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The conductivity of the half filled Landau level

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It is shown that the thermodynamic instability at the half filling of Ll leads to the vortex lattice formation with the electronic spectrum analogous to that of graphene with two Dirac–Fermi points on Brillouin cell boundary. This result is used for the explanation of the observed current generated by SAW (surface acoustic wave) in the heterostructure on the surface of piezoelectric GaAs. Using the existence of two Fermi points instead Fermi surface suggested in the previous theoretical works, permit the explanation of the experimental results.

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The theory of electron states with Fractional Hall Effect developed in a short time after the experimental discovery (Tsui, Stoermer, Gossard, 1982) was used mostly the projection method assuming that the states must be constructed exclusively from the states of the first Ll like the famous Laughlin function [1, 2] for 1/3 fraction. Indeed there is no explicit calculation of the possible fractions using this assumption. The phenomenological theory of "composite" fermions [3] assuming that the electrons are "dressed" by some additional magnetic flux gives some part of the observed electron densities [3] as well as a bit more refined theory of Chern–Simons field [4].

Later it was shown in the work [5] that the electron states with the partially filled Ll are thermodynamic unstable due to the formation of the quantized vortices lowering the electron free energy in the external magnetic field. In spite of triviality in the argumentation of [5] the result is quite general and we repeat it. The electron free energy in the external magnetic field [6] is $F = E - \int \mathbf{A}\mathbf{j} d^2 r$ where E is the internal electron energy, $\mathbf{A}$ is the external vector-potential, $\mathbf{j}$ is the electrical current density. Suppose we know the internal energy E at $\mathbf{j} = 0$ which can be obtained by the minimization of the average for the electron hamiltonian. In any case $\delta E/\delta\psi = 0$ where ψ is the electron wave function at zero temperature. Now we change the electron wave function. Irrespective to the details of the hamiltonian the variation of E will be of the second order in the variation of the wave function. But the change of the average current is the quantity of the first order in this variation and we can minimize the free energy creating a nonzero current. It is quite evident for the free noninteracting electrons. This phenomenon has a close analogy in the formation of vortices in a rotating vessel [7] with liquid He_3. In a large enough sample the vortices form the periodic vortex lattice.It is a kind of a phase transition.

We shall consider strong magnetic fields when the kinetic energy term is dominating compare to Coulomb interaction term. That gives the possibility to neglect Coulomb interaction as a quantity proportional to square root of magnetic field small compare to the main term proportional to magnetic field including the interaction with the vortex lattice.

The lattice periodicity in magnetic field is not enough to have a band energy structure for electrons because the translations change the hamiltonian and one must use so called ray representations of the periodic space groups. The simple energy band structure arise only for the rational magnetic flux per unit cell of the vortex lattice $l/n\Phi_0$,where $\Phi_0 = 2\pi e/c\hbar$ is the unit of the flux, (l, n) are co prime numbers. The requirement defining the electron density for the filled energy bands [5] has the form

$$Bs + K\Phi_0 = \frac{l}{n}\Phi_0, \tag{1}$$

where B is the external magnetic field, s is the unit cell area, K is the vortex circulation number. The preference has $|K| = 1$ for each individual vortex because it gives the minimal electron energy. The electron density of the fully filled vortex lattice is

$$n_e = \frac{B}{\Phi_0}\frac{l - nK}{n}. \tag{2}$$

That covered all observed fractions in FQHE. Unfortunately there is no attempts to observe the vortex lattice directly. It is a difficult experimental problem.

But some specific properties of the electrons in the vortex lattices can be observed not only by FQHE which

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require the energy gap at the boundary of the filled band. There are the specific cases when the vortex velocity is fully compensating the external magnetic field e.g.

$$Bs - 2\Phi_0 = 0. \qquad (3)$$

It is easy to see that this case corresponds to the half filled Ll density. This equation corresponds to two vortices with the unit circulation in unit cell. There are also other cases with the larger number of the vortices in unit cell but we restrict our consideration by this case in connection with the experimental results [8].

