows that the absorption correction to the dipole cross section is

$$\delta\sigma \sim R_c^{-2}\int\limits_{R_c^2} d\rho^2 K_1^2(\rho/R_c)\frac{\sigma(\xi,\rho)\sigma(\xi,r)}{8\pi B} \sim \sim \frac{\sigma(\xi,R_c)\sigma(\xi,r)}{8\pi B}. \qquad (12)$$

With growing ξ the dipole cross section $\sigma(\xi, r)$ increases approaching the unitarity bound, $\sigma = 8\pi B$. Untill $\sigma(\xi, R_c)/8\pi B \ll 1$

$$\frac{\delta\sigma_{\rm tot}}{\sigma_{\rm tot}} \sim \frac{\sigma(\xi, R_c)}{8\pi B}. \qquad (13)$$

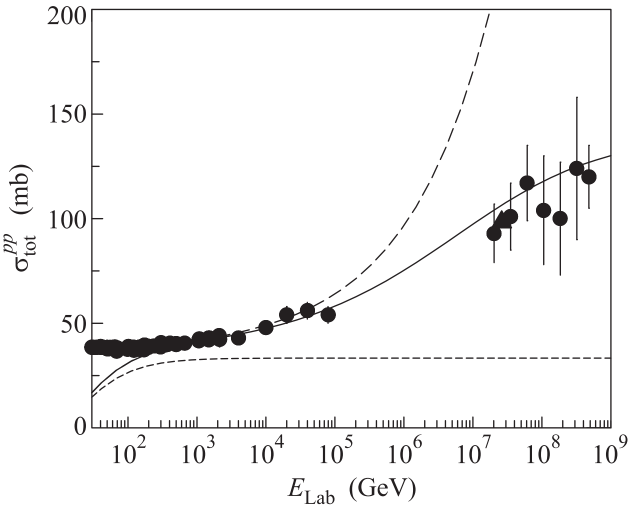
The Eq. (13) explains why the absorption correction $\delta\sigma_{\rm tot}$ dominated by the dipoles of sizes $r \sim R_c$ grows with energy faster than $\sigma_{\rm tot}$ dominated by $r \sim r_p$. The point is that the local pre-asymptotic pomeron intercept $\Delta(\xi, r)$ in the parameterization

$$\sigma(\xi, r) \propto \exp[\Delta(\xi, r)\xi] \qquad (14)$$

depends on r [23] and for $R_c \ll r_p$ [23]

$$\Delta(\xi, R_c) > \Delta(\xi, r_p). \qquad (15)$$

6. Comparison with experimental data. In [9] we found that the choice $R_c = 0.26$ Fm leads to a very good description of the DIS data on the proton structure function $F_2(x, Q^2)$ at small x. Applying the color dipole factorization to $\sigma_{\rm tot}^{pp}$ we observe (see Figure) that hard



The CD BFKL description of the experimental data [24] on $\sigma_{\rm tot}^{pp}$. Dashed line corresponds to $\sigma_{\rm tot}^{pp}$ obtained within the linear CD BFKL with color screening. The account of the absorption corrections results in $\sigma_{\rm tot}^{pp}$ shown by the solid line. Doted line - the nonperturbative contribution to $\sigma_{\rm tot}^{pp}$. The black triangle corresponds to $\sigma_{\rm tot}^{pp}$ as measured by the LHC [24]

effects in the pp scattering do exhaust completely the observed rise of $\sigma_{\rm tot}^{pp}(E_{\rm Lab})$ at moderately large $E_{\rm Lab}$. However, if the CD BFKL evolution is treated to a linear approximation, then the predicted $\sigma_{\rm tot}^{pp}(E_{\rm Lab})$ would exhibit too rapid a rise at superhigh energies. The real issue is whether there exists a mechanism to tame this excessive growth of $\sigma_{\rm tot}^{pp}(E_{\rm Lab})$ at very high energies. We addressed this issue resorting to the BK-equation [6, 7] reformulated to incorporate the effects of the finite correlation length of perturbative gluons (see Eq. (8)). Shown by the solid line in Figure is the pp total cross section evaluated with the account of the absorption effects. The agreement with data is quite reasonable. In Eq. (8) the plausible choice of the Regge parameter is $1/x = W^2/M^2$, with $W^2 \approx 2m_pE_{\rm Lab}$ and $M^2 = 1.5$ GeV^2.

The emerging hierarchy of perturbative and nonperturbative components of the pp total cross section is as follows. At moderately high energies, $E_{\rm Lab} < < (10^4{-}10^5)$ GeV, the bulk of the total cross section comes from the nonperturbative large dipoles. The purely perturbative component of the total cross section is still the subdominant one, is capable of describing a growth of the total cross section, and receives only marginal nonlinear corrections. At superhigh energies, $E_{\rm Lab} > (10^4{-}10^5)$ GeV, the perturbative component starts taking over and its unitarization becomes a central issue. In [8–10] it was demonstrated that the non-linear, i.e., unitarity effects are very sensitive to the IR regularization which defines a transition between the nonperturbative and perturbative domains. Empirically, the bulk of the total cross section at $E_{\rm Lab} \sim (10^4{-}10^5)$ GeV comes from the nonperturbative domain and our analysis suggests rather small, infrared cutoff for the perturbative contribution, $R_c = 0.26$ Fm. In an alternative approach to the purely perturbative non-linear analysis, Ref. [25] suggests a rather soft infrared regularization at distances ≈ 0.8 Fm. Normally, the QCD perturbation theory would completely break down, and the purely perturbative BFKL and BK analyses would be nonsensical, at such a large distances. The authors [25] circumvent the problem by enforcing a surprisingly small running QCD coupling which is $\alpha_S \approx 0.45$ at $r = 0.8$ Fm.