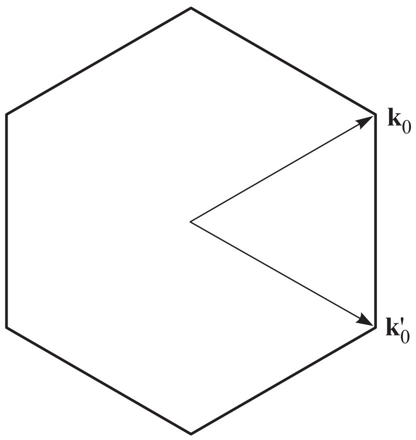

Fig. 1. The Brillouin cell in the reciprocal lattice for the half-filled Ll

On the surface of the piezoelectric GaAs was constructed a high quality heterostructure $Al_xGa_{1-x}As/GaAs$. In the volume of piezoelectric GaAs was generated SAW (surface acoustic wave) which produced the electric field acting on 2DES at strong magnetic field. The finite conductivity of 2DES gives the additional dissipation which can be registered to measure the conductivity at various electron densities. The results show the ohmic conductivity at the 1/2 and 1/4 of Ll fillings. The strongest conductivity is at 1/2 filling. The experimental results in the full extent were not explained by the theory of the composite fermions [3], or by Chern–Simons field [4], both suggesting the existence of the Fermi surface. We shall try to construct the physical picture and calculate the proper conductivity in the model of the vortex lattice. We can choose the gauge with $\mathbf{A}_{\text{eff}} \equiv \mathbf{A}_{\text{eff},y} = \mathbf{A}_{\text{ext}} + \mathbf{A}_v$ where the effective vector-potential is the sum of the external vector potential and the contribution of vortices [5]. If the total flux through the unit cell vanish the magnetic translations transform into en ordinary abelian group of translations. The main term in the hamiltonian with a strong magnetic field shall be

$$H = \int \hat{\psi}^+ \frac{\hbar^2}{2m} \left[-i\boldsymbol{\nabla} - \frac{e}{c\hbar}\mathbf{A}_{eff,y} \right]^2 \hat{\psi} d^2r. \qquad (4)$$

The Fourier transformation of the periodic function $A_{\text{eff},y}(\mathbf{r})$ defines the reciprocal lattice. We suppose the simplest hexagonal lattice for the vortices with the simplest Brillouin cell in the form of the hexagon with two nonequivalent vectors $\mathbf{k}_0$ and $\mathbf{k}'_0$ on its boundary. It means that (see e.g. [9]) the space group of 2d vortex crystal in the vicinity of $\mathbf{k}_0$ and $\mathbf{k}'_0$ inevitably has two dimensional representations and the gap between two subsequent bands in these points vanish. The space group representations is well known in their vicinity. Therefore we have not a Fermi surface as supposed in [3, 4] but two Fermi points

$$\mathbf{k}_0 = (k_0^x, k_0^y); \quad \mathbf{k}'_0 = (k_0^x, -k_0^y), \qquad (5)$$

where $k_0^x = 2\pi/3a$ and $k_0^y = 2\pi/3\sqrt{3}a$ here a is the length of the unit cell side. The electron wave functions have two dimensional representation

$$\psi^+ = (\psi_1^+, \psi_2^+)$$

as a line and

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

as a column well known for the electrons in graphene. But the length scale in the vortex lattice is of the order of the magnetic length $l_B = \sqrt{c\hbar/eB}$ large compare to the inter atomic scale in graphene at existing magnetic fields. That is important because the electric field generated by SAW in [8] may have the wave length comparable to the size of the unit cell of the vortex lattice.

The two dimensional representation of the space group near the points $\mathbf{k}_0, \mathbf{k}'_0$ gives Dirac like spectrum at the electron chemical potential equal to the energy at these points

$$H_0 = v_f \int \psi^+ \hat{p}_l \sigma_l \psi d^2r. \qquad (6)$$

Here $\sigma_l = (\sigma_x, \sigma_y)$ are 2×2 Pauli matrices, v_f is a constant defining the energy spectrum

$$\epsilon = \mp v_f p. \qquad (7)$$

Where $p_l = \pi_l - k_{0l}$ or $p'_l = \pi_l - k'_{0l}$ and π_l is the local quasimomentum.

In order to find the electron conductivity one must perform the second quantization of the electron wave function putting

$$\psi(\mathbf{r},t) = \sum_{\mathbf{p}} \exp[i(\mathbf{k}_0+\mathbf{p})\mathbf{r}] \times$$
$$\times \left[\psi_-(\mathbf{p})a_-(\mathbf{p},t) + \psi_+(\mathbf{p})a_+(\mathbf{p},t)\right], \quad (8)$$

$$\psi^+(\mathbf{r},t) = \sum_{\mathbf{p}} \exp[-i(\mathbf{k}_0+\mathbf{p})\mathbf{r}] \times$$
$$\times \left[\psi_-^+(\mathbf{p})a_-^+(\mathbf{p},t) + \psi_+^+(\mathbf{p})a_+^+(\mathbf{p},t)\right], \quad (9)$$

where two-component functions $\psi_\mp(\mathbf{p}), \psi_\mp^+(\mathbf{p})$ are normalized and orthogonal. The quantities $a_\mp(\mathbf{p},t), a_\mp^+(\mathbf{p}',t)$ give standard Fermi operators in Heisenberg representation with the undisturbed hamiltonian H_0. We suppose that all states with negative energy are filled. An analog representation is in the vicinity of $\mathbf{k}_0'$.

The current is induced by the electrical potential of the SAW in the form

$$\phi(y,t) = \left[\exp\left(i\kappa y - i\omega t\right) + \exp\left(-i\kappa y + i\omega t\right)\right]\phi_0, \quad (10)$$

where ϕ_0 is real and $\omega(\kappa) = c_0\kappa$ where c_0 is the velocity of SAW.

The correspondent term in the Hamiltonian is

$$H' = \int e\phi(y,t)\hat\rho d^2r, \quad (11)$$

where $\hat\rho(\mathbf{r}) = \psi^+(\mathbf{r})\psi(\mathbf{r})$ is the density operator.

The value of the electrical potential is supposed to be small compare to the other terms in the total Hamiltonian

$$H = \int d^2r\psi^+(\mathbf{r})v_f\sigma_l p_l\psi(\mathbf{r}) + e\int d^2r\phi(y,t)\psi^+(\mathbf{r})\psi(\mathbf{r}). \quad (12)$$

Treating the potential term as a small perturbation we can use Kubo expression for the average current [10]

$$\langle j_y(\mathbf{r},t)\rangle =$$
$$= -\frac{ie^2}{\hbar}\int_{-\infty}^{t} dt_1 \int d^2r_1 \langle[\hat j_y(\mathbf{r},t)\hat\rho(\mathbf{r}_1,t_1)]\rangle\phi(y_1,t_1), \quad (13)$$

where $\hat j_y(\mathbf{r},t)$ is the current operator in Heisenberg representation with the unperturbed Hamiltonian H_0 and $\hat\rho(\mathbf{r}_1,t_1)$ is the same for the density operator. The square brackets denote the commutator for the two inside operators. The angle brackets denote the average over the state with all negative energies filled.

The external electrical potential of SAW generate some transitions from the occupied states in the vicinity of $\mathbf{k}_0$ and $\mathbf{k}_0'$ to unoccupied states with the positive energies. The most important are the transitions from the vicinity of $\mathbf{k}_0'$ (negative energy) to the vicinity of $\mathbf{k}_0$ (positive energy) and the inverse transition from the vicinity of $\mathbf{k}_0$ to the vicinity of $\mathbf{k}_0'$, both possible at a large enough wave vector κ of the SAW.

But this model suggests the infinite life time of the created electron-hole pairs and give the divergent time integral in Kubo formula. Indeed the electron-hole pairs have a finite life time which can be taken into account by the exponentially decreasing factor $\exp(t_1/\tau)$ with some phenomenological electron-hole life time τ connected with the annihilation process on the impurities. That correspond to the results by so called "cross" technique taking into account the scattering processes [11].

We can write the full expression for the average current in this approximation using the definitions (6)–(11):

$$\langle j_y(0,0)\rangle = -\frac{ie^2v_f}{h}\int_{-\infty}^{0}\exp\frac{t_1}{\tau}dt_1 \times$$
$$\times\int d^2r_1\left\langle\left[\psi^+(0,0)\frac{p_y}{p}\psi(0,0), \psi^+(\mathbf{r}_1,t_1)\psi(\mathbf{r}_1,t_1)\right]\right\rangle \times$$
$$\times\,\phi(y_1,t_1). \quad (14)$$

Here we use the uniformity of the current in time and space and put $r = t = 0$.