Conclusions. We explored the consequences for high energy total cross sections from the BFKL dynamics with finite correlation length of perturbative gluons, R_c. We use very restrictive perturbative two-gluon exchange as a parameter-free boundary condition for BFKL and BK evolution in the color dipole basis. With the account of the BK absorption and under plausible assertions on the color dipole structure of the proton, our parameter-free description of the total proton-proton cross section agrees reasonably well with the LHC determinations.

V.R.Z. thanks the Dipartimento di Fisica dell'Università della Calabria and the Istituto Nazionale di Fisica Nucleare – gruppo collegato di Cosenza for their warm hospitality while a part of this work was done. The work was supported in part by the Ministero Italiano dell'Istruzione, dell'Università e della Ricerca, by the RFBR grants # 11-02-00441 and 12-02-00193.

1. E. A. Kuraev, L. N. Lipatov, and V. S. Fadin, Sov. Phys. JETP **44**, 443 (1976); **45**, 199 (1977); Ya. Ya. Balitskii and L. N. Lipatov, Sov. J. Nucl. Phys. **28**, 822 (1978).
2. E. V. Shuryak, Nucl. Phys. B **203**, 93 (1982); T. Schäfer and E. V. Shuryak, Rev. Mod. Phys. **70**, 323 (1998); A. E. Dorokhov and I. O. Cherednikov, Annals Phys. **314**, 321 (2004).
3. M. D'Elia, A. Di Giacomo, and E. Meggiolaro, Phys. Lett. B **408**, 315 (1997); A. Hasenfratz and C. Nieter, Phys. Lett. B **439**, 366 (1998); E.-M. Ilgenfritz, B. V. Martemyanov, and M. Muller-Preussker, Phys. Rev. D **62**, 096004 (2000).
4. O. V. Kancheli, JETP Lett. **18**, 274 (1973) [Pisma ZhETF **18**, 465 (1973)].
5. N. N. Nikolaev and V. I. Zakharov, Phys. Lett. B **55**, 397 (1975); V. I. Zakharov and N. N. Nikolaev, Sov. J. Nucl. Phys. **21**, 227 (1975).
6. I. Balitsky, Nucl. Phys. B **463**, 99 (1996); Phys. Rev. D **60**, 014020 (1999).
7. Yu. V. Kovchegov, Phys. Rev. D **60**, 034008 (1999); Phys. Rev. D **61**, 074018 (2000).
8. R. Fiore, P. V. Sasorov, and V. R. Zoller, *BFKL, BK and the Infrared Published in AIP Conf.Proc.* **1523**, 324 (2012); arXiv:1210.7438 [hep-ph].
9. R. Fiore, P. V. Sasorov, and V. R. Zoller, JETP Lett. **96**, 687 (2013).
10. R. Fiore and V.R. Zoller, *UHE neutrinos: fusing gluons within diffraction cone*, talk at *Low x* meeting, Santiago de Compostela, Spain, June 2–7 (2011); arXiv:1111.0516 [hep-ph].
11. R. Kolevatov and K. Boreskov, arXiv:1311.7654 [hep-ph].
12. S. Ostapchenko, Phys. Rev. D **81**, 114028 (2010).
13. N. N. Nikolaev, B. G. Zakharov, and V. R. Zoller, JETP Lett. **59**, 6 (1994); Phys. Lett. B **328**, 486 (1994).
14. N. N. Nikolaev and B. G. Zakharov, J. Exp. Theor. Phys. **78**, 598 (1994); Z. Phys. C **64**, 631 (1994).
15. N. N. Nikolaev and B. G. Zakharov, Phys. Lett. B **327**, 149 (1994).
16. N. N. Nikolaev, B. G. Zakharov, and V. R. Zoller, JETP Lett. **66**, 138 (1997); N. N. Nikolaev, J. Speth, and V. R. Zoller, Phys. Lett. B **473**, 157 (2000).
17. N. N. Nikolaev, B. G. Zakharov, and V.R. Zoller, JETP Lett. **66**, 137 (1997).
18. J. Nemchik, N. N. Nikolaev, E. Predazzi, B. G. Zakharov, and V. R. Zoller, JETP **86**, 1054 (1998).
19. R. Fiore, N. N. Nikolaev, and V. R. Zoller, JETP Lett. **90**, 319 (2009); N. N. Nikolaev and V. R. Zoller, Phys. Atom. Nucl. **73**, 672 (2010).

20. N. N. Nikolaev, B. G. Zakharov, and V. R. Zoller, JETP Lett. **60**, 694 (1994).
21. N.N . Nikolaev, B. G. Zakharov, and V.R. Zoller, Phys. Lett. B **366**, 337 (1996).
22. I. P. Ivanov, N. N. Nikolaev, and A. A. Savin, Phys. Part. Nucl. **37**, 1 (2006).
23. N. N. Nikolaev, B. G. Zakharov, and V. R. Zoller, J. Exp. Theor. Phys. **78**, 806 (1994).
24. J. Beringer et al. (Particle Data Group), Phys. Rev. D **86**, 010001 (2012).
25. J. L. Albacete, N. Armesto, J. G. Milhano, P. Quiroga-Arias, and C. Salgado, Eur. Phys. J. C **71**, 1705 (2011); J. L. Albacete, N. Armesto, J. G. Milhano, and C. Salgado, Phys. Rev. D **80**, 034031 (2009).