It is easy to show that there are two kinds of the processes corresponding to the different terms in the commutator and generated by different terms in the real electric potential. Therefore it is possible to consider only one process given by the matrix element with the transition $\mathbf{k}_0' \to \mathbf{k}_0$ with the corresponding term in the electric potential with $\kappa \geq k_{y0} - k_{y0}'$. The terms in the commutator defining the average current are

$$n_y\sum_{all\mathbf{p}}\langle a_-^+(\mathbf{p}')\psi_-^+(\mathbf{p}')a_+(\mathbf{p})\psi_+(\mathbf{p})a_+^+(\mathbf{p}_1',t_1)\psi_+^+(\mathbf{p}_1') \times$$
$$\times\, a_-(\mathbf{p}_1,t_1)\psi_-(\mathbf{p}_1)\exp i(\mathbf{p}_1-\mathbf{p}_1')\mathbf{r}_1\rangle,$$

where n_y is the unit vector in y direction.

Putting this expression into the formula for the average current and performing the integrations over $\mathbf{r}_1$ and t_1 and using the absence of the excitations above the filled states with a negative energy one get for one electron close to $\mathbf{k}_0'$

$$\langle j_y\rangle = \frac{2e^2}{\hbar}v_f\phi_0\left[\frac{\omega(\kappa)\tau^2}{1+\omega^2\tau^2}\right]|\psi_+^+(k_{0y}'+\kappa)\psi_-(k_{0y}')|^2 S.$$

The product of the normalization factors is equal to S^{-2}, where S is the sample area and assuming $\omega\tau \ll 1$ we get for the current generated by one electron close to $\mathbf{k}_0'$

$$\langle j_y\rangle = \frac{2e^2}{\hbar}v_f c_0\kappa\tau^2\frac{\phi_0}{S}.$$

This expression give the contribution to the total average current by one electron with quasimomentum close to $\mathbf{k}_0'$ (and $\mathbf{k}_0$ as well). If one takes the electron with the other momentum $\mathbf{p}$ and the energy $\epsilon(p) \ll \hbar\omega(\kappa)$ the contribution will be practically the same. The increasing of the electron energy far from the critical points gives the additional oscillation diminishing their contribution to the current and give the invalidation of used Dirac spectrum. Therefore the total contribution to the current can be estimated as

$$\langle j_y \rangle = \frac{2e^2}{\hbar} v_f c_0 \tau^2 \nu \kappa \frac{\phi_0}{S} = \frac{2e^2}{\hbar\lambda^2} v_f c_0 \tau^2 \kappa \phi_0,$$

where $\nu = S/\lambda^2$ is the number of electrons in the "effective"vicinity of the critical points with a small energy. Here λ is the averaged wave length of the electrons in the "effective" domain near the critical points. It is impossible to have an electron-hole life time τ shorter then $1/\omega$ that gives the maximal estimate $1/\omega = \tau$. That gives the crude estimate for the current independent on τ

$$j_y = \frac{(2\pi)^3 e^2}{\hbar} \kappa \phi_0.$$

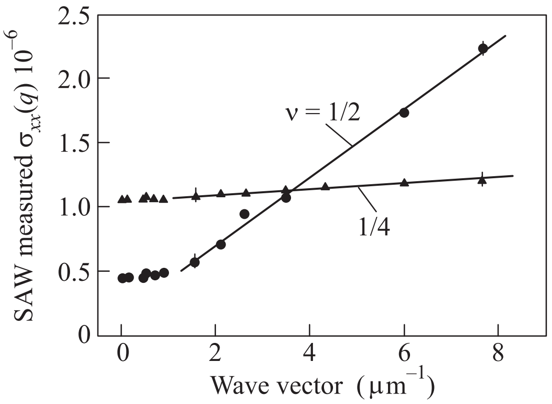

Fig. 2. Reproduced from the work [7] by the permit of R.L. Willett

The direct calculation of the current for the other realization of the electrical potential

$$\phi(\mathbf{r}, t) = \phi_0 \left[\exp(-i\omega t)\exp(-i\boldsymbol{\kappa}\mathbf{r}) + \exp(i\omega t \exp(i\boldsymbol{\kappa}\mathbf{r})\right] \tag{15}$$

instead (10) replacing $\boldsymbol{\kappa} \to -\boldsymbol{\kappa}$, $\omega \to \omega$ we obtain the same formula for the current but with the negative κ (the direction of the current is opposite).

Thus the conductivity is proportional to the wave vector of the SAW. That is confirmed by the experimental results of [8] giving the explicit linear dependence on the wave vector in a wide region beginning from some finite value of κ. It must be noted that in our consideration we have neglected by thermal effects. That is possible by using Gibbs average in Kubo formula in a more complete consideration. Thus we have an essential difference in the physical situation when we have two Fermi points instead one Fermi surface.

